CONSISTENT ESTIMATION IN LARGE HETEROGENEOUS PANELS WITH MULTIFACTOR STRUCTURE AND ENDOGENEITY

By

Giovanni Forchini
(University of Surrey)
Bin Jiang
(Monash University)
&
Bin Peng
(University of Technology Sydney)

DP 03/15

School of Economics
University of Surrey
Guildford
Surrey GU2 7XH, UK
Telephone +44 (0)1483 689380
Facsimile +44 (0)1483 689548
Web www.econ.surrey.ac.uk
ISSN: 1749-5075
Consistent Estimation in Large Heterogeneous Panels with Multifactor Structure and Endogeneity

G. Forchini\textsuperscript{1} \hspace{1cm} Bin Jiang\textsuperscript{2} \hspace{1cm} Bin Peng\textsuperscript{3}

University of Surrey \hspace{1cm} Monash University \hspace{1cm} University of Technology Sydney

8 June 2015

Abstract

The set-up considered by Pesaran (Econometrica, 2006) is extended to allow for endogenous explanatory variables. A class of instrumental variables estimators is studied and it is shown that estimators in this class are consistent and asymptotically normally distributed as both the cross-section and time-series dimensions tend to infinity.

Keywords: Heterogeneous panels, multifactor structure, endogeneity, instrumental variables estimator, common correlated effects estimator, common correlated effects mean group estimator

\textsuperscript{1} Address for correspondence: Giovanni Forchini, School of Economics, University of Surrey, Guildford, Surrey GU2 7XH, United Kingdom. E-mail: G.Forchini@surrey.ac.uk. This research was partially supported by Australian Research Council grant DP0985432.

\textsuperscript{2} Address for correspondence: Bin Jiang, Department of Econometrics and Business Statistics, Monash University, Caulfield East, VIC 3145, Australia. E-mail: Bin.Jiang@monash.edu.

\textsuperscript{3} Address for correspondence: Bin Peng, Economics Discipline Group, University of Technology Sydney, Australia, 2007. E-mail: Bin.Peng@uts.edu.au.
1. Introduction

Pesaran (2006) has introduced a very popular panel data model with cross-sectional heterogeneity exhibited through a factor structure in the errors in which the unobserved factors also affect the observed explanatory variables. The model has been extended in several directions (e.g. Bai (2009), Kapetanios, Pesaran and Yamagata (2011) and Pesaran and Tosetti (2011)) and used to analyse, for instance, health care expenditures (e.g. Baltagi and Moscone (2010) and Hauck and Zhang (2014)), energy policies (e.g. Arouri et al. (2012)) and house prices (e.g. Holly, Pesaran and Yamagata (2010)).

Economic models are often characterised by endogeneity of some of the explanatory variables, and endogeneity usually invalidates estimation methods that do not take it into account. Panel data models with endogenous variables and a factor structure in the errors have been studied by Ahn, Lee and Schmidt (2001), Harding and Lamarche (2011), Ahn, Lee and Schmidt (2013), Robertson and Sarafidis (2015) and Forchini, Jiang and Peng (2015). Ahn, Lee and Schmidt (2001, 2013) and Robertson and Sarafidis (2015) derive and study consistent estimators of the structural parameters in panel data models in which the factors are not regarded as unobservable random variables but as unknown parameters and their results do not extend to the model introduced by Pesaran (2006).

Harding and Lamarche (2011) introduce endogeneity in the model of Pesaran (2006) and propose a two steps instrumental variables procedure to estimate it. In the first step, Pesaran (2006)’s model is augmented with the cross-sectional means of the dependent variable and the independent variables affected by the shocks; in the second step, the augmented model is estimated using instrumental variables with instruments given by the cross-sectional means of the instruments, and the dependent variables not affected by the shocks. Forchini, Jiang and Peng (2015) investigate endogeneity in panel data with unobservable factors in the errors, in the explanatory variables and in the instruments when the time-dimension is fixed and have shown that classical estimators such as the two-stage least squares (TSLS) and the limited maximum likelihood estimator are inconsistent when the factor loadings in the errors and the exogenous variables are correlated given the common factors.

In this paper, we suggest a class of instrumental variables (IV) estimators for the model of Pesaran (2006) when some of the explanatory variables are endogenous. These are analog estimators
exploiting the compatibility restrictions between the structural equation and reduced form, and extend the IV estimators from a standard set-up to the model of Pesaran (2006). We show that estimators in this class are consistent and asymptotically normal as the cross-section and time-series dimensions tend to infinity.

The rest of the paper is organized as follows. Section 2 describes the model with endogenous explanatory variables, defines its reduced form and discusses their relationship. The assumptions needed to construct an asymptotic theory are stated in Section 3 and Section 4 describes the class of estimators considered. Consistency and asymptotic normality are obtained in Section 5. Section 6 investigates the small sample performance of the proposed estimators and Section 7 concludes. Proofs of all results are in the appendix.

2. Model

Consider the panel data model

\[ y_{1i} = \lambda_i' d_i + \beta_i' y_{2i} + \Theta_i' x_{1i} + u_i, \]

where \( y_{1i} \) and \( y_{2i} \) contain observations on the endogenous variables, \( d_i \) is a vector of observed common effects, \( x_{1i} \) is a vector of observed unit-specific exogenous regressors. In equation (1) and in the rest of the paper, the brackets appearing under the vectors or matrices the first time they are used denote their dimensions. In order to introduce a multifactor structure we consider the reduced form

\[ y_i = \begin{pmatrix} y_{1i} \\ y_{2i} \end{pmatrix} = \begin{pmatrix} \alpha_i' \\ \alpha_{2i}' \end{pmatrix} \begin{pmatrix} \pi_{1i} \\ \Pi_{12i} \end{pmatrix} \begin{pmatrix} \xi_{1i} \\ \xi_{2i} \end{pmatrix} + \begin{pmatrix} x_{1i} \\ x_{2i} \end{pmatrix} + \begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix}, \]

where \( x_{2i} \) denotes a vector of observed instruments. We assume that the reduced form errors have a multifactor error structure of the form

\[ \begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix} = \begin{pmatrix} \gamma_{1i} \\ \gamma_{2i} \end{pmatrix} f_i + \begin{pmatrix} e_{1i} \\ e_{2i} \end{pmatrix}, \]
in which \( f_i \) is a vector of unobserved common effects, \( \gamma_{1i} \) and \( \gamma_{2i} \) are factor loadings and \( \varepsilon_{1it} \) and \( \varepsilon_{2it} \) are errors which are independent of \( f_i \), \( d_i \), \( x_{1it} \) and \( x_{2it} \) for each \( i \) and \( t \). Moreover,

