Abstract

This paper addresses two main questions. First, it examines the implications of financial frictions, embedded in a New Keynesian DSGE model with a banking sector, for the conduct of welfare-optimal monetary policy. Assuming only one monetary instrument, we analyse how financial frictions affect the gains from commitment and how different are the optimized simple rules with and without financial frictions, when monetary policy responds only to non-financial variables. Then we proceed to investigate whether there is a welfare benefit from monetary policy responding to financial variables, such as spreads, leverage or Tobin Q.

Second, this paper studies the role of a macro-prudential instrument, a subsidy for net worth financed by a tax on loans, for improving welfare outcomes. Assuming both monetary and macro-prudential instruments, we study the welfare gains from using the macro-prudential instrument and whether there are additional gains from commitment. Moreover, we examine if monetary policy and macro-prudential regulation should jointly target financial and non-financial variables, rules that may need to be implemented within one policy institution.

JEL Classification: C11, C52, E12, E32.

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1 Introduction

The financial crisis of 2007 and 2008 raised the debate regarding the role of monetary policy and traditional regulatory and prudential frameworks on promoting macroeconomic stability. Apparently, both monetary policy and financial regulation failed to prevent the financial crisis, the most severe one since the Great Depression. Some economists argue that the financial crisis was a consequence of an excessively lax monetary policy stance that contributed to the increasing of housing price inflation (Taylor (2010, 2007)). On the other hand, a large literature gives more weight to the failure of financial regulation in mitigating the risks that were building up across the system (Blanchard et al. (2010), Fund (2011)).

In the aftermath of the financial crisis, there are different implications of this debate for the role of monetary policy and macro-prudential regulation in promoting financial stability. It is clear that the achievement of financial stability is crucial for the pursuit of macroeconomic stability. However, it is not consensual what policy should target financial stability, given the close nature between financial stability and macroeconomic stability.

The implementation of a macro-prudential approach to deal with systemic risks raises some concerns about how it should interact with monetary policy, since both policies target macroeconomic stability. Another related issue regards the design of new institutional arrangements of macro-prudential policy. The literature has not offered yet a consensual view about this issue, since there are both advantages and shortcomings associated to the combination or separation of policies. Despite that, there is a convergent opinion towards an institutional mandate in which the central bank is responsible for macro-prudential policy or, at least, towards a regime that promotes cooperation between both policies. Notwithstanding, research has not shown so far whether the plausible welfare gains from cooperation are large enough to justify a combined institutional regime that would, to some extent, undertake the risk of conducting conflicting policies and jeopardizing the reputation and credibility of the central bank when a banking crisis emerge.

This paper contributes to this literature by analysing two main questions. First, it examines the implications of financial frictions, embedded in a New Keynesian DSGE model

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1 According to Galati and Moessner (2012), the term “macro-prudential” firstly appeared in the late 1970’s in unpublished documents prepared by the Cooke Committee, the precursor of the present Basel Committee on Banking Supervision, and in a document of the Bank of England.
with a banking sector, for the conduct of welfare-optimal monetary policy. Assuming only one monetary instrument, we analyse how financial frictions affect the gains from commitment and how different are the optimized simple rules with and without financial frictions, when monetary policy responds only to non-financial variables. Then we proceed to investigate whether there is a welfare benefit from monetary policy responding to financial variables, such as spreads, leverage or Tobin Q.

Second, this paper studies the role of a macro-prudential instrument, a subsidy for net worth financed by a tax on loans, for improving welfare outcomes. Assuming both monetary and macro-prudential instruments, we study the welfare gains from using the macro-prudential instrument and whether there are additional gains from commitment. Moreover, we examine if monetary policy and macro-prudential regulation should jointly target non-financial and financial regulation, or if monetary policy should be assigned to non-financial variables and macro-prudential policy to financial variables.

The paper is organized as follows. Section 2 reviews the main literature concerning the role of monetary policy and macro-prudential regulation in promoting financial stability. Section 3 presents the core New Keynesian model and the extension to embed a banking model. Section 4 sets out the results of the policy exercises for these two models. Section 5 considers macro-prudential regulation and Section 6 concludes. An Appendix assesses how our analysis would change if we carried our local approximation about a stochastic rather than a deterministic steady state.

2 Background Literature

The origins of the financial crisis of 2007/2008 remain under study as economists are still trying to understand what went wrong, especially with the conduct of macroeconomic policy. The way monetary policy strategy was being implemented before the crisis is currently under scrutiny by the economics profession. In particular, the financial crisis has revived the old debate concerning whether monetary policy should respond to asset price bubbles (lean-against-the-wind) by increasing the interest rates\(^2\) or focus solely on

\(^2\)Galí (2011) does not agree completely with the view that a tighter monetary policy (i.e. an increase in nominal interest rates) can have an impact on asset price bubbles, especially if we consider that they are of the rational type (i.e. consistent with the rational expectations hypothesis). Considering that an asset price bubble has two components, a fundamental component and a bubble component, Galí (2011) claims that raising interest rates may have the opposite effect and increase the size and volatility of the bubble.
output and inflation stability. In addition, macro-prudential regulation is being discussed as an alternative tool to deal with financial imbalances. In this context, there are questions about the design of the new institutional frameworks of banking regulation and monetary policy that require reflection. The main literature is reviewed in this section.

2.1 Monetary policy and asset price bubbles

The general doctrine that has been followed in the previous years was that monetary policy should focus solely on inflation and output stability\(^3\), because it would be ineffective in leaning against asset price bubbles. There are several reasons for this explored in the literature (see, for example, Mishkin (2011)). One of the main arguments is that bubbles are difficult to identify and even if the central bank is able to detect them, private markets will do it too, since it is assumed that the central banks have no informational advantage over the markets. Thus, once a bubble is detected, it is rather difficult to develop further since the markets are also aware of it. Second, admitting that a bubble has been detected, it is argued that interest rates may be ineffective in reducing the bubble, since private investors expect to profit from buying bubble-driven assets, even when interest rates are increasing. Third, there is no convergence in the theoretical literature about the implications for asset prices of raising interest rates. Given the uncertainty around the true impact of monetary policy in reducing asset price bubbles, even because monetary policy targets the general level of prices, leaning against the bubble would be as costly as cleaning after the bubble bursts. Lastly, there are some concerns about policy communication impact: the public might misunderstood the policy objectives and this confusion could lead to a confidence break in the central bank conduct.

The fact that monetary policy would not react, at least explicitly, to financial variables does not mean that policymakers were not aware of the importance of financial stability in promoting the stability of prices and output. The prevalent view was that output and inflation stability, per se, would be sufficient to ensure the stability of asset prices and, therefore, asset price bubbles would be less likely to happen. Under this common view, it

\(^3\)Naturally, there were economists that disagreed with this view. Borio and White (2003), for example, suggested that monetary policy should work together with prudential policy in leaning against the build up of financial imbalances, for two reasons. First, it would contain the downside risks for the macroeconomy in a context of economic downturn. Second, it would partly prevent the risk of monetary policy loosing effectiveness, especially in the situation when the policy rate may reach the zero lower bound and central banks need to implement unconventional actions, which efficacy would be less certain.
is understandable that conventional macroeconomic frameworks did not include features related to banking sectors to allow for the study of the impact of financial frictions on economic activity.

The financial crisis and the Great Recession that followed questioned the basic principles of the science of monetary policy and revived, in particular, the debate concerning that price and output stability would be sufficient to ensure financial stability. Mishkin (2011) goes further in his analysis of the origins of the financial crisis arguing that “a period of price and output stability might actually encourage credit-driven bubbles because it leads to market participants to underestimate the amount of risk in the economy”.

Thus, there is a growing support for the leaning against asset price bubbles approach, instead of cleaning after the bubble burst. Notwithstanding, there are different views about how to design and implement such a policy.

For example, Mishkin (2011) suggests that monetary policy should lean against credit-driven bubbles only (rather than responding to irrational exuberance bubbles), since “it is much easier to identify credit bubbles than it is to identify whether asset prices are deviating from fundamental values”, pointing out that the argument that is difficult to detect asset price bubbles is not valid any more in the case of credit bubbles. Following the idea of Woodford (2010) that financial intermediation played a significant role during the financial crisis, Cúrdia and Woodford (2010) suggest a Taylor Rule that also reacts contemporaneously to credit spreads. By developing a New Keynesian model with credit frictions, they conclude that a modified Taylor Rule of this kind can not only decrease the distortions originated by a financial shock, but also improve the economy reaction to several shocks. Current macroeconomic models used to implement monetary policy lacked

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4Following Mishkin (2011), pages 68-75, the nine basic principles in which science of monetary policy is anchored are: “inflation is always and everywhere a monetary phenomenon”; “price stability has important benefits”, “there is no long-run trade-off between unemployment and inflation”; “expectations play a crucial role in the macroeconomy”; “the Taylor Principle is necessary for price stability”; “the time-inconsistency problem is relevant to monetary policy”; “central bank independence improves macroeconomic performance”; “credible commitment to a nominal anchor promotes prices and output stability” and “financial frictions play an important role in the business cycle”.

5Some economists still agree with this view, but they argue that an excessively lax monetary policy was the main cause of the housing bubble (Taylor (2010, 2007)). Borio and Zhu (2008) discuss the existence of a risk-taking channel in the monetary policy transmission mechanism, that might explain how a too low policy rate may distort the risk perceptions of economic agents and lead to the creation of asset price bubbles. Given this, once the conduct of monetary policy follows a Taylor rule, in which output and inflation are the only targeted dimensions of economy activity, a macro-prudential approach to the financial system is not necessary.

financial intermediation features and also they did not include financial frictions, such as default and systemic liquidity risk (Viñals (2012)). Viñals (2012) considers that monetary policy rules should lean more than clean, by reacting to financial variables, such as credit and indebtedness, but only in the pursuit of price stability, since he believes that financial stability should not be added to the objectives list of monetary policy and it should be primarily a prudential policy goal.

However, there are dissent voices claiming that it is a mistake for monetary policy to counteract asset price bubbles. For instance, Blanchard et al. (2010) and Bean et al. (2010) state that monetary policy is apparently insufficient to deal with excessive leverage and risk-taking and asset-price booms without inducing adverse collateral effects on economic activity. Mishkin (2011) argues that monetary policy should only focus on price stability in order to comply with the Tinbergen (1939) principle, that states that the number of instruments should equal the number of policy targets. Therefore, a number of economists suggests the use of macro-prudential regulation to respond to asset price bubbles and mitigate the dissemination of systemic risks across the financial system (Bean et al. (2010); Blanchard et al. (2010); Fund (2011); Viñals (2012); Mishkin (2011)).