\[
\begin{align*}
X_{it} &= \begin{pmatrix} x_{1it} \\ x_{2it} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{1i} \\ \mathbf{A}_{2i} \end{pmatrix}_{(k \times m)} d_i + \begin{pmatrix} \mathbf{\Gamma}_{1i} \\ \mathbf{\Gamma}_{2i} \end{pmatrix}_{(k \times m)} f_i + \begin{pmatrix} \mathbf{v}_{1it} \\ \mathbf{v}_{2it} \end{pmatrix}_{(k \times l)}
\end{align*}
\]

where \( \mathbf{A}_{1i} \), \( \mathbf{A}_{2i} \), \( \mathbf{\Gamma}_{1i} \) and \( \mathbf{\Gamma}_{2i} \) are factor loading matrices and \( \mathbf{v}_{1it} \) and \( \mathbf{v}_{2it} \) are the specific components of \( x_{1it} \) and \( x_{2it} \). The terms \( \mathbf{v}_{1it} \) and \( \mathbf{v}_{2it} \) are independent of \( d_i \) and \( f_i \).

Notice that (2), (3) and (4) form a multivariate version of the model of Pesaran (2006). Notice also that the factors affect all exogenous variables including the instruments and that we allow for random coefficients and for the instruments to depend on the common factors.

Compatibility of the structural equation (1) and the reduced form (2) implies that

\[
\begin{align*}
\pi_{1it} &= \Pi_{1it} \beta_i \\
\alpha_{it} &= \alpha_i \beta_i + \lambda_i \\
\pi_{1it} &= \Pi_{1it} \beta_i + \theta_i
\end{align*}
\]

and

\[
\begin{align*}
u_{it} &= e_{1it} - \beta^* e_{2it} = (\gamma_{1it} - \gamma_{2it}) f_i + \varepsilon_{1it} - \beta^* e_{2it}.
\end{align*}
\]

The latter expression shows that the structural error \( u_q \) has a factor structure with factor loadings \( \gamma_{1it} - \gamma_{2it} \). The relationship in (5) expresses the over-identifying restrictions for each individual structural parameter \( \beta_i \), and the vector of coefficients \( \beta_i \) is identified if and only if \( \Pi_{2it} \) has rank \( p \leq k_2 \); equation (6) defines the factor loadings associated with the observed common factors in terms of the reduced form factor loadings, and equation (7) defines the vector of parameters \( \gamma_i \) in terms of the reduced form parameters.

3. Assumptions and notation

We make the same assumptions as Pesaran (2006), apart from Assumption 4 which needs to be slightly modified. We use \( \|\| \) to denote the Frobenius norm as in Pesaran (2006).
Assumption 1. The vector of common effects $g_t = (d_t', f_t')'$ is covariance stationary with absolute summable autocovariances, and is distributed independently of the unit-specific errors $(e_{it}, e_{is})'$ and $(v_{it}, v_{is})'$ for all $i$, $t$ and $s$.

Assumption 2. The reduced form errors $e_it = (e_{it}, e_{is})'$ and $v_it = (v_{it}, v_{is})'$ are independent for all $i$, $t$ and $s$. Moreover for each $i$, $e_it$ and $v_it$ follow linear stationary processes with absolute summable autocovariances, $e_it = \sum_{l=0}^{k_i} A_{il} z_{it-l}$ and $v_it = \sum_{l=0}^{k_i} S_{il} v_{it-l}$, where $(z_{it}, v_{it})'$ are $(k_i + k_s + p+1) \times 1$ vectors of identically, independently distributed random variables with mean zero, identity covariance matrix, and finite fourth order cumulants. In particular they are such that the covariance matrices of $e_it = (e_{it}, e_{is})'$ and $v_it = (v_{it}, v_{is})'$ are uniformly bounded.

Assumption 3. The factor loadings $(\gamma_{it}, \gamma_{is}, \Gamma_{it}, \Gamma_{is})$ are independently and identically distributed across $i$, and of the specific errors $e_it = (e_{it}, e_{is})'$ and $v_it = (v_{it}, v_{is})'$, the common factors $g_it = (d_t', f_t')'$ and they have finite means $E(\gamma_{it}, \gamma_{is}, \Gamma_{it}, \Gamma_{is}) = (\gamma_t, \gamma_s, \Gamma_t, \Gamma_s)$ and covariance matrices that are uniformly bounded.

Assumption 4. (a) The reduced form parameter $\Pi_i = \begin{pmatrix} \pi_{1i} & \Pi_{12i} \\ \Pi_{21i} & \pi_{22i} \end{pmatrix}$ follows the random coefficient model $\Pi_i = \Pi + \Pi_i$, where $\Pi = \begin{pmatrix} \pi_{1i} & \Pi_{12i} \\ \Pi_{21i} & \pi_{22i} \end{pmatrix}$, vec$(\Pi_i) - iid (0, \Omega_{\Pi})$, $\|\Pi\|$ and $\|\Omega_{\Pi}\|$ are uniformly bounded and the deviation $\Pi_i$ is distributed independently of $\gamma_i, \Gamma_i, e_it, v_it$ and $g_it$ for all $i, j, t$.
(b) The structural parameters $\beta_i$ and $\gamma_i$ follow the random coefficient models $\beta_i = \beta + \tilde{\beta}_i$ and $\theta_i = \theta + \tilde{\theta}_i$ where vec$(\tilde{\beta}_i) - iid (0, \Omega_{\beta})$, vec$(\tilde{\theta}_i) - iid (0, \Omega_{\theta})$, $\|\beta\|$, $\|\theta\|$, $\Omega_{\beta}$ and $\Omega_{\theta}$ are uniformly...
bounded and the deviations $\tilde{\beta}_i$ and $\tilde{\theta}_i$ are distributed independently of $\gamma_j$, $\Gamma_j$, $\varepsilon_\rho$, $\nu_\rho$, and $g_i$, for all $i, j, t$.

Let $y^{(j)}_a$ be the $j$-th component of $y_a$, $j = 1, \ldots, p + 1$, and $z^{(j)}_{at} = (y^{(j)}_a, x^*_a)'$, and $\Sigma^{(j)} = \sum_{t=1}^{N} \omega_t z^{(j)}_{at}$.

Moreover, let $D$ be the $T \times n$ matrix of observations on $d_t$ and $Z^{(j)}_{at}$ be the $T \times (k_t + k_{z_t} + 1)$ matrix of observations on $x_t$, and $y^{(j)}_t$ be the $T \times (p + 1)$ matrix of observations on the exogenous variables $x^*_a$. Finally let

$$\tilde{H}^{(j)}_{at} = (D, Z^{(j)}_{at})$$

and $\tilde{H}^{(j)}_{at} = I_T - \tilde{H}^{(j)}_{at} \left( \tilde{H}^{(j)}_{at}, \tilde{H}^{(j)}_{at} \right)^{-1}$ denotes the generalized inverse of $\tilde{H}^{(j)}_{at}$.

Similarly, the $j$-th components of $\varepsilon_a = (\varepsilon_{1a}, \varepsilon_{2a})'$ and of $\gamma = (\gamma_1, \gamma_2)$ are denoted respectively by $\varepsilon^{(j)}_a$ and $\gamma^{(j)}$. Finally, $\varepsilon_i^{(j)}$ denotes the $T \times (p + 1)$ matrix of the errors $\varepsilon_a$.

**Assumption 5.** (a) $\frac{1}{T} X_i'M_iX_i$ and $\frac{1}{T} X_i'M_iX_i$ are non-singular for each $j = 1, \ldots, p + 1$, where $M_i = I_T - G (G' G)^{-} G'$ and $G$ is the $T \times (n + m)$ matrix of observations on $g_i = (d_t, f_t)'$, and $(G' G)^{-}$ denotes the generalized inverse of $G' G$; (b) $\frac{1}{T} \sum_{t=1}^{N} \omega_t X_i'M_i^{(j)}X_i$ is non-singular for each $j = 1, \ldots, p + 1$ and for scalar weights $\omega_t$ satisfying $\omega_t = O \left( \frac{1}{N} \right)$, $\sum_{t=1}^{N} \omega_t = 1$ and $\sum_{t=1}^{N} |\omega_t| < K$ where $K$ is a finite constant.

Assumptions 1, 2, 3 and 5 are the same assumptions used by Pesaran (2006) and they are fully discussed in that work. Assumption 4 is different from Assumption 4 of Pesaran (2006) because the model we consider involves structural and reduced form parameters for both of which we impose random coefficient structures. We discuss what this entails briefly.
Using the compatibility conditions between structural equation and reduced form (5), (6) and (7), we can write

\[ \Pi_i = \begin{bmatrix} \theta_i & \pi_{12i} \\ 0 & \Pi_{22i} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\beta_i & I_p \end{bmatrix}^{-1} = \begin{bmatrix} \theta_i & \pi_{12i} \\ 0 & \Pi_{22i} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \beta_i & I_p \end{bmatrix} \]

Due to Assumption 4, this can be rewritten as

\[ \Pi + \hat{\Pi}_i = \begin{bmatrix} \theta & \pi_{12i} \\ 0 & \Pi_{22i} \end{bmatrix} + \hat{\Pi}_i \begin{bmatrix} \hat{\beta}_i & \pi_{12i} \\ 0 & \hat{\Pi}_{22i} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \beta & I_p \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \hat{\beta}_i & 0 \end{bmatrix} \]

Thus, by taking expectations of both sides of the equation above we must have

\[ (9) \quad \Pi = \begin{bmatrix} \theta & \pi_{12i} \\ 0 & \Pi_{22i} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \beta & I_p \end{bmatrix}, \]

and

\[ (10) \quad \hat{\Pi}_i = \begin{bmatrix} \hat{\beta}_i + \hat{\pi}_{12i} & \hat{\pi}_{12i} \\ \hat{\Pi}_{22i} + \hat{\beta}_i & \hat{\Pi}_{22i} \end{bmatrix} \begin{bmatrix} \hat{\beta}_i \\ \hat{\Pi}_{22i} \end{bmatrix} \]

Notice that the condition \( E(\hat{\Pi}_i) = 0 \) holds only if \( E(\hat{\pi}_{12i}) = 0 \) and \( E(\hat{\Pi}_{22i}) = 0 \) (cf. Kelejian (1974)). Therefore, Assumption 4, entails that \( \hat{\beta}_i \) is not correlated to \( \hat{\pi}_{12i} \) and \( \hat{\Pi}_{22i} \). Notice also that (9) expresses the standard over-identifying restrictions which in this case apply to the means of the coefficients rather than the coefficients themselves.

Finally, the notation \( (N,T) \to \infty \) indicates that \( N \) and \( T \) tend to infinity jointly.

4. Estimators

Provided \( T \) is sufficiently large we can estimate the matrix \( \Pi_i \) for each unit \( i \) by aggregating the common correlated effects (CCE) estimators for the slope coefficients in the reduced form for each individual endogenous variable:

\[ (11) \quad \hat{\Pi}_i = \left( X_i^\prime \hat{\mathbf{M}}_n^{(1)} X_i \right)^{-1} X_i^\prime \hat{\mathbf{M}}_n^{(1)} y_i^{(1)} \ldots \left( X_i^\prime \hat{\mathbf{M}}_n^{(p+1)} X_i \right)^{-1} X_i^\prime \hat{\mathbf{M}}_n^{(p+1)} y_i^{(p+1)} \right]. \]
One possible choice for estimating $\Pi$ is the common correlated effects mean group (CCEMG) estimator

$$\hat{\Pi}_{MG} = \left[ \frac{1}{N} \sum_{i=1}^{N} \left( X_i, \tilde{M}_\omega^{(i)} X_i \right)^{-1} X_i, \tilde{M}_\omega^{(i)} y_i^{(i)} \right] \cdots \left( \frac{1}{N} \sum_{i=1}^{N} \left( X_i, \tilde{M}_\omega^{(p+1)} X_i \right)^{-1} X_i, \tilde{M}_\omega^{(p+1)} y_i^{(p+1)} \right).$$

An alternative is the common correlated effects pooled (CCEP) estimator:

$$\hat{\Pi}_P = \left[ \left( \sum_{i=1}^{N} \omega_i X_i, \tilde{M}_\omega^{(i)} X_i \right)^{-1} \sum_{i=1}^{N} \omega_i X_i, \tilde{M}_\omega^{(i)} y_i^{(i)} \right] \cdots \left( \sum_{i=1}^{N} \omega_i X_i, \tilde{M}_\omega^{(p+1)} X_i \right)^{-1} \sum_{i=1}^{N} \omega_i X_i, \tilde{M}_\omega^{(p+1)} y_i^{(p+1)} \right],$$

where we set the aggregating weights equal to the pooling weights (cf. Pesaran (2006, p. 986)). Notice that the columns of (12) and (13) are the CCEMG and CCEP estimators applied to the reduced form for each individual endogenous variables.