2.2 Macro-prudential policy

As Blanchard et al. (2010) point out, financial regulation has played a key role in the crisis, by contributing to amplify the effects that converted the U.S. housing bubble into a major world economic crisis. The financial regulation framework, by being characterised by a limited perimeter of action, encouraged banks to create off-balance-sheet entities to avoid some prudential rules and increase leverage (shadow banking). Moreover, mark-to-market rules, together with constant capital ratios, forced financial institutions to reduce their balance-sheets, aggravating fire sales, and deleveraging. The financial crisis has also shown the lack of effective mechanisms to deal with systemically important institutions, because neither macroeconomic policymakers nor prudential regulators were responsible for promoting the stability of the financial system as a whole (Fund (2011)). Therefore, it is argued that the policy toolkit lacks instruments to deal directly with systemic risks and financial stability. For this reason, macro-prudential regulation has been mentioned as an alternative or, at least, as a complementary approach to a monetary policy that
leans-against-the-wind.

There are several arguments for the use of macro-prudential policies instead of monetary policy. Blanchard et al. (2010) claim that that there is uncertainty about the effectiveness of monetary policy rules in dealing with asset prices: even assuming that raising interest rates would reduce excessive asset prices, it would be likely that it would do so at the expense of a large output gap. Viñals (2012) argues that macro-prudential policy is needed to countervail the building up of systemic risks across the financial system as a whole. The success of macro-prudential policies would prevent monetary policy of cleaning up after the eruption of financial crises. Woodford (2010) states that macro-prudential policy is important, since it helps monetary policy focusing only in output and inflation stabilization. In its absence, monetary policy should pay attention to the behaviour of some financial variables, in particular multiple interest rates and credit spreads. Mishkin (2011) argues that “the recent financial crisis provides strong support for a systemic regulator”, but he also points out that the effectiveness of macro-prudential policy in constraining credit bubbles is not clear, since, by affecting the financial institutions more directly, it is more subject than monetary policy to political pressures.

Many of these authors consider that macro-prudential policy should be used as a complement and not as a substitute of a monetary policy stance that leans-against-the-wind (Viñals (2012); Mishkin (2011); Borio (2011)). Both policies are intimately related to each other, since both are concerned about financial stability. To rely solely in macro-prudential policy to promote financial stability could be imprudent, as argued by Mishkin (2011), since the effectiveness of macro-prudential measures is yet to be proven. However, the same can be argued for monetary policy.

Given that countries are already implementing a macro-prudential oversight of their financial systems, one of the challenges that emerges is the need for coordination with monetary policy, in order to ensure the effectiveness of both policies in dealing with financial imbalances. An important concern among scholars refers to the design of the new institutional arrangements governing macro-prudential and monetary policy: should macro-prudential powers be assigned to central banks or to an independent agency instead? The advantages of a combined institutional mandate are based on the sharing

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7Mishkin (2011), page 103.
of information and expertise. According to the Fund (2011), informational gains can be explored by both policies, since macro-prudential policy may be interested on the financial stability risks associated with a given monetary policy stance and, likewise, monetary policymakers may prefer to be informed of the macro-prudential authority’s plan of action when calibrating monetary policy. In what regards the gains from sharing expertise, it is argued that the central bank role as a lender of last resort and as a monetary policymaker is extremely important for the definition of macro-prudential policy measures that aim to reduce the likelihood of banking distress.

However, problems can emerge from the combination of functions. The major argument is related to the conflict of interest that may arise if both policies were implemented together (Fund (2011), Blanchard et al. (2010); Beau et al. (2011)). The price stability is the primary goal of monetary policy and financial stability objectives must have a secondary role. In other words, changes in monetary policy, such as changes in interest rates, should not be recommended by the macro-prudential authority, because they can conflict with the principal monetary policy goal and jeopardize the monetary policy independence (Blanchard et al. (2010); Fund (2011)). Beau et al. (2011) consider that the conflict of interest outcome will depend on the type and dissemination of supply and demand imbalances across the financial system and the real economy. For example, in the case of an economy characterised by an asset bubble and by downside risks to price stability, macro-prudential policy would limit credit and liquidity growth, but this actions could have adverse effects in aggregate activity and could increase downside risks to price stability. If the prices fall as a consequence of macro-prudential policy, than that may require the intervention of the central bank, by lessening the monetary policy stance. Hence, the necessary measures to control financial stability would have a negative impact on price stability, resulting in a conflicting outcome. In turn, an expansionary monetary policy can also impact adversely on financial stability. For instance, lower interest rates can create incentives for banks to take more risk, when they operate in an environment featuring asymmetric information and limited responsibility. Another argument against the combination of policies is associated to organizational costs, since it is argued that a single agency would be more complex and, consequently, less accountable (Blanchard et al. (2010)). Moreover, reputation risks are also a serious disadvantage that has to be considered. The reputation of the central
bank is more likely to suffer, than to benefit, from bank supervision, specially in periods of banking distress (Goodhart and Schoenmaker (1995)). Garicano and Lastra (2010) also argue that “the wider is the role of the central bank, the more subject it could become to political pressures, thus threatening its independence”.

In the growing literature about this topic, a convergent stance is apparently emerging towards the defence of a single mandate, in which the central bank is the natural choice to play the macro-prudential role or, at least, towards a regime that promotes cooperation between the two policies. Garicano and Lastra (2010) argue that macro-prudential measures should be allocated to the central bank, because, in their opinion, this institutional arrangement seems to provide relevant benefits and, at the same time, it avoids the main organizational costs. In particular, they defend that the multitasking, informational economies of scope and reputation risks apply typically to microprudential policy, as well as the conflicts of interest arise from the connections of that function and monetary policy. In turn, the role of lender of last resort is a function that is more related with macro-prudential supervision.

The next subsection covers some of the recent theoretical literature that analyses the interactions between macro-prudential regulation and monetary policy.

### 2.3 Interactions between macro-prudential and monetary policy

A main topic in the design of an effective institutional mandate for macro-prudential policy is how it should interact with monetary policy, since both promote macroeconomic stability and affect real macroeconomic variables. According to Galati and Moessner (2012), the interaction between these policies depends on whether financial imbalances play a role in the monetary policy framework and they also argue that the challenge of combining both policies is similar, to some extent, to the challenge of coordinating monetary policy and fiscal policy.

A number of papers offer some preliminary insights and suggest different ways of combining the macro-prudential tool with the monetary policy instrument. Bean et al. (2010)) extend a New-Keynesian DSGE model based on Gertler and Karadi (2009) to incorporate both physical capital and a simple banking sector, in order to analyse how

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9 Brunnermeier et al. (2009); Fund (2011); Blanchard et al. (2010); Garicano and Lastra (2010)
the macro-prudential policy tools might impact on the conduct of monetary policy. Financial frictions are set on the assets side of the banks’ balance sheets: in order to avoid the misuse of funds by borrowers, the bank has an incentive to monitor them, incurring into a monitoring effort cost. This requires that the real profits from lending exceed the effort of monitoring. As a macro-prudential policy instrument, it was selected a (lump-sum) levy/subsidy on the banking sector, which is used to influence the amount of bank’s capital that is carried to the next periods. Monetary policy rules react to aggregate demand and credit supply simultaneously. First, they analyse the conduct of monetary and macro-prudential policy when a single policymaker is in charge of both functions, under three types of shocks: technology, monitoring effort and mark-up shocks. Then, they compare the outcomes with the ones resulting from a distinct arrangement, in which macro-prudential policy is delegated to a different agency. Under a single a cooperative solution and when adjustments in credit supply are necessary, the results suggest that macro-prudential policy is more effective than a monetary policy that leans-against-the wind, since it works directly on that credit friction. In the case of a non-cooperative arrangement, the results show that under technology and monitoring effort shocks a conflict of interest does not emerge. However, under a mark-up shock, a “push-me, pull-you” outcome arises, since macro-prudential policy moves to maintain bank capital, not considering its effects on inflation outcomes. In turn, monetary policy raises the interest rate more aggressively to contain inflation, not taking into account the capital gap. Thus, initially, the policy instruments are moved more abruptly in opposite directions than it is the case of a single agency, suggesting that both policies should be coordinated, since they are not merely substitutes.

Angelini et al. (2011) develop a dynamic general equilibrium model with a banking sector following Gerali et al. (2010) to analyse the interactions of the macro-prudential regulation and monetary policy, to determine what are the gains from cooperation in terms of economic stabilisation. In this model, macro-prudential policy is concerned with “excessive” lending and cyclical fluctuations of the economy, so it minimises a loss function whose elements are variances of the loans-to-output ratio and of the output. There are two alternative macro-prudential instruments: a capital requirement and a loan-to-value ratio. The interplay between macro-prudential and monetary policies is modeled in a co-
operative case, in which both authorities jointly and simultaneously set the parameters of their respective policy rules to minimise the weighted average of their objective functions, and a non-cooperative context, in which each authority minimises its loss function taking the policy rule of the other as given. The outcomes of three shocks are analysed: technology, financial (i.e. destruction of bank capital) and housing shocks. Under a technology shock, the results suggest that the gains from cooperation are small. In normal times, the contribution to macroeconomic stability of macro-prudential policy is negligible. However, in the non-cooperative case, conflicts may arise, due to the macro-prudential authority’s incentive to stabilise the loans-to-output ratio, neglecting the impact of its behaviour on the objectives of the monetary authority. In particular, macro-prudential policy becomes procyclical and monetary policy countercyclical. In this situation, it is also observed a substantial increase in the variability of the policy instruments. In the presence of financial and housing market shocks, advantages of macro-prudential policy become more relevant. In the cooperative game, the central bank deviates from strict adherence to its objectives to help macro-prudential policy achieving its goals. Hence, when the economy is hit by sector shocks, its possible to assist to a higher inflation volatility.

A formal dynamic game framework between the monetary and regulatory institutions is adopted in DePaoli and Paustian (2012) which is similar to the two-country monetary game of Levine and Currie (1987) and Currie et al. (1996). Financial frictions are introduced through the mechanism of a borrowing constraint on firms, owned and run by entrepreneurs, that is tightened as net worth and profits decrease. Cooperative and non-cooperative equilibria, with and without commitment, are compared. Under cooperation the combined authority minimizes the true quadratic loss function derived from an approximation to the utility of the household. In the absence of cooperation, different welfare loss functions or “mandates” are assigned to the two policymakers in a manner that conforms to statements about monetary policy and regulation objectives, namely that the former are only concerned with price distortions and the latter with credit distortions. They find that with this particular choice of mandates cooperation with commitment results in significant welfare gains when firms are subject to cost-push shocks, but under discretion

10They acknowledge the subjective character of such a mandate which distinguishes their set-up from two-country monetary policy games where the choice by each central bank of their own representative household as the basis for their welfare objective is not problematic.
the familiar counterproductive cooperative prevails that goes back to the seminal paper of Rogoff (1985).

The literature suggests that there are gains from coordination of policies, because when the instruments are set separately by different institutions, a “push-me, pull-you” effect is likely to arise, under specific economic situations. Thus, the “conflict of interest” argument seems to have some support in analytical frameworks.

3 The Model: A NK Model with a Banking Sector

This section sets out the benchmark NK DSGE model with investment costs, sticky prices and exogenous technology, government spending, price mark-up and preference shocks. In this model with no financial frictions the expected return on capital is equal its expected cost, the expected real interest rate. Our modelling strategy for introducing a banking sector is conceptually straightforward. We replace the latter arbitrage condition with a banking sector as in Gertler and Kiyotaki (2010) that introduces a wedge between the expected cost of loans and the return on capital.

3.1 The Core NK Model: Model I

In a cashless version of the model, household behaviour is described by

\[ \Lambda_t = \Lambda(C_t, L_t) = \frac{((C_t - \chi C_{t-1})^{(1-\phi)}L_t^{\phi})^{1-\sigma_c} - 1}{1 - \sigma_c} \]

\[ \Lambda_{C,t} = (1 - \phi)(C_t - \chi C_{t-1})^{(1-\phi)(1-\sigma_c)-1}(1 - h_t)\phi^{(1-\sigma_c)} \]

\[ R_{t+1}^{ex} = \frac{R_{n,t-1}}{\Pi_t} \]  
\[ \Lambda_{C,t} = \beta E_t [R_{t+1}^{ex} \Lambda_{C,t+1}] \]  
\[ \Lambda_{L,t} = \phi(C_t - \chi C_{t-1})^{(1-\phi)(1-\sigma_c)}L_t^{\phi(1-\sigma_c)-1} \]

\[ \Lambda_{L,t} = \frac{W_t}{P_t} \]  
\[ L_t = 1 - h_t \]

where \( C_t \) is real consumption, \( L_t \) is leisure, \( R_{n,t} \), our monetary policy instrument, is the gross nominal interest rate set in period \( t \) to pay out interest in period \( t+1 \) and \( R_{t+1}^{ex} \) is the corresponding ex post gross real interest rate adjusted for gross inflation \( \Pi_t = \frac{P_t}{P_{t-1}} \) where
$P_t$ is the retail price level. $h_t$ are hours worked and $\frac{W_t}{P_t}$ is the real wage. Single period utility $\Lambda_t$ is an increasing non-separable function of consumption relative to external habit, $\chi C_{t-1}$, and leisure $L_t$ and has a functional form consistent with a balanced growth path.

The Euler consumption equation, (2), where $\Lambda_{C,t} = \frac{\partial \Lambda_t}{\partial C_t}$ is the marginal utility of consumption and $E_t[\cdot]$ denotes rational expectations based on agents observing all current macroeconomic variables (i.e., ‘complete information’), describes the optimal consumption-savings decisions of the household. It equates the marginal utility from consuming one unit of income in period $t$ with the discounted marginal utility from consuming the gross income acquired, $R_t$, by saving the income. For later use it is convenient to write the Euler consumption equation as

$$1 = R_t E_t [D_{t,t+1}] \quad (5)$$

where $D_{t,t+1} \equiv \beta \Lambda_{C,t+1}^{1+1}$ is the real stochastic discount factor over the interval $[t,t+1]$. (3) equates the real wage with the marginal rate of substitution between consumption and leisure.

Firm behaviour is given by

$$Y_t^W = F(A_t, h_t, K_t) = (A_t h_t)^\alpha K_t^{1-\alpha} \quad (6)$$

$$Y_t = (1-c)Y_t^W \quad (7)$$

$$\frac{P_t^W}{P_t} F_{h,t} = \frac{P_t^W}{P_t} \frac{\alpha Y_t^W}{h_t} = \frac{W_t}{P_t} \quad (8)$$

$$K_t = (1-\delta)K_{t-1} + (1-S(X_t))I_t \quad (9)$$

Equation (6) is a Cobb-Douglas production function for the wholesale sector that is converted into differentiated goods in (7) at a cost $cY_t^W$. Equation (8), where $F_{h,t} \equiv \frac{\partial F_t}{\partial h_t}$, equates the marginal product of labour with the real wage. $P_t$ and $P_t^W$ are the aggregate price indices in the retail and wholesale sectors respectively. Capital accumulation is given by (9). Note here $K_t$ is end-of-period $t$ capital stock.

To determine investment, it is convenient to introduce capital producing firms that at time $t$ convert $I_t$ of output into $(1-S(X_t))I_t$ of new capital sold at a real price $Q_t$. They
then maximize with respect to \{I_t\} expected discounted profits

$$E_t \sum_{k=0}^{\infty} D_{t,t+k} \left[ Q_{t+k}(1 - S(I_{t+k}/I_{t+k-1}))I_{t+k} - I_{t+k} \right]$$

where \(D_{t,t+k} = \beta^k \left( \frac{\Lambda_C;_{t+1}}{\Lambda_C;_t} \right)\) is the real stochastic discount rate over the interval \([t, t+k]\).

Defining \(X_t = \frac{I_t}{I_{t-1}}\) results in the first-order condition

$$Q_t(1 - S(X_t) - X_t S'(X_t)) + E_t \left[ D_{t,t+1} Q_{t+1} S'(X_{t+1})X_{t+1}^2 \right] = 1$$

Demand for capital by firms must satisfy

$$E_t[R_{t+1}] = E_t[R_{k,t+1}]$$

where the return on capital is given by

$$R_{k,t} = \frac{Z_t + (1 - \delta)Q_t}{Q_{t-1}}$$

$$Z_t = (1 - \alpha)\frac{P_t^W Y_t^W}{P_t K_{t-1}}$$

In (10) the right-hand-side is the expected gross return to holding a unit of capital in from \(t\) to \(t+1\). The left-hand-side is the expected gross return from holding bonds, the opportunity cost of capital. We complete this set-up with the functional form

$$S(X) = \phi_X(X_t - (1 + g))^2$$

where \(g\) is the balanced growth rate. Note that along a balanced growth path \(X_t = 1 + g\) and investment costs disappear. This is a convenient property because then the steady state is unchanged from introducing investment costs.

There are two ways of introducing sticky prices used in NK DSGE models. The first is through Calvo contracts, but here we adopt the easier way is through Rotemberg contracts. The retail sector uses a homogeneous wholesale good to produce a basket of differentiated
goods for consumption

\[ C_t = \left( \int_0^1 C_t(m)^{(\zeta_t-1)/\zeta_t} dm \right)^{\zeta_t/(\zeta_t-1)} \]  

(14)

where \( \zeta_t \) is the elasticity of substitution. For each differentiated good \( m \), the consumer chooses \( C_t(m) \) at a price \( P_t(m) \) to maximize (14) given total expenditure \( \int_0^1 P_t(m)C_t(m)dm \).

This results in a set of consumption demand equations for each differentiated good \( m \) with price \( P_t(m) \) of the form

\[ C_t(m) = \left( \frac{P_t(m)}{P_t} \right)^{-\zeta_t} C_t \]  

(15)

where \( P_t = \left[ \int_0^1 P_t(m)^{1-\zeta_t} dm \right]^{1/(1-\zeta_t)} \). \( P_t \) is the aggregate price index. \( C_t \) and \( P_t \) are Dixit-Stigliz aggregates. Demand for investment and government services takes the same form so in aggregate

\[ Y_t(m) = \left( \frac{P_t(m)}{P_t} \right)^{-\zeta_t} Y_t \]  

(16)

Retail firms face quadratic price adjustment costs \( \xi_t \left( \frac{P_{t+1}}{P_{t-1}} - 1 \right)^2 Y_t \), as in Rotemberg (1982) – where parameter \( \xi \) measures the degree of price stickiness. For each producer \( m \), given its real marginal cost common to all firms \( MC_t(m) = MC_t \), the objective is at time \( t \) to choose \( \{P_t(m)\} \) to maximize discounted profits

\[ E_t \sum_{k=0}^{\infty} D_{t,t+k} \left[ \frac{P_{t+1}(m)Y_{t+k}(m)}{P_{t+k}} - \frac{P_{t+k}Y_{t+k}(m)}{P_{t+k}} MC_{t+k} - \frac{\xi_t}{2} \left( \frac{P_{t+k}(m)}{P_{t+k-1}(m)} - 1 \right)^2 Y_{t+k} \right] \]  

subject to (16). The solution to this is

\[ 1 - \zeta_t + \zeta_t MC_t - \xi_t(\Pi_t - 1)\Pi_t + \xi E_t \left[ D_{t,t+1}(\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] = 0 \]  

(18)

The resource constraint is

\[ Y_t = C_t + G_t + I_t + \frac{1}{2} \xi_t(\Pi_t - 1)^2 Y_t \]  

(19)

and real marginal costs in the retail sector are given by

\[ MC_t = \frac{P^W_t}{P_t} \]  

(20)
and the real side of the model is completed with a balanced budget constraint with lump-sum taxes.