Partition the CCEMG and CCEP estimators of the reduced form parameters conformably to $\Pi_i$ in equation (2). The panel instrumental variables (IV) estimators based on, respectively, the CCEMG and CCEP estimators of the reduced form parameters are

$$\hat{\beta}_{IV - MG} = \left( \hat{\Pi}_{22MG}, \hat{H} \hat{\Pi}_{22MG} \right)^{-1} \left( \hat{\Pi}_{22MG}, \hat{H} \hat{\pi}_{21MG} \right)$$

and

$$\hat{\beta}_{IV - P} = \left( \hat{\Pi}_{22P}, \hat{H} \hat{\Pi}_{22P} \right)^{-1} \left( \hat{\Pi}_{22P}, \hat{H} \hat{\pi}_{21P} \right),$$

where $\hat{H}$ is any positive definite random matrix converging in probability to a positive definite matrix $H$. Similarly, we can estimate the coefficients of the exogenous variables as

$$\hat{\theta}_{IV - MG} = \hat{\pi}_{11MG} - \hat{\Pi}_{12MG} \hat{\beta}_{IV - MG},$$

and

$$\hat{\theta}_{IV - P} = \hat{\pi}_{11P} - \hat{\Pi}_{12P} \hat{\beta}_{IV - P}.$$

Estimators analogous to these have been studied by Forchini, Jiang and Peng (2015) for the case where the reduced form parameters are estimated using OLS and $T$ is fixed. They find that the classical panel TSLS and LIML are not consistent if the factor loadings in the errors and the exogenous variables are dependent conditional on the factors as in the case considered here.
It is important to compare the estimators proposed above with the IV estimator proposed by Harding and Lamarche (2011). The structural equation (1) is augmented using the cross-sectional means $\bar{y}_i$, $\bar{y}_2$ and $\bar{X}_1$

$$y_{ii} = y_{2i}/\beta + \bar{y}_2/\eta_2 + D\lambda_i + X_i/\theta + \bar{X}_i/\eta_3 + \bar{y}_i/\eta_1 + \bar{u}_i,$$

Notice that the endogenous variables are $y_{2i}$ and $\bar{y}_2$. The two stage procedure is implemented as follows:

1) $[y_{2i}, \bar{y}_2]$ is regressed on $d_i$, $\bar{X}_2$ and the constant to obtain the predicted values

$$\hat{y}_{2i}, \hat{\bar{y}}_2 = P_{[D, \bar{X}_2]}[y_{2i}, \bar{y}_2];$$

2) $[\hat{y}_{2i}, \hat{\bar{y}}_2]$ is replaced in the augmented equation

$$y_{ii} = P_{[D, \bar{X}_2]}y_{2i}/\beta + P_{[D, \bar{X}_2]}\bar{y}_2/\eta_2 + D\lambda_i + X_i/\theta + \bar{X}_i/\eta_3 + \bar{y}_i/\eta_1 + \bar{u}_i,$$

which is estimated using OLS.

In order to estimate $\begin{pmatrix} \beta \\ \theta \end{pmatrix}$ we premultiply the above equation by $M_{[\eta_{i, \bar{y}_2}, \eta_{i, \bar{X}_2}, \Pi]}$ to obtain

$$M_{[\eta_{i, \bar{y}_2}, \eta_{i, \bar{X}_2}, \Pi]}y_{ii} = M_{[\eta_{i, \bar{y}_2}, \eta_{i, \bar{X}_2}, \Pi]}(P_{[D, \bar{X}_2]}y_{2i}, X_i)(\hat{\beta}, \hat{\theta})$$

Adding up over $i$ we have

$$\sum_{i=1}^{N}(P_{[D, \bar{X}_2]}y_{2i}, X_i)M_{[\eta_{i, \bar{y}_2}, \eta_{i, \bar{X}_2}, \Pi]}y_{ii} = \sum_{i=1}^{N}(P_{[D, \bar{X}_2]}y_{2i}, X_i)M_{[\eta_{i, \bar{y}_2}, \eta_{i, \bar{X}_2}, \Pi]}(P_{[D, \bar{X}_2]}y_{2i}, X_i)(\hat{\beta}, \hat{\theta})$$

or

$$\begin{pmatrix} \hat{\beta} \\ \hat{\theta} \end{pmatrix} = \left(\sum_{i=1}^{N}(P_{[D, \bar{X}_2]}y_{2i}, X_i)M_{[\eta_{i, \bar{y}_2}, \eta_{i, \bar{X}_2}, \Pi]}(P_{[D, \bar{X}_2]}y_{2i}, X_i)\right)^{-1}\sum_{i=1}^{N}(P_{[D, \bar{X}_2]}y_{2i}, X_i)M_{[\eta_{i, \bar{y}_2}, \eta_{i, \bar{X}_2}, \Pi]}y_{ii}$$

Notice that the IV estimator proposed by Harding and Lamarche (2011) can be thought of as the CCEP estimator of

$$y_{ii} = P_{[D, \bar{X}_2]}y_{2i}/\beta + P_{[D, \bar{X}_2]}\bar{y}_2/\eta_2 + D\lambda_i + X_i/\theta + \bar{X}_i/\eta_3 + \bar{y}_i/\eta_1 + \bar{u}_i.$$

Some of the estimators in the class we considered can also be interpreted as two stage procedures. Suppose that $\Pi_i = \Pi$, $\beta_i = \beta$ and $\theta_i = \theta$ as in Harding and Lamarche (2011). Then,
1) The reduced form for $y_2$ is estimated using (12) or (13) and the predicted value for $y_2$ is obtained as $\hat{y}_{2t} = \hat{\beta}_2' x_{it} + \hat{\Pi}_{12}' x_{it} + \hat{\Pi}_{22}' x_{it}.