The nominal interest rate is given by the following Taylor-type rule\textsuperscript{11}

\[
\log \left( \frac{R_{n,t}}{R_n} \right) = \rho_r \log \left( \frac{R_{n,t-1}}{R_n} \right) + \theta_x \log \left( \frac{\Pi_t}{\Pi} \right) + \theta_{r,y} \log \left( \frac{Y_t}{Y} \right) \tag{21}
\]

Finally there are four exogenous AR1 shocks to technology, government spending, the discount factor (in probit form so that $\beta \in [0,1]$) and $\zeta_t$, the latter giving rise to a mark-up shock:

\[
\begin{align*}
\log A_t - \log \bar{A}_t &= \rho_A (\log A_{t-1} - \log \bar{A}_{t-1}) + \epsilon_{A,t} \\
\log G_t - \log \bar{G}_t &= \rho_G (\log G_{t-1} - \log \bar{G}_{t-1}) + \epsilon_{G,t} \\
\log x_t - \log x &= \rho_x (\log x_{t-1} - \log x) + \epsilon_{x,t} \\
\beta_t &= \exp(x_t) \left( 1 + \exp(x_t) \right) \\
\log \zeta_t - \log \bar{\zeta}_t &= \rho_\zeta (\log \zeta_{t-1} - \log \bar{\zeta}) + \epsilon_{\zeta,t}
\end{align*}
\]

### 3.1.1 The Steady State

The balanced growth steady state is as follows. The consumption Euler equation gives

\[
\frac{\bar{A}_{C,t+1}}{A_{C,t}} = \left( \frac{\bar{C}_{t+1}}{C_t} \right)^{(1-\varphi)(1-\sigma_c)-1} = (1 + g)^{(1-\phi)(1-\sigma_c)-1} = \beta R_n \Pi \tag{22}
\]

On the balanced-growth path (bgp) consumption, output, investment, capital stock, the real wage and government spending are growing at a common growth rate $g$ driven by exogenous labour-technical change $\bar{A}_{t+1} = (1 + g)\bar{A}_t$, but labour input $h$ is constant. It is convenient to stationarize the bgp by defining stationary variables such as $Y \equiv \frac{\bar{Y}_t}{\bar{A}_t}$\textsuperscript{12}

\textsuperscript{11}We also explore the role for a rule that includes feedback from financial variables including Tobin’s Q and, for the banking model, leverage and spread.

\textsuperscript{12}The full model can also be stationarized in the same way by dividing $Y_t$, $Y_t^W$, $C_t$, etc by $\bar{A}_t$.  

15
Then the stationarized bgp is given by

\[ Y^W = h^\alpha K^{1-\alpha} \]  
\[ \frac{\varrho C(1-\chi)}{(1-\varrho)(1-h)} = W \]  
\[ \frac{\alpha Y^W}{h} = W \]  
\[ \frac{(1-\alpha)Y^W}{K} = R_n - 1 + \delta \]  
\[ I = (\delta + g)K \]  
\[ Y = C + I + G \]

which together with (22) defines the bgp.

3.1.2 Calibration of Fundamental Parameters

To calibrate these dimensionless parameters and \( \delta \), if we have data for \( R, g, \) factor shares \( \alpha \) and \( 1 - \alpha \), \( \frac{C}{Y}, \frac{I}{Y} \) and \( h \) we can pin down \( \varrho \) and \( \varrho \) as follows. From (24) we obtain

\[ \varrho = \frac{1 - h}{1 + \left( \frac{c_y(1-c)(1-\chi)}{\alpha} - 1 \right) h} ; \quad \text{where } c_y = \frac{C}{Y} \]

so \( \varrho \) is pinned down and increases with the habit parameter \( \chi \) (see Figure 1). From (26) and (27) we have

\[ \frac{I}{Y} = \frac{(\delta + g)K}{Y} = \frac{(\delta + g)(1-\alpha)}{(1-c)(R-1+\delta)} \]

from which \( \delta \) is obtained.

Then (22) can be used to calibrate one out of the two remaining parameters \( \beta \) and \( \sigma_c \). Since there is a sizeable literature on the micro-econometric estimation of the latter risk-aversion parameter, it is usual to calibrate \( \beta \). From the price mark-up in the retail sector

\[ \mu = \frac{1}{MC} = \frac{1}{1 - \frac{1}{\zeta}} \]

Hence the parameter \( \zeta \) can be calibrated using data on the mark-up, \( \mu \), from

\[ \zeta = \frac{1}{1 - \frac{1}{\mu}} \]
For example if the mark-up is 15%, $\mu = 1.15$ and $\zeta = 7.67$.

Finally can impose a free entry condition on retail firms in this steady state which drives steady-state monopolistic profits, $[(1 - c) P - P^W] Y$ to zero.\(^{13}\) This implies that $PY = P^W Y^W$ and the costs of converting wholesale to retail goods are given by

$$c = \frac{1}{\zeta}$$

<table>
<thead>
<tr>
<th>Calibrated parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Growth Rate</td>
<td>$g$</td>
<td>0.025/4</td>
</tr>
<tr>
<td>Labour Share</td>
<td>$\alpha$</td>
<td>0.67</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Growth rate</td>
<td>$g$</td>
<td>0</td>
</tr>
<tr>
<td>Substitution elasticity of goods</td>
<td>$\zeta$</td>
<td>7</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>$c$</td>
<td>$\frac{1}{\zeta} = 0.1429$</td>
</tr>
<tr>
<td>preference parameter</td>
<td>$\varrho$</td>
<td>$1 + h - \frac{1}{\alpha(1 + h - \frac{1}{1+h-1}h)}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Implied steady state relationship</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Government expenditure-output ratio</td>
<td>$g_y$</td>
<td>0.2</td>
</tr>
<tr>
<td>Consumption-output ratio</td>
<td>$c_y$</td>
<td>0.64</td>
</tr>
<tr>
<td>Investment-output ratio</td>
<td>$i_y$</td>
<td>$1 - g_y - c_y$</td>
</tr>
</tbody>
</table>

Table 1: Calibrated Parameters

\(^{13}\)This assumes there is strategic interaction between forms in a Bertrand equilibrium that determines the number of firms. However this is ignored by the firms in the Rotemberg price setting.
3.1.3 Calibration of Shocks

Consider shocks to $\zeta$ and $\beta$. How do we set priors on the standard deviation of these shocks which cannot be observed? First for mark-up differentiate (31) to obtain

$$\frac{d\mu}{\mu} = \frac{1}{\zeta - 1} \frac{d\zeta}{\zeta} \quad (33)$$

so that a 1% shock to the mark-up is equivalent to a $(\zeta - 1)$% shock to $\zeta$.

For the preference shock in a no-growth steady state $R = \frac{1}{\beta}$ so that

$$\frac{dR_n}{R_n} = -\frac{d\beta}{\beta} \quad (34)$$

or a 1% change in the gross interest rate translates into a 1% change in $\beta$. However our shock is to a probit transformation of $\beta$ defined by

$$\beta \equiv \frac{\exp(x)}{1 + \exp(x)} \in [0, 1] \text{ for } x \in [-\infty, \infty] \quad (35)$$

Hence differentiating

$$\frac{d\beta}{dx} = \frac{\exp(x)}{(1 + \exp(x))^2} = \frac{\beta}{1 + \exp(x)} = (1 - \beta)\beta \quad (36)$$

Hence

$$\frac{d\beta}{\beta} = (1 - \beta)x \frac{dx}{x} = (1 - \beta)\log \left[ \frac{\beta}{1 - \beta} \right] \frac{dx}{x} \quad (37)$$

If $\beta = 0.99$, then a 1% change in $\beta$ is equivalent to a $1/0.046 = 21.7\%$ in $x$.

These two calculations suggest that if we believe the sds of the mark-up and the discount factor are of the order 1%, then we should set priors of around 7% for $\zeta$ and 22% for $x$. Reflecting the Bayesian estimation literature in what follows we set standard deviations for the technology, government spending mark-up and discount factor at 0.5%, 2.5%, 0.5% and 0.1% of their steady state values respectively, the latter two implying standard deviations of 3.5% and 2.2% for $\zeta$ and $x$ respectively.
3.2 The NK Model with Financial Frictions: Model II

The modelling strategy is conceptually straightforward. We replace (10) with a banking sector that introduces first a wedge between the expected ex-ante cost of loans from households, $R_{t+1}^{ex}$ and the return on capital $R_{k,t}$.

In the model, which closely follows Gertler and Kiyotaki (2010) - henceforth GK - but embeds in a NK model with sticky prices. Financial frictions affect real activity via the impact of funds available to bank but there is no friction in transferring funds between banks and nonfinancial firms. Given a certain deposit level a bank can lend frictionlessly to nonfinancial firms against their future profits. In this regard, firms offer to banks a perfect state contingent security.

The activity of the bank can be summarized in two phases. In the first one banks raises deposits and equity from the households. In the second phase banks uses the deposits to make loans to firms.

In particular, we have the following sequence of events:

1. Banks raise deposits, $d_t$ from households at a real deposit net rate $R_{t+1}$ over the interval $[t, t+1]$, the ‘time period $t$’.

2. Banks make loans to firms.

3. Loans are $s_t$ at a price $Q_t$. The asset against which the loans are obtained is end-of-period capital $K_t$. Capital depreciates at a rate $\delta$ in each period.

The level of the loans depends on the level of the deposits and the net worth of the intermediary. This implies a banking sector’s balance sheet of the form:\footnote{\textsuperscript{14}In a slight departure from notation elsewhere, lower case denotes the representative bank. Upper case variables later denote aggregates}

\[ Q_t s_t = n_t + d_t \]  \hspace{1cm} (38)

where $s_t$ are claims on non-financial firms to finance capital acquired at the end of period $t$ for use in period $t+1$ and $Q_t$ is the price of a unit of capital so that the assets of the bank. Therefore $Q_t s_t$ are the assets of the bank. The liabilities of the bank are household deposits $d_t$ and net worth $n_t$.\footnote{\textsuperscript{14}}
Net worth of the bank accumulates according to:

\[ n_t = R_{k,t}Q_{t-1}s_{t-1} - R_{t}^{ex}d_{t-1} \]  

(39)

where real returns on bank assets are given by

\[ R_{k,t} = \frac{[Z_t + (1 - \delta)Q_t]}{Q_{t-1}} \]

\( Z_t \) is the gross return (marginal product) of capital and \( Z_t + (1 - \delta)Q_t \) represents the net return after depreciation.

Banks exit with probability \( 1 - \sigma_B \) per period and therefore survive for \( i - 1 \) periods and exit in the \( i \)th period with probability \( (1 - \sigma_B)\sigma_B^{i-1} \). Given the fact that bank pays dividends only when it exists, the banker’s objective is to maximize expected discounted terminal wealth

\[ V_t = E_t \sum_{i=0}^{\infty} (1 - \sigma_B)\sigma_B^i \Lambda_{t,t+1+i}n_{t+1+i} \]  

(40)

where \( \Lambda_{t,t+i} = \beta^i \frac{\Lambda_C_{t+1,i} P_{t+i}}{\Lambda_C \beta_i P_t} \) is the stochastic discount factor, subject to an incentive constraint for lenders (households) to be willing to supply funds to the banker.

To understand this dynamic problem better we can substitute for \( d_t \) from (38) and rewrite (39) as

\[ n_t = R_{t}^{ex} n_{t-1} + (R_{k,t} - R_{t}^{ex})Q_{t-1}s_{t-1} \]  

(41)

which says that net worth at the end of period \( t \) equals the gross return at the real riskless rate plus the excess return over the latter on the assets. With these returns and \( Q_t \) exogenous the the bank, given \( n_{t-1} \) at the beginning of period \( t \) net worth in all future periods is determined by its choice of \( \{s_{t+i}\} \) subject to a borrowing constraint.

To motivate an endogenous constraint on the bank’s ability to obtain funds, we introduce the following simple agency problem. We assume that after a bank obtains funds, the bank’s manager may transfer a fraction of assets to her family. In the recognition of this possibility, households limit the funds they lend to banks. Moreover we assume that the fraction of funds that a banker can divert depends on the composition of the bank’s liabilities.

Divertable assets consists of total gross assets \( Q_t s_t \) minus interbank borrowing \( b_t \). If
a bank diverts assets for its personal gain, it defaults on its debt and shuts down. The creditors may re-claim the remaining fraction $1 - \Theta$ of funds. Because its creditors recognize the bank’s incentive to divert funds, they will restrict the amount they lend. In this way a borrowing constraint may arise. In order to ensure that bankers do not divert funds the following incentive constraint must hold:

$$V_t \geq \Theta_t Q_t s_t$$  \hspace{1cm} (42)

The incentive constraint states that for households to be willing to supply funds to a bank, the bank’s franchise value $V_t$ must be at least as large as its gain from diverting funds.

The optimization problem for the bank is to choose a path for borrowing, $\{s_{t+1}\}$ to maximize $V_t$ subject to (38) and (39) or equivalently (41) and (42). To solve this problem we guess a linear solution of the form:

$$V_t = V_t(s_t, d_t) = \nu_{s,t} s_t - \nu_{d,t} d_t$$  \hspace{1cm} (43)

where $\nu_{s,t}/Q_t$, and $\nu_{d,t}$ are time-varying parameters that are the marginal values of the asset at the end of period $t$. Now eliminate $d_t$ from (43) using (38) to obtain

$$V_t = V_t(s_t, n_t) = \mu_{s,t} Q_t s_t + \nu_{d,t} n_t = \mu_{s,t} Q_t s_t + \nu_{d,t} n_t$$  \hspace{1cm} (44)

where $\mu_{s,t} \equiv \nu_{s,t}/Q_t - \nu_{d,t}$ is the excess value of bank assets over deposits.

Next write the Bellman equation for a given path for $n_t$ in the form

$$V_{t-1}(s_{t-1}, n_{t-1}) = E_t \Lambda_{t+1}[(1 - \sigma_B)n_t + \sigma_B \max_{s_t} V_t(s_t, n_t)]$$  \hspace{1cm} (45)

Then we perform the optimization $\max_{s_t} V_t(s_t, n_t)$ subject to the IC constraint (42). The Lagrangian for this problem is

$$\mathcal{L}_t = V_t + \lambda_t [V_t - \Theta Q_t s_t] = (1 + \lambda_t)V_t - \lambda_t \Theta Q_t s_t$$  \hspace{1cm} (46)

where $\lambda_t > 0$ if the constraint binds and $\lambda_t = 0$ otherwise.
The first order conditions for the optimization problem are:

\[ s_t : (1 + \lambda_t)\mu_{s,t} = \lambda_t \Theta \]
\[ \lambda_t : \mu_{s,t}Q_{t} = \nu_{d,t}n_t \geq \Theta Q_{t} \]

We now define \( \phi_t \) be the leverage ratio of the representative bank that satisfies the incentive constraint:

\[ Q_t = \phi_t n_t \]  \( (47) \)

where \( \phi_t \) is given by

\[ \phi_t = \frac{\nu_{d,t}}{\Theta - \mu_{s,t}} \]  \( (48) \)

Using (47) we can write (44) as

\[ V_t = [\mu_{s,t}\phi_t + \nu_{d,t}]n_t \]  \( (49) \)

and hence (45) becomes

\[ V_t(s_t, n_t) = E_t \Lambda_{t,t+1}[1 - \sigma_B + \sigma_B(\mu_{s,t+1}\phi_{t+1} + \nu_{d,t+1})]n_{t+1} \]
\[ = E_t \Lambda_{t,t+1}\Omega_{t+1}n_{t+1} \]
\[ = E_t \Lambda_{t,t+1}\Omega_{t+1}[R_{k,t+1}Q_t - R_{t+1}^{ex}d_t] \]  \( (50) \)

defining \( \Omega_t = 1 - \sigma_B + \sigma_B(\nu_{d,t} + \phi_t \mu_{s,t}) \), the shadow value of a unit of net worth, and using (39).

Comparing (50) with (43) and equating coefficients of \( s_t \) and \( d_t \), we arrive at the determination of \( \nu_{s,t} \) and \( \nu_{d,t} \):

\[ \nu_{d,t} = E_t \Lambda_{t,t+1}\Omega_{t+1}R_{t+1}^{ex} \]
\[ \nu_{s,t} = E_t \Lambda_{t,t+1}\Omega_{t+1}Q_tR_{k,t+1} \]

Hence

\[ \mu_{s,t} = \frac{\nu_{s,t}}{Q_t} - \nu_{d,t} = E_t \Lambda_{t,t+1}\Omega_{t+1}(R_{k,t+1} - R_{t+1}^{ex}) \]  \( (51) \)
At the aggregate level the banking sector balance sheet is:

\[ Q_t S_t = N_t + D_t \]

At the aggregate level net worth is the sum of existing (old) bankers and new bankers:

\[ N_t = N_{o,t} + N_{n,t} \]

Net worth of existing bankers equals earnings on assets held in the previous period net cost of deposit finance, multiplied by a fraction \( \sigma_B \), the probability that they survive until the current period:

\[ N_{o,t} = \sigma_B \{(Z_t + (1 - \delta)Q_t)S_{t-1} - R_{t}^{ex}D_{t-1}\} \]

Since new bankers cannot operate without any net worth, we assume that the family transfers to each one the fraction \( \xi_B/(1 - \sigma_B) \) of the total value assets of exiting entrepreneurs. This implies:

\[ N_{n,t} = \xi_B[Z_t + (1 - \delta)Q_t]S_{t-1} \]

Note that in aggregate

\[ \phi_t = \frac{Q_t S_t}{N_t^o} \neq \frac{Q_t S_t}{N_t} \]

as in Gertler et al. (2010) and related papers. In fact from (52) we have \( N_{n,t} = \xi_B R_{k,t} Q_{t-1} S_{t-1} = \xi_B R_{k,t} \phi_t N_{o,t} \) so

\[ N_t = (1 + \xi_B R_{k,t} \phi_t) N_{o,t} \quad \text{(54)} \]

\[ Q_t S_t = \phi_t N_{o,t} = \frac{\phi_t N_t}{(1 + \xi_B R_{k,t} \phi_t)} \quad \text{(55)} \]

### 3.2.1 Summary of the Banking Model

The complete model is the NK model plus the banking sector is illustrated in Figure 8. It is derived above with the demand for capital relationship replaced with the following
equations that represent the banking sector:

\[ S_t = K_t \]
\[ (1 + \lambda_t)\mu_{s,t} = \lambda_t \Theta \]
\[ Q_t S_t = \frac{\phi_t N_t}{(1 + \xi_B R_{k,t} \phi_t)} \]
\[ \phi_t = \frac{\nu_{d,t}}{\Theta - \mu_{s,t}} \]
\[ N_t = R_{k,t}(\sigma_B + \xi_B)Q_{t-1}S_{t-1} - \sigma_B R_{t+1}^e D_{t-1} \]
\[ D_t = Q_t S_t - N_t \]

\[ \nu_{d,t} = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1}^{ex} \]
\[ \mu_{s,t} = E_t \Lambda_{t,t+1} \Omega_{t+1}(R_{k,t+1} - R_{t+1}^{ex}) \]
\[ \Omega_t = 1 - \sigma_B + \sigma_B (\nu_{d,t} + \phi_t \mu_{s,t}) \]
\[ R_{k,t} = \frac{Z_t + (1 - \delta)Q_t}{Q_{t-1}} \]
\[ Z_t = \frac{(1 - \alpha)P_t W Y_t W}{K_{t-1}} \]

Figure 1 illustrates the model.

3.2.2 Steady State of the Banking Model

The main difference with respect to the basic NK code is that in this model we solve for two unknowns: hours worked and capital. The balanced-growth growth steady state of
the banking sector is:

\[ S = K \]

\[ Q = 1 \]

\[ \Lambda = \beta \]

\[ (1 + \lambda)\mu_s = \lambda \Theta \]

\[ QS = \frac{\phi N}{(1 + \xi_B R_k \phi)} \]

\[ \phi = \frac{\nu_d}{\Theta - \mu_s} \]

\[ N = R_k (\sigma_B + \xi_B) QS - \sigma_B R^e D \]

\[ D = QS - N \]

\[ \nu_d = \Lambda \Omega R^e \]

\[ \mu_s = \Lambda \Omega (R_k - R) \]

\[ \Omega = 1 - \sigma_B + \sigma_B (\nu_d + \phi \mu_s) = 1 - \sigma_B + \Theta \sigma_B \phi t \]

\[ R_k = \frac{Z + (1 - \delta)Q}{Q \frac{25}{K}} \]

\[ Z = \frac{(1 - \alpha) P Y^W}{K} \]

\[ R^e = \frac{R_n}{\Pi} = \frac{1}{\beta} (1 + g)^{(1 - \theta)(1 - \sigma_c) - 1} \]
3.2.3 Calibration of the Banking Model

The parameters of the banking sector are calibrated in the following way. Following GK choose the value of $\sigma_B$ so that that bankers survive 8 years (32 periods) on average. Then $\frac{1}{1-\sigma_B} = 32$. The values of $\Theta_B$ and $\xi_B$ are computed to hit an economy wide leverage ratio of three and to have an average credit spread of 88 basis points per year. Then we obtain

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_B$</td>
<td>0.9685</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.3118</td>
</tr>
<tr>
<td>$\xi_B$</td>
<td>0.000733</td>
</tr>
</tbody>
</table>

Table 2. Calibrated Parameters

4 Optimal Monetary Policy

What are the policy implications of financial frictions for monetary policy? In this section we address this question. We examine three monetary policy regimes: the ex ante optimal policy with commitment (the Ramsey problem), the time consistent optimal policy (discretion) and a Taylor-type interest rate rule of the form (21) with welfare-optimized feedback parameters.$^{15}$ Notice that this is a Taylor-type rules as in Taylor (1993) that responds to deviations of output from its deterministic steady state values and not from its flexi-price outcomes. Such a rules has the advantage that it can be implemented using readily available macro-data series rather than from model-based theoretical constructs (see Schmitt-Grohe and Uribe (2007)). We first consider policy ignoring zero-lower bound (ZLB) issues for the nominal interest rate.