2) The predicted values for $y_2$ are replaced into (1), which is also augmented as in Pesaran (2006) to eliminate cross sectional dependence, $y_{it} = D\lambda_i + \hat{\gamma}_{2i}\beta + x_{it}\theta + \bar{X}_{iu}\eta_i + \bar{y}_{iu}\eta_j + \tilde{u}_i$. The last equation is then estimated using OLS.

It is easy to show that this yields an estimator of the form (14) or (15) with

$$(18) \hat{\Pi}^e = \sum_{i=1}^{N} X_{2i}' M_{[D,\bar{x},\bar{y}]} X_{2i} - \sum_{i=1}^{N} X_{2i}' M_{[D,\bar{x},\bar{y}]} \left( \sum_{i=1}^{N} X_{iu}' M_{[D,\bar{x},\bar{y}]} X_{iu} \right)^{-1} \sum_{i=1}^{N} X_{iu}' M_{[D,\bar{x},\bar{y}]} X_{2i}.$$ 

Notice that the first stage of the procedure of Harding and Lamarche (2011) is different from that in our procedure because we take into account the fact that the instruments may also be affected by the common shocks.

5. Main results

First we show that the CCEMG and the CCEP estimators of the reduced form coefficients are consistent and asymptotically normal when both the cross-section and the time dimensions go to infinity. Notice that the reduced form is just a multivariate linear model, so that the results of Pesaran (2006) apply with obvious modification.

**Proposition 1.** Given Assumptions 1-5,

(a) the CCEMG estimator of the reduced form parameter $\Pi$ is consistent and asymptotically normal with $\sqrt{N} vec(\hat{\Pi}_{MG} - \Pi) \rightarrow^d N(0, \Sigma_{MG})$ as $(N,T) \rightarrow \infty$ and the asymptotic covariance matrix $\Sigma_{MG}$ can be estimated by $\hat{\Sigma}_{MG} = \frac{1}{N-1} \sum_{i=1}^{N} \left( vec(\hat{\Pi}_i) - vec(\hat{\Pi}_{MG}) \right) \left( vec(\hat{\Pi}_i) - vec(\hat{\Pi}_{MG}) \right)'$;

(b) the CCEP estimator of the reduced form parameter $\Pi$ is consistent and asymptotically normal with $\left( \sum_{i=1}^{N} \sigma_i^2 \right)^{-\frac{1}{2}} vec(\hat{\Pi}_p - \Pi) \rightarrow^d N(0, \Sigma_p)$ as $(N,T) \rightarrow \infty$ and the asymptotic covariance matrix $\Sigma_p$ can be estimated by $\hat{\Sigma}_p = \left( \sum_{i=1}^{N} \sigma_i^2 \right)^{-\frac{1}{2}} \hat{\Phi}^{-1} \hat{R} \hat{\Phi}^{-1}$ with
\[
\Phi = \begin{bmatrix}
\sum_{i=1}^{N} \omega_i \frac{1}{T} X_i^\prime \widetilde{M}_{\omega}^{(1)} X_i & 0 \\
\vdots & \ddots & \vdots \\
0 & \sum_{i=1}^{N} \omega_i \frac{1}{T} X_i^\prime \widetilde{M}_{\omega}^{(p+1)} X_i 
\end{bmatrix},
\]

and

\[
\hat{K} = \sum_{i=1}^{N} \bar{\omega}_i^2 \begin{bmatrix}
\frac{1}{T} X_i^\prime \widetilde{M}_{\omega}^{(1)} X_i & 0 \\
\vdots & \ddots & \vdots \\
0 & \frac{1}{T} X_i^\prime \widetilde{M}_{\omega}^{(p+1)} X_i 
\end{bmatrix} \left( \text{vec}(\hat{\Pi}_i) - \text{vec}(\hat{\Pi}_p) \right) \times \\
\begin{bmatrix}
\frac{1}{T} X_i^\prime \widetilde{M}_{\omega}^{(1)} X_i & 0 \\
\vdots & \ddots & \vdots \\
0 & \frac{1}{T} X_i^\prime \widetilde{M}_{\omega}^{(p+1)} X_i 
\end{bmatrix},
\]

where \( \bar{\omega}_i = \omega_i / \sqrt{N^{-1} \sum_{i=1}^{N} \omega_i^2} \).

It follows easily from Proposition 1 that:

**Proposition 2.** Given Assumptions 1-5 and the over-identifying restrictions

(a) the CCEMG estimators of the structural parameters \( \beta \) and \( \theta \) are consistent as \( (N,T) \to \infty \), and

\[
\sqrt{N} \left( \hat{\beta}_{IV-MG} - \beta \right) \to^D \left( (1,-\beta') \otimes \left( \Pi_{22} \right) (0,I_{k}) \right) N(0,\Sigma_{MG})
\]

and

\[
\sqrt{N} \left( \hat{\theta}_{IV-MG} - \theta \right) \to^D \left( (1,-\beta') \otimes \left( \Pi_{22} \right) (0,I_{k}) \right) N(0,\Sigma_{MG}).
\]

(b) the CCEP estimators of the structural parameters \( \beta \) and \( \theta \) are consistent as \( (N,T) \to \infty \), and

\[
\sqrt{N} \left( \hat{\beta}_{IV-P} - \beta \right) \to^D \left( (1,-\beta') \otimes \left( \Pi_{22} \right) (0,I_{k}) \right) N(0,\Sigma_{P})
\]

and

\[
\sqrt{N} \left( \hat{\theta}_{IV-P} - \theta \right) \to^D \left( (1,-\beta') \otimes \left( \Pi_{22} \right) (0,I_{k}) \right) N(0,\Sigma_{P}).
\]
Therefore, if we have endogenous variables in the model of Pesaran (2006) and instruments are available, it is possible to use the CCEMG and the CCEP estimators of the reduced form parameters to estimate consistently the structural coefficients. These estimators are also asymptotically normal. Notice also that the matrices $\Sigma_{MG}$ and $\Sigma_{P}$ do not have a Kronecker product structures like in the classical instrumental variables model in a cross section. Therefore, it is not possible to choose a value of $H$ which makes the IV estimator asymptotically efficient.

6. Small sample performance

We now study the small sample performance for two of the IV estimators in the class of estimators proposed in the previous sections and compare them with existing estimators. The data generating process is described below. We choose $p = k_1 = n = 1$, $k_2 = 2$ and $m = 3$.

Factors

The factors are stationary AR(1) processes: $f_t = 0.8f_{t-1} + \text{i.i.d. } N\left(0, \Sigma_f\right)$, where $\Sigma_f$ is an $m \times m$ positive definite matrix generated at the start of the simulations from a Wishart distribution $W_m(I_m, m)$ for $t = -50, \ldots, -1, 0, 1, 2, \ldots, T$ with $f_{-50} = 0$.

Factor loadings

The factor loadings in the errors and those in the exogenous variables are allowed to be correlated. Thus, we assume that

$$\text{vec}(\gamma_{12}, \gamma_2, \Gamma_{12}, \Gamma_2) \sim \text{i.i.d. } N\left(i_{(3+k_1)k_2}, \Omega \otimes I_m\right)$$

where

$$\Omega = \begin{pmatrix}
\sigma^2_{\gamma_{12}} I_{k_1} & \rho_{\gamma_{12}} \sigma_{\gamma_{2}} \sigma_{\gamma_{12}} I_{(1+k_1)k_2} \\
\rho_{\gamma_{12}} \sigma_{\gamma_{2}} \sigma_{\gamma_{12}} I_{(1+k_1)k_2} & \sigma^2_{\gamma_{2}} I_{k_2}
\end{pmatrix}.$$
The parameter $-1 < \rho_{\gamma_{t}, \gamma_{t}} < 1$ captures the correlation between the factor loadings in the errors and those in the exogenous variables. In the simulations we let $\sigma_{ij}^{2} = \sigma_{ji}^{2} = 1$ and $\rho_{\gamma_{t}, \gamma_{t}} = 0.2, 0.8$.

Exogenous variables

The exogenous variables are

$$
\begin{pmatrix}
  x_{1t} \\
  x_{2t}
\end{pmatrix} =
\begin{pmatrix}
  A_{11} & \overset{\cdots}{0} \\
  A_{21} & \overset{\cdots}{0}
\end{pmatrix}
\begin{pmatrix}
  d_{t} \\
  f_{t}
\end{pmatrix} +
\begin{pmatrix}
  \Gamma_{11} & \overset{\cdots}{0} \\
  \Gamma_{21} & \overset{\cdots}{0}
\end{pmatrix}
\begin{pmatrix}
  v_{1t} \\
  v_{2t}
\end{pmatrix},
\tag{13}
$$

where $d_{t} = 0.5d_{t-1} + \text{i.i.d. } N(0, \Sigma_{d_{t}})$, where $\Sigma_{d_{t}}$ is generated at the start of the simulations as $W_{u}(I_{n}, n)$ for $t = -50, -1, 0, 1, 2, ..., T$ with $d_{-50} = 0$. The term $(v_{1t}, v_{2t})'$ follows a stationary process for each unit $i$ and are independent over $i$

$$
\begin{pmatrix}
  v_{1t} \\
  v_{2t}
\end{pmatrix} = 0.5
\begin{pmatrix}
  v_{1t-1} \\
  v_{2t-1}
\end{pmatrix} + \text{i.i.d. } N
\begin{pmatrix}
  0 \\
  0
\end{pmatrix},
\begin{pmatrix}
  \sigma_{v_{1i}}^{2} & \rho_{v_{1i}, v_{1j}} \sigma_{v_{1i}} \sigma_{v_{1j}} & \rho_{v_{1i}, v_{1j}} \sigma_{v_{1i}} \sigma_{v_{1j}} & \cdot & \cdot & \cdot \\
  \rho_{v_{1i}, v_{1j}} \sigma_{v_{1i}} \sigma_{v_{1j}} & \sigma_{v_{1i}}^{2} & \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{pmatrix}
\begin{pmatrix}
  1 \\
  1 \\
  \cdot \\
  \cdot \\
  \cdot \\
  \cdot
\end{pmatrix}
\begin{pmatrix}
  l_{i} \\
  l_{i}
\end{pmatrix},
\tag{14}
$$

for $t = -50, ..., 0, 1, 2, ..., T$ with $\begin{pmatrix}
  v_{1t-50} \\
  v_{2t-50}
\end{pmatrix} = 0$ and $l_{i}$ is a $1 \times k_{2}$ matrix of ones and $-1 < \rho_{v_{1i}, v_{1j}} < 1$. In the simulations we take $\sigma_{v_{1i}}^{2} = \sigma_{v_{1j}}^{2} = 1$ and $\rho_{v_{1i}, v_{1j}} = 0.5$. The components of $(A_{11}')$ are independently uniformly distributed over the interval $(−1, 2)$.

Error terms

We use the compatibility conditions to generate the structural error $u_{it}$ and the reduced form error $e_{2it}$ as follows

$$
\begin{pmatrix}
  u_{it} \\
  e_{2it}
\end{pmatrix} =
\begin{pmatrix}
  \tilde{\gamma}_{1i} \\
  \tilde{\gamma}_{2i}
\end{pmatrix}
\begin{pmatrix}
  f_{t} \\
  \tilde{e}_{2it}
\end{pmatrix},
\tag{15}
$$

where $\tilde{\gamma}_{1i} = \gamma_{1i}, \tilde{\gamma}_{1i}' = \gamma_{1i}' - \beta_{1i}' \gamma_{2i}'$, $\tilde{\gamma}_{2i} = e_{2i}$ and $\tilde{\gamma}_{1i} = e_{1i} - \beta_{1i}' \tilde{e}_{2it}$. Instead of generating the original quantities $\gamma_{1i}, \gamma_{2i}, e_{1i}$ and $e_{2i}$ we generate directly $\tilde{\gamma}_{1i}, \tilde{\gamma}_{2i}, \tilde{e}_{1it}$ and $\tilde{e}_{2it}$. The idiosyncratic errors $\begin{pmatrix}
  \tilde{e}_{1it} \\
  \tilde{e}_{2it}
\end{pmatrix}$ are generated independently over $i$ as stationary processes.
\[
\begin{pmatrix}
\hat{e}_{1t} \\
\hat{e}_{2t}
\end{pmatrix} = 0.5 \begin{pmatrix}
\hat{z}_{1,t-1} \\
\hat{z}_{2,t-1}
\end{pmatrix} + \text{i.i.d. } \mathcal{N}\left(0, \begin{pmatrix}
\sigma_{\hat{e}_1}^2 & \rho_{\hat{e}_1, \hat{e}_2} \sigma_{\hat{e}_1} \sigma_{\hat{e}_2} \\
\rho_{\hat{e}_1, \hat{e}_2} \sigma_{\hat{e}_1} \sigma_{\hat{e}_2} & \sigma_{\hat{e}_2}^2
\end{pmatrix}\right)
\]
for \( t = -50, \ldots, 0, 1, 2, \ldots, T \) and \( \begin{pmatrix}
e_{10} \\
e_{20}
\end{pmatrix} = 0 \). Notice that the square of the quantity \(-1 < \rho_{\hat{e}_1, \hat{e}_2} < 1\) measures the amount of classical endogeneity when no shock is present. We let \( \sigma_{\hat{e}_1}^2 = \sigma_{\hat{e}_2}^2 = 1 \) and choose \( \rho_{\hat{e}_1, \hat{e}_2} = 0.2, 0.8 \) to reflect moderate and strong endogeneity respectively.