4.1 Optimal Policy Ignoring the Interest Rate Zero Lower Bound

Tables 3 and 4 set out the welfare outcomes and forms of the simple rules using the inter-temporal household utility for models I and II. The tables report the welfare outcomes relative to the ex ante optimal policy for each model separately and the steady-state

$^{15}$Full details of these policy regimes can be found in Currie and Levine (1993) and Levine et al. (2012).
variance of the nominal interest rate $\sigma_r^2$. The intertemporal welfare $\Omega_0$ is expressed in terms of a consumption equivalent increase relative to the steady state.\textsuperscript{16} Table 4 reports the optimized simple rule of the form (21) with an additional feedback from Tobin’s Q. Figures 5-12 show the impulse responses to our four shocks for the two models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Policy</th>
<th>Frictions</th>
<th>$\Omega_0$</th>
<th>$\sigma_r^2$</th>
<th>$c_e$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Commit</td>
<td>Product</td>
<td>0</td>
<td>0.225</td>
<td>0</td>
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<tr>
<td></td>
<td>Discretion</td>
<td>Product</td>
<td>2.979</td>
<td>0.057</td>
<td>0.03</td>
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<tr>
<td>II</td>
<td>Commit</td>
<td>Product, Financial</td>
<td>0</td>
<td>0.841</td>
<td>0</td>
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<tr>
<td></td>
<td>Discretion</td>
<td>Product, Financial</td>
<td>3.437</td>
<td>4.294</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 3. Optimal Rules with and without Commitment: No ZLB.

<table>
<thead>
<tr>
<th>Model</th>
<th>Frictions</th>
<th>$\Omega_0$</th>
<th>Rule $[\rho_r, \theta_\pi, \theta_y, \theta_q]$</th>
<th>$\sigma_r^2$</th>
<th>$c_e$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Product</td>
<td>1.655</td>
<td>$[0.775, 2.253, 0.008, 0]$</td>
<td>0.096</td>
<td>0.02</td>
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<tr>
<td>II</td>
<td>Product, Financial</td>
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<td>$[1.000, 1.902, 0.012, 0]$</td>
<td>0.047</td>
<td>0.007</td>
</tr>
<tr>
<td>II</td>
<td>Product, Financial</td>
<td>0.697</td>
<td>$[1.000, 1.003, -0.003, 0.035]$</td>
<td>0.029</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Table 4. Optimized Simple Rules: No ZLB

For monetary policy alone our results can be summarized as follows:

- There are modest gains from commitment in the basic NK model of $c_e = 0.03\%$ in consumption equivalent terms which rise to $c_e = 0.04\%$ in model II with financial frictions (FF).

- The costs of simplicity are also small, $c_e = 0.02\%$ at most.

- High variances of the nominal interest rate, $\sigma_r^2$, indicate that ZLB considerations arise in the model with FF for optimal and discretionary policy. As we will see, these contribute an increase in the gains from commitment.

\textsuperscript{16}To derive the welfare in terms of a consumption equivalent percentage increase ($c_e \equiv \frac{\Delta C}{C} \times 10^2$), expanding $\Lambda(C_t, 1 - N_t)$ as a Taylor series, $\Delta \Lambda = \Lambda C \Delta C = \Lambda C c_e x 10^{-2}$. Losses $X$ reported in the Table are of the order of variances expressed as percentages and have been scaled by $1 - \beta$. Thus $X \times 10^{-4} = \Delta \Lambda$ and hence $c_e = \frac{X \times 10^4}{C \Lambda C}$. For the steady state of this model, $\Lambda C = 1.01$. It follow that a welfare loss difference of $X = 1$ gives a consumption equivalent percentage difference of about $0.01\%$.
• The optimized simple rule is more aggressive in the model with FF and is close to a price-level rule.\textsuperscript{17}

• We have also explored but find no benefit from targeting spread, or leverage; however modest gains from targeting Tobin’s Q are reported in Table 4.

4.2 Interest Rate Zero Lower Bound Considerations

Table 4 indicates that the aggressive nature of the optimal and discretionary rules in the model with FF leads to high interest rate variances resulting in a ZLB problem. From the table with our zero-inflation steady state and nominal interest rate of 1% per quarter, optimal policy variances between 0.722 and 3.098 of a normally distributed variable imply a probability per quarter of hitting the ZLB in the range $[0.121, 0.284]$. At the upper end of these ranges the ZLB would be hit every year on average. In this subsection we address this issue.

Our LQ set-up for a given set of observed policy instruments $w_t$ considers a linearized model in a general state-space form:

$$
\begin{bmatrix}
  z_{t+1} \\
  E_{t}x_{t+1}
\end{bmatrix}
= A
\begin{bmatrix}
  z_t \\
  x_t
\end{bmatrix}
+ Bw_t
+ \begin{bmatrix}
  u_{t+1} \\
  0
\end{bmatrix}
$$

(56)

where $z_t, x_t$ are vectors of backward and forward-looking variables, respectively, $w_t$ is a vector of policy variables, and $u_t$ is an i.i.d. zero mean shock variable with covariance matrix $\Sigma_u$.

Let $y_t^T \equiv [z_t \ x_t \ w_t]$. Then welfare-based quadratic large-distortions approximation to welfare loss function at time $t$ by $E_t[\Omega_t]$ where

$$
\Omega_t = \frac{1}{2} \sum_{i=0}^{\infty} \beta^i [y_{t+i}^TQy_{t+i}]
$$

(57)

\textsuperscript{17}There has been a recent interest in the case for price-level rather than inflation stability. Gaspar et al. (2010) provide an excellent review of this literature. The basic difference between the two regimes in that under an inflation targeting mark-up shock leads to a commitment to use the interest rate to accommodate an increase in the inflation rate falling back to its steady state. By contrast a price-level rule commits to a inflation rate below its steady state after the same initial rise. Under inflation targeting one lets bygones be bygones allowing the price level to drift to a permanently different price-level path whereas price-level targeting restores the price level to its steady state path. The latter can lower inflation variance and be welfare enhancing because forward-looking price-setters anticipates that a current increase in the general price level will be undone giving them an incentive to moderate the current adjustment of its own price.
where $Q$ is a matrix. In the absence of a lower bound constraint on the nominal interest rate the policymaker’s optimization problem is to minimize $\Omega_0$ given by (57) subject to (56) and given $z_0$. If the variances of shocks are sufficiently large, this will lead to a large nominal interest rate variability and the possibility of the nominal interest rate becoming negative. We can impose a lower bound effect on the nominal interest rate by modifying the discounted quadratic loss criterion as follows. Consider first the ZLB constraint on the nominal on the nominal interest rate. Rather than requiring that $R_t \geq 0$ for any realization of shocks, we impose the constraint that the mean rate should at least $k$ standard deviation above the ZLB. For analytical convenience we use discounted averages. Define $\bar{R}_n \equiv E_0 \left[(1 - \beta) \sum_{t=0}^{\infty} \beta^t R_{n,t}\right]$ to be the discounted future average of the nominal interest rate path $\{R_{n,t}\}$. Our ‘approximate form’ of the ZLB constraint is a requirement that $\bar{R}_n$ is at least $k_r$ standard deviations above the zero lower bound; i.e., using discounted averages that

$$\bar{R}_n \geq k_r \sqrt{(R_{n,t} - \bar{R}_n)^2} = k_r \sqrt{R_{n,t}^2 - (\bar{R}_n)^2} \quad (58)$$

Squaring both sides of (58) we arrive at

$$E_0 \left[(1 - \beta) \sum_{t=0}^{\infty} \beta^t R_{n,t}^2\right] \leq K_r \left[E_0 \left[(1 - \beta) \sum_{t=0}^{\infty} \beta^t R_{n,t}\right]\right]^2 \quad (59)$$

where $K_r = 1 + k_r^{-2} > 1$.

We now maximize $\sum_{t=0}^{\infty} \beta^t[U(X_{t-1}, W_t)]$ subject to the additional constraint (59) alongside the other dynamic constraints in the Ramsey problem. Using the Kuhn-Tucker theorem this results in an additional term $w_r \left(\bar{R}^2 - K(\bar{R}_n)^2\right)$ in the Lagrangian to incorporate this extra constraint, where $w_r > 0$ is a Lagrangian multiplier. From the first order conditions for this modified problem this is equivalent to adding terms $E_0(1 - \beta) \sum_{t=0}^{\infty} \beta^t w_r (R_{n,t}^2 - 2K \bar{R}_n R_{n,t})$ where $\bar{R}_n > 0$ is evaluated at the constrained optimum. It follows that the effect of the extra constraint is to follow the same optimization as before, except that the single period loss function terms of in log-linearized variables is

\[\text{This follow the treatment of the ZLB in Woodford (2003) and Levine et al. (2008)}\]
replaced with

\[ L_t = y_t^T Q y_t + w_r (r_{n,t} - r_n^*)^2 \]  \hspace{1cm} (60)

where \( r_n^* = (K - 1) \bar{R}_n > 0 \) is a nominal interest rate target for the constrained problem.

In our LQ approximation of the non-linear optimization problem we have linearized around the Ramsey steady state which has zero inflation. With a ZLB constraint, the policymaker’s optimization problem is now to choose an unconditional distribution for \( r_{n,t} \), shifted to the right by an amount \( r_n^* \), about a new positive steady-state inflation rate, such that the probability of the interest rate hitting the lower bound is extremely low. This is implemented by choosing the weight \( w_r \) for each of our policy rules so that

\[ z_0(p) \sigma_r < R^* \]

where \( z_0(p) \) is the critical value of a standard normally distributed variable \( Z \) such that \( \text{prob} (Z \leq z_0) = p \), \( R^*_n = (1 + \pi^*) R_n + \pi^* \) is the steady state nominal interest rate, \( R \) is the shifted steady state real interest rate, \( \sigma_r^2 = \text{var}(R) \) is the unconditional variance and \( \pi^* \) is the new steady state positive net inflation rate. Given \( \sigma_r \) the steady state positive inflation rate that will ensure \( R_{n,t} \geq 0 \) with probability \( 1 - p \) is given by

\[ \pi^* = \max \left[ \frac{z_0(p) \sigma_r - R_n + 1}{R_n} \times 100, 0 \right] \]  \hspace{1cm} (61)

In our linear-quadratic framework we can write the intertemporal expected welfare loss at time \( t = 0 \) as the sum of stochastic and deterministic components, \( \Omega_0 = \tilde{\Omega}_0 + \bar{\Omega}_0 \). By increasing \( w_r \) we can lower \( \sigma_r \) thereby decreasing \( \pi^* \) and reducing the deterministic component, but at the expense of increasing the stochastic component of the welfare loss. By exploiting this trade-off, we then arrive at the optimal policy that, in the vicinity of the steady state, imposes a ZLB constraint, \( R_{n,t} \geq 0 \) with probability \( 1 - p \). Figures 3 – 4 and Table 5 show this solution to the problem for optimal commitment and discretionary policy regimes with \( p = 0.005 \); i.e., a stringent ZLB requirement that the probability of hitting the zero lower bound is only once every 200 quarters or 50 years. Note that the low interest rate variances for optimized simple rules imply there are no ZLB concerns when policy is implemented using them.

For the commitment and discretionary regimes as the penalty on the interest rate variance \( w_r \) increases, the variance \( \sigma_r^2 \) falls and the steady-state inflation shift \( \pi^* \) necessary to enforce the ZLB falls to zero. However the effectiveness of this outcome depends
Figure 3: Model II: Imposition of ZLB for Optimal Policy

Figure 4: Model II: Imposition of ZLB for Time-Consistent Policy
crucially on the ability of the monetary authority to commit to a particular interest rate rule. Absent such commitment a higher penalty on interest rate adjustments is necessary resulting in an increase in the gains to commitment to 0.07%.

<table>
<thead>
<tr>
<th>Model</th>
<th>Policy</th>
<th>Frictions</th>
<th>Ω₀</th>
<th>σ²ₜ</th>
<th>wᵣ</th>
<th>π⁺</th>
<th>prob(ZLB)</th>
<th>eᵣ(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Commit</td>
<td>Product</td>
<td>0.185</td>
<td>0.11</td>
<td>0.035</td>
<td>0</td>
<td>&lt;0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>I</td>
<td>Discretion</td>
<td>Product</td>
<td>2.979</td>
<td>0.057</td>
<td>0</td>
<td>0</td>
<td>&lt;0.005</td>
<td>0.03</td>
</tr>
<tr>
<td>II</td>
<td>Commit</td>
<td>Product, Financial</td>
<td>0.176</td>
<td>0.11</td>
<td>0.013</td>
<td>0</td>
<td>&lt;0.005</td>
<td>0.002</td>
</tr>
<tr>
<td>II</td>
<td>Discretion</td>
<td>Product, Financial</td>
<td>6.754</td>
<td>0.13</td>
<td>0.16</td>
<td>0</td>
<td>&lt;0.005</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 5. Optimal Rules with and without Commitment: with ZLB\(^{19}\)

5 Macro-prudential Regulation

Much of the macro-prudential regulation literature focuses on the capital-loan ratio which in the context of the model of this paper is \(\frac{N_t}{Q_t S_t}\), at the aggregate which approximately is \(\frac{1}{\phi_t}\), the inverse of the leverage ratio. To incentivize the banks to raise to lower this ratio about their laissez-faire choice we now turn to a Pigovian subsidies revenue-neutral regime of taxes and subsidies on net worth and loans. For example a counter-cyclical regime to raise capital requirements in an economic upturn will see a increase in a subsidy for net worth, \(\tau^s\), financed by a tax on loans, \(\tau_t\). The balance sheet of the bank now is generalized to

\[
(1 + \tau_t)Q_t s_t = (1 + \tau^s_t) n_t + d_t
\]

which says that net worth plus subsidies plus deposits can be used to finance loans net of tax. The net worth of the bank now accumulates according to:

\[
n_t = \frac{(R_{k,t} - \tau_{t-1})Q_{t-1}s_{t-1} - R^{ex}_{t} d_{t-1}}{1 - \tau^s_t}
\]

The assumed form of the value function, (43), now becomes

\[
V_t = V_t(s_t, n_t) = \nu_{s,t} s_t - \nu_{d,t} [(1 + \tau_t)Q_t s_t - (1 + \tau^s_t) n_t] = \mu_{s,t} Q_t s_t + \mu_{d,t} n_t
\]

\(^{19}\)A utility difference of 100 is equivalent to \(C_{AC} = 1.01\) for this model and parameter values.
where \( \mu_{s,t} \equiv \frac{\nu_{s,t}}{Q_t} - \nu_{d,t}(1 + \tau_t) \) is the excess value of bank assets over deposits net of the tax on assets and \( \mu_{d,t} \equiv \nu_{d,t}(1 + \tau_t^s) \) is the shadow price of deposits with a subsidy for net worth.

The first order conditions follow as before with \( \mu_{d,t} \) replacing \( \nu_{d,t} \) so leverage \( \phi_t \) and the value function now become

\[
\phi_t = \frac{\mu_{d,t}}{\Theta - \mu_{s,t}}
\]

\[
V_t = [\mu_{s,t}\phi_t + \mu_{d,t}]n_t
\]

Hence (45) becomes

\[
V_t(s_t, n_t) = E_t \Lambda_{t,t+1}[1 - \sigma_B + \sigma_B(\mu_{s,t+1}\phi_{t+1} + \mu_{d,t+1})]n_{t+1}
\]

\[
= E_t \Lambda_{t,t+1}\Omega_{t+1}n_{t+1}
\]

\[
= E_t \Lambda_{t,t+1}\Omega_{t+1} \left[ \frac{(R_{k,t+1} - \tau_t)Q_t s_t - R_{s,t}^e d_t}{1 - \tau_t^s \tau_{t+1}} \right]
\]

(67)

defining \( \Omega_t = 1 - \sigma_B + \sigma_B(\nu_{d,t} + \phi_t\mu_{s,t}) \), the shadow value of a unit of net worth, and using (63).

Comparing (67) with (43) and equating coefficients of \( s_t \) and \( d_t \) as before, we arrive at the determination of \( \nu_{s,t} \), and \( \nu_{d,t} \):

\[
\nu_{d,t} = E_t \left[ \frac{\Lambda_{t,t+1}\Omega_{t+1}R_{s,t}^e}{1 - \tau_t^s \tau_{t+1}} \right]
\]

\[
\nu_{s,t} = E_t \left[ \frac{\Lambda_{t,t+1}\Omega_{t+1}Q_t(R_{k,t+1} - \tau_t)}{1 - \tau_t^s \tau_{t+1}} \right]
\]

Hence

\[
\mu_{s,t} \equiv \frac{\nu_{s,t}}{Q_t} - \nu_{d,t}(1 + \tau_t^s) = E_t \left[ \frac{\Lambda_{t,t+1}\Omega_{t+1}(R_{k,t+1} - R_{s,t}^e - \tau_t - \tau_t^s R_{s,t}^e)}{1 - \tau_t^s \tau_{t+1}} \right]
\]

(68)

\[
\mu_{d,t} \equiv (1 + \tau_t^s)\nu_{d,t} = E_t \left[ \frac{\Lambda_{t,t+1}\Omega_{t+1}(1 + \tau_t^s)R_{s,t}^e}{1 - \tau_t^s \tau_{t+1}} \right]
\]

(69)

Aggregation follows as before and we now add a balanced budget condition

\[
\tau_t Q_t S_t = \tau_t^s N_t
\]

(70)
We also add costs of administering the tax regime so that the resource constraint (19) now becomes

\[ Y_t = C_t + G_t + I_t + \frac{1}{2} \xi (\Pi_t - 1)^2 Y_t + \psi \tau^2 Q_t S_t \]  

(71)

In (71) we have added a cost of raising \( \tau Q_t S_t \) by taxing loans that is quadratic in the tax rate. To achieve a steady state of the Ramsey problem we found that \( \psi \geq 0.85 \). Putting \( \psi = 0.85 \) resulted in a cost of the scheme of around 0.2% of GDP.

The complete model is summarized by

\[
\begin{align*}
S_t &= K_t \\
(1 + \lambda_t) \mu_{s,t} &= \lambda_t \Theta \\
Q_t S_t &= \frac{\phi_t N_t}{(1 + \xi B R_{k,t} \phi_t)} \\
\phi_t &= \frac{\mu_{d,t}}{\Theta - \mu_{s,t}} \\
N_t &= \frac{(R_{k,t} - \tau_{t-1})(\sigma_B + \xi B)Q_{t-1}S_{t-1} - \sigma_B R_{t-1}^{ext} D_{t-1}}{1 - \tau^s_t} \\
D_t &= Q_t S_t - N_t \\
\tau_t Q_t S_t &= \tau_t^s N_t \\
\mu_{d,t} &= E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \frac{(1 + \tau_t^s) R_{t+1}^{ext}}{1 - \tau_{t+1}^s} \right] \\
\mu_{s,t} &= E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \frac{(R_{k,t+1} - R_{t+1}^{ext} - \tau_t - \tau_t^s R_{t+1}^{ext})}{1 - \tau_{t+1}^s} \right] \\
\Omega_t &= 1 - \sigma_B + \sigma_B (\mu_{d,t} + \phi_t \mu_{s,t}) \\
R_{k,t} &= \frac{Z_t + (1 - \delta) Q_t}{Q_{t-1}} \\
Z_t &= \frac{(1 - \alpha) P^W Y^W}{K_{t-1}}
\end{align*}
\]

with \( \tau_t^s \) or \( \tau_t \) exogenous. Clearly in the absence of taxes of subsidies, \( \tau_t = \tau_t^s = 0 \) we get back to the previous set-up.