Reduced form for \( y_2 \)

The reduced form for \( y_2 \) is generated as follows:

\[
y_{2t} = \alpha_{2t}' d_t + \left[ \Pi_{12}, \Pi_{22} \right] \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} + e_{2t}
\]
where the component of \( \alpha_{2t} \) are generated independently from a uniform distribution over the interval \((-1, 2)\). The random coefficients satisfies

\[
\left[ \Pi_{12}, \Pi_{22} \right] \sim \left[ 0, \sqrt{c} I_{k_1 \times k_2} \right] + \mathcal{N}\left(0, I_p \otimes \left(\frac{1}{1+k_2} \Sigma_{\Pi}\right)\right)
\]
where \( \Sigma_{\Pi} \) is generated once at the beginning of the simulations as \( \Sigma_{\Pi} = W_{1+k_2} \left(1+k_2, I_{1+k_2}\right) \). The parameter \( c \) measures the strength of the instruments in the absence of shocks. Moreover, we set \( c = 0.8 \) so that relatively weak instruments are considered.

Structural equation

Finally, the structural equation is generated as follows

\[
y_{1t} = \lambda_t' d_t + \beta_t y_{2t} + x_{1t} + u_t,
\]
where \( \beta_t = 1 + i.i.d. \mathcal{N}(0, 1) \) and \( \lambda_t \) are independently uniformly distributed over the interval \((1, 2)\).

We consider two types of estimators in our class corresponding to different choices of \( \hat{H} \). For the IV-MG and IV-P estimators, we take \( \hat{H} \) equal to identity matrix. For the TSLS-MG and TSLS-P estimators, \( \hat{H} \) is defined in (18). We compare these estimators with the OLS (which ignores both
common shocks and endogeneity), the CCEMG and the CCEP estimators of Pesaran (2006) (which ignore the classical endogeneity), the panel TSLS of the structural parameters (which does not take the common shocks into account), and the estimator of Harding and Lamarche (2011) denoted by IV-HL. Notice that by neglecting either classical endogeneity or the endogeneity induced by the shocks, the OLS, CCEMG, CCEP and TSLS estimators are not consistent. The IV-HL estimator is also inconsistent when the shocks affect the instruments.

The outcomes of the simulation experiments based on 10000 replications are summarized in Tables 1 to 4. The tables capture different strengths of correlation between the factor loadings in the errors and in the exogenous variables and different levels of endogeneity. The IV-MG, IV-P, TSLS-MG and TSLS-P have very small bias independently of the degree of endogeneity and correlation between the factor loadings. The IV-HL appears to have small bias when the correlation between the factor loadings is small but can have very large bias when this is not the case. This is not unexpected since the IV-HL implicitly assumes that the instruments do not depend on the common factors. The OLS, TSLS, CCEMG and CCEP estimators can be seriously biased as one would expect since they have not been devised for this particular model.

The performance of the IV-MG, IV-P, TSLS-MG and TSLS-P estimators in terms of MSE is not as impressive as the one in terms of bias. This is partly due to the choice of the matrix \( \hat{H} \) or to the slow convergence towards the asymptotic limits. The IV-HL estimator seems to perform well in terms of MSE when the sample size and the correlation among the factors are small. The OLS, CCEMG and CCEP estimators may have very low MSE. It often happens in structural equations models that an estimator like the OLS estimator that does not take into account simultaneity and is thus inconsistent has very low MSE due to the fact that it converges quickly as the sample size grows to infinity (and thus has low variability) and the bias is small for certain combinations of the parameters. We suspect this is what happens here.

In order to see what the distributions of the various estimators look like, we plot the densities of the difference between the estimator and the true parameter \( \beta \) in Figures 1 to 4 for \( N=T=100 \) in the set-up described above. The densities of the IV-MG and TSLS-MG estimators are almost identical in
all situations. The same applies to the densities of the IV-P and TSLS-P estimators. They are correctly centred around zero. The densities of the other estimators are shifted to the right and their relative positions change depending on the situations considered.

7. Conclusions

The paper has extended the model of Pesaran (2006) to allow for endogenous explanatory variables and for the instruments influenced by the common shocks. We have exploited the compatibility conditions between structural equation and reduced form to construct a class of estimators analogous to the classical IV estimator in cross-sectional structural equations models. Since the reduced form parameters can be estimated consistently using the CCEMG and the CCEP estimators, the structural parameters can also be consistently estimated. Moreover, since the estimators in our class have asymptotically normal distribution, tests and confidence intervals can be easily constructed. A simulation experiment, suggests that in small samples the IV estimators proposed perform reasonably well especially in terms of bias.

Appendix: proofs

Proof of Proposition 1

(a) Notice that

\[
\sqrt{N} \text{vec}(\hat{\Pi}_{MG} - \Pi) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \text{vec}(\tilde{\Pi}_i) + \left[ \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left( \frac{1}{T} X_i', \tilde{M}_m^{(i)} X_i \right)^{-1} \left( \frac{1}{T} X_i', \tilde{M}_m^{(i)} \varepsilon_i^{(i)} \right) \right] + \left[ \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left( \frac{1}{T} X_i', \tilde{M}_s^{(i)} X_i \right)^{-1} \left( \frac{1}{T} X_i', \tilde{M}_s^{(i)} \varepsilon_i^{(i)} \right) \right] + \ldots + \left[ \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left( \frac{1}{T} X_i', \tilde{M}_s^{(i+1)} X_i \right)^{-1} \left( \frac{1}{T} X_i', \tilde{M}_s^{(i+1)} \varepsilon_i^{(i+1)} \right) \right].
\]
Either equation (56) or the equation just above (60) of Pesaran (2006) can be obtained for each component of the matrices above. Thus, as \((N,T) \to \infty\), \(\sqrt{N} \text{vec} \left( \hat{\Pi}_{MG} - \Pi \right) \to N(0, \Sigma_{MG})\) and the asymptotic covariance matrix \(\Sigma_{MG}\) can be estimated using \(\hat{\Sigma}_{MG}\) specified in Proposition 1.