Taking \( \tau_t \) as the tax instrument alongside the nominal interest rate, we examine either optimal or discretionary policy as before or with a simple Taylor-type rule

\[
\log \left( \frac{1 + \tau_t}{1 + \tau} \right) = \rho \tau \log \left( \frac{1 + \tau_{t-1}}{1 + \tau} \right) + \theta_{\tau,y} \log \left( \frac{Y_t}{Y} \right)
\]  

(72)

34
The zero-growth steady state of the banking sector with this tax regime in place is:

\[ S = K \]
\[ Q = 1 \]
\[ \Lambda = \beta \]
\[ (1 + \lambda)\mu_s = \lambda \Theta \]
\[ QS = \frac{\phi N}{(1 + \xi B R_k \phi)} \]
\[ \phi = \frac{\mu_d}{\Theta - \mu_s} \]
\[ N = \frac{(R_k - \tau)(\sigma_B + \xi_B)QS - \sigma_B R^e x D}{1 - \tau^s} \]
\[ D = QS - N \]
\[ \mu_d = \frac{\Lambda \Omega (1 + \tau^s) R^e x}{1 - \tau^s} \]
\[ \mu_s = \frac{\Lambda \Omega (R_k - R^e x - \tau - \tau^s R^e x)}{1 - \tau^s} \]
\[ \Omega = 1 - \sigma_B + \sigma_B (\nu_d + \phi \mu_s) = 1 - \sigma_B + \Theta \sigma_B \phi \]
\[ \tau QS = \tau^s N \]
\[ R_k = \frac{Z + (1 - \delta)Q}{Q} \]
\[ Z = \frac{(1 - \alpha)B^W Y^W}{K} \]
\[ R^e x = \frac{R_n}{\Pi} = \frac{1}{\beta} (1 + g)^{(1 - \varphi)(1 - \sigma_c) - 1} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>Instruments</th>
<th>Policy</th>
<th>( \Omega_0 )</th>
<th>( \sigma^2_\tau )</th>
<th>( \sigma^2_\tau )</th>
<th>( c_n(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>MP+MR</td>
<td>Commit</td>
<td>0</td>
<td>0.180</td>
<td>221</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>MP+MR</td>
<td>Discretion</td>
<td>5.731</td>
<td>5.868</td>
<td>517</td>
<td>0.06</td>
</tr>
<tr>
<td>II</td>
<td>MP only</td>
<td>Commit</td>
<td>0.882</td>
<td>0.320</td>
<td>0</td>
<td>0.01</td>
</tr>
<tr>
<td>II</td>
<td>MP only</td>
<td>Discretion</td>
<td>5.221</td>
<td>8.744</td>
<td>0</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 6. Optimal Rules with and without Commitment: No ZLB.

Monetary Policy (MP) and Macro-prudential Policy (MP). In Ramsey steady state: \( \tau = 0.018 \), \( \Lambda = -1.00327 \), \( \phi = 1.6367 \) with tax and \( \Lambda = -1.00613 \), \( \phi = 3.0 \) without tax.
Table 7. Optimized Simple Rules: No ZLB.

Monetary Policy (MP) and Macro-prudential Regulation (MR).

Tables 6 and 7 set out the results. These can be summarized as follows:

- In the steady state reduces the leverage from $\phi = 3.00$ to $\phi = 1.64$ with a tax on loans $\tau = 0.018$. The steady state utility rises from $\Lambda = -1.00613$ to $\Lambda = -1.00327$, an approximate increase of 0.3%. With $\sigma_c = 2$ and $\varrho = 0.854$ (calibrated to hit hours worked $h = 0.4$) this is equivalent to a permanent consumption increase of about 2%.

- Stabilization gains from macro-prudential regulation are by contrast small, $c_e = 0.01\%$.

- The simple rule that mimics the optimal tax policy is counter-cyclical with considerable persistence. In log-linear form it is given by

$$\tau_t = 0.927\tau_{t-1} + 0.57y_t$$

(73)

along-side an optimal monetary rule

$$\Delta r_t = 5.00\pi_t - 0.0027y_t$$

(74)

which is very close to a price-level rule.

- As before nominal interest rate variances of optimized simple rules are small so these pose no ZLB concerns. However this is not the case with optimal commitment and discretionary policies whether MR is in place or not. With or without MR variances under discretion are high, particularly in the former case. So the main benefit of MR is to reduce the welfare costs of imposing a ZLB constraint under discretion.

<table>
<thead>
<tr>
<th>Model</th>
<th>Instruments</th>
<th>$\Omega_0$</th>
<th>Rule $[\rho_r, \theta_{\pi}, \theta_{\tau_y}, \theta_{\tau_y}, \theta_{\tau}]$</th>
<th>$\sigma_r^2$</th>
<th>$\sigma_{\tau}^2$</th>
<th>$c_e(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>II MP+MR</td>
<td>0.570</td>
<td>[1.000, 4.670, −0.068, 0.574, 0.927]</td>
<td>0.081</td>
<td>79.2</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>II MP only</td>
<td>1.230</td>
<td>[1.000, 5.000, −0.0027, 0, 0]</td>
<td>0.076</td>
<td>0</td>
<td>0.01</td>
<td></td>
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</tbody>
</table>
Finally we examined if monetary policy and macro-prudential regulation should jointly target financial and non-financial variables. In fact we found no significant benefit from rules of this character, suggesting that the benefit from combining MP and MR within one institution may also be insignificant.

6 Conclusions

This paper has considered both monetary policy alone and in conjunction with macro-prudential regulation. For the latter we find modest gains from commitment and costs of simplicity in the basic NK model, but these increase when we introduce financial frictions (FF) ZLB considerations arise particularly in the model with FF for optimal and discretionary policy contributing to an increase in the gains from commitment. The optimized simple rule is more aggressive in the model with FF and is close to a price-level rule. We find no welfare benefit from targeting spread, or leverage but modest gains from targeting Tobin’s Q.

Turning to macro-prudential regulation through a tax on loans and a subsidy for the bank’s net worth, we find significant gains from using this instrument in the deterministic steady state equivalent to a permanent consumption increase of about $c_e = 2\%$. Stabilization gains from macro-prudential regulation are by contrast small, $c_e = 0.01\%$. The simple rule that mimics the optimal tax policy is counter-cyclical with considerable persistence alongside an interest rate rule that is very close to a price-level rule. For optimal commitment and discretionary policies whether MR is in place or not nominal interest rate variances under discretion are high, particularly in the latter case. So the main benefit of MR is to reduce the welfare costs of imposing a ZLB constraint under discretion.

Our LQ framework is based on a local approximation to the non-linear optimal policy problem about the deterministic Ramsey steady state. Given that most of the welfare gains from macro-prudential regulation come from a more efficient steady state the question arises as to how our conclusions might change if we approximate about the stochastic steady state as in Gertler et al. (2010). This is discussed in an Appendix.
References


Juillard, M. (2011). Local approximation of DSGE models around the risky steady state. wp.comunite 0087, Department of Communication, University of Teramo.


A The Stochastic Steady State

So far our LQ framework is based on an approximation to the non-linear optimal policy problem about the deterministic Ramsey steady state. This Appendix discusses how our results may change if we approximate about the stochastic steady state (sss) as in Gertler et al. (2010).

Juillard (2011) and Coeurdacier et al. (2011) provide the clearest discussion of how to handle the stochastic steady state. Ignoring the time dimension for the moment, consider two random variables $x$, $y$ with joint cumulative distribution $p(x, y)$. Then a second order Taylor series approximation to the expected value of $f(x, y)$ is given by

\[
\int f(x, y) dp(x, y) \approx \int \left\{ f(\bar{x}, \bar{y}) + \frac{\partial f}{\partial x} (x - \bar{x}) + \frac{\partial f}{\partial y} (y - \bar{y}) 
+ \frac{1}{2} \frac{\partial^2 f}{\partial x^2} (x - \bar{x})^2 + \frac{1}{2} \frac{\partial^2 f}{\partial y^2} (y - \bar{y})^2 + \frac{\partial^2 f}{\partial x \partial y} (x - \bar{x})(y - \bar{y}) \right\} dp(x, y)
\]

\[
= f(\bar{x}, \bar{y}) + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} \text{var}(x) + \frac{1}{2} \frac{\partial^2 f}{\partial y^2} \text{var}(y) + \frac{\partial^2 f}{\partial x \partial y} \text{cov}(x, y) \quad (A.1)
\]

where $\bar{x}, \bar{y}$ are the means of $x, y$.

It is clear from this expression that any approximation of the system about the deterministic steady state will be inconsistent with the stochastic steady state because of the additional variance and covariance terms in the approximation above. The implication is that for different policy rules for the instruments, the variances and covariances of the variables will be different; furthermore this implies that the steady states will differ. Therefore, in order to generate approximate linear representations of a given rule, it is necessary to approximate it about its stochastic steady state. This suggests the following iterative procedure, as followed by Gertler et al. (2010), to obtain the impulse response functions for a given rule:
1. For a given \( sss \), obtain the approximate linear representation in deviations from the \( sss \), and solve for the dynamics (using the saddlepath requirement for an RE solution) and the covariance matrix of the system.

2. Use the variances and covariances to solve for the new \( sss \) by incorporating them as in (A.1) into any equation that contains an expectation of the form \( E_t f(x_{t+1}, y_{t+1}) \).

3. If this is equal to the previous \( sss \), stop. Otherwise return to 1.

The procedure for the approximation to optimal policy is very similar. Firstly one obtains the first-order conditions as described earlier; for a given set of covariances one can evaluate the \( sss \), after which one can obtain the an expression for the welfare loss using the standard LQ approach, and also the dynamics of the linear approximation about the \( sss \). After obtaining the optimal policy for this approximation, one can calculate the covariances, and if these are equal to the previous iteration’s covariances, then one can stop. If not, restart the next iteration with the current covariances.

Note that this is a very different approach from the usual second-order solution to a Ramsey problem, which would usually be computed about a deterministic steady state.

The use of covariances is standard in the finance literature, and is crucial to obtaining equilibrium relationships between asset returns. Thus in a model of macroeconomics and banking, Wickens (2011) establishes a no-arbitrage condition between the expected return and cost of capital in his equation (6).
Figure 5: Core NK Model: IRFs with Technology Shock

Figure 6: Core NK Model: IRFs with Government Spending Shock
Figure 7: Core NK Model: IRFs with Mark-up Shock

Figure 8: Core NK Model: IRFs with Preference Shock
Figure 9: Banking Model: IRFs with Technology Shock

Figure 10: Banking Model: IRFs with Government Spending Shock
Figure 11: Banking Model: IRFs with Mark-up Shock

Figure 12: Banking Model: IRFs with Preference Shock