(b) Similarly, we can write

\[
\text{vec} \left( \hat{\Pi}_p - \Pi \right) = \begin{bmatrix}
\left( \sum_{i=1}^N \omega_i \frac{1}{T} X_i', \tilde{M}_{\omega}^{(1)} X_i \right)^{-1} & 0 \\
0 & \left( \sum_{i=1}^N \omega_i \frac{1}{T} X_i', \tilde{M}_{\omega}^{(p+1)} X_i \right)^{-1} \\
\left( \sum_{i=1}^N \omega_i \frac{1}{T} X_i', \tilde{M}_{\omega}^{(1)} F_{\gamma_i}^{(1)} \right) & \left( \sum_{i=1}^N \omega_i \frac{1}{T} X_i', \tilde{M}_{\omega}^{(p+1)} e_i^{(p+1)} \right) \\
\left( \sum_{i=1}^N \omega_i \frac{1}{T} X_i', \tilde{M}_{\omega}^{(p+1)} F_{\gamma_i}^{(p+1)} \right) + \left( \sum_{i=1}^N \omega_i \frac{1}{T} X_i', \tilde{M}_{\omega}^{(p+1)} e_i^{(p+1)} \right)
\end{bmatrix}
\]

Proceeding as in the proof of Theorem 3 of Pesaran (2006), we can conclude that every component of \(\text{vec} \left( \hat{\Pi}_p - \Pi \right)\) satisfies (B.5) of Pesaran (2006), so that asymptotic normality follows by taking \((N,T) \to \infty\). \(\square\)

**Proof of Proposition 2.**

(a) For the CCEMG estimator, notice that \(\hat{\beta}_{IV-MG} = \left( \hat{\Pi}_{22MG}', \hat{H} \hat{\Pi}_{22MG} \right)^{-1} \left( \hat{\Pi}_{22MG}' H \hat{\pi}_{21MG} \right)\) is a continuous function of \(\hat{\Pi}_{22MG}\), \(\hat{\pi}_{21MG}\) and \(\hat{H}\). By construction \(\hat{H} \to^p H\) and by Proposition 1, \(\hat{\Pi}_{22MG} \to^p \Pi_{22}\) and \(\hat{\pi}_{21MG} \to^p \pi_{21} = \Pi_{22} \beta\), thus \(\hat{\beta}_{IV-MG} \to^p \left( \Pi_{22}' H \Pi_{22} \right)^{-1} \left( \Pi_{22}' H \Pi_{22} \beta \right) = \beta\). It follows from Proposition 1 and consistency of \(\hat{\beta}_{IV-MG}\) that

\[
\hat{\beta}_{IV-MG} = \hat{\pi}_{11MG} - \hat{\Pi}_{12MG} \hat{\beta}_{IV-MG} \to^p \pi_{11} - \Pi_{12} \beta = \theta.
\]
Notice that if the over-identifying restrictions hold,

\[
\sqrt{N} (\hat{\beta}_{IV-MG} - \beta) = \left( \hat{\Pi}_{22MG}' \hat{H} \hat{\Pi}_{22MG} \right)^{-1} \left( \hat{\Pi}_{22MG}' \hat{H} \right) \sqrt{N} \left( \hat{\sigma}_{21MG}, \hat{\Pi}_{22MG} \right) \left( \frac{1}{-\beta} \right) \\
= \left( \hat{\Pi}_{22MG}' \hat{H} \hat{\Pi}_{22MG} \right)^{-1} \left( \hat{\Pi}_{22MG}' \hat{H} \right) \sqrt{N} \left( \hat{\sigma}_{21MG}, \hat{\Pi}_{22MG} \right) - \left( \pi_{21}, \pi_{22} \right) \left( \frac{1}{-\beta} \right),
\]

so that

\[
\sqrt{N} (\hat{\beta}_{IV-MG} - \beta) = \left( (1, -\beta') \otimes \left( \hat{\Pi}_{22MG}' \hat{H} \hat{\Pi}_{22MG} \right)^{-1} \left( \hat{\Pi}_{22MG}' \hat{H} \right) \left( 0, I_{k_i} \right) \right) \sqrt{N} \left( \text{vec} \left( \hat{\Pi}_{MG} \right) - \text{vec} \left( \Pi \right) \right)
\]

and the first part of (a) follows. For the estimator of the exogenous variables, using the over-identifying restrictions and (19) we can write

\[
\sqrt{N} (\hat{\theta}_{IV-MG} - \theta) = \sqrt{N} \left( (\hat{\pi}_{11MG}, \hat{\Pi}_{12MG}) - (\pi_{11}, \Pi_{12}) \right) \left( \frac{1}{-\beta} \right) - \hat{\Pi}_{12MG} \sqrt{N} \left( \hat{\beta}_{IV-MG} - \beta \right) \\
= \left( (I_{k_i}, 0) - \hat{\Pi}_{12MG} \left( \hat{\Pi}_{22MG}' \hat{H} \hat{\Pi}_{22MG} \right)^{-1} \left( \hat{\Pi}_{22MG}' \hat{H} \right) \left( 0, I_{k_i} \right) \right) \sqrt{N} \left( \text{vec} \left( \hat{\Pi}_{MG} \right) - \Pi \right) \left( \frac{1}{-\beta} \right).
\]

Noticing that

\[
\sqrt{N} (\hat{\theta}_{IV-MG} - \theta) = \left( (1, -\beta') \otimes \left( \left( I_{k_i}, 0 \right) - \hat{\Pi}_{12MG} \left( \hat{\Pi}_{22MG}' \hat{H} \hat{\Pi}_{22MG} \right)^{-1} \left( \hat{\Pi}_{22MG}' \hat{H} \right) \left( 0, I_{k_i} \right) \right) \right) \sqrt{N} \text{vec} \left( \hat{\Pi}_{MG} - \Pi \right)
\]

the result follows.

(b) The same argument applies to the CCEP estimators of the coefficients of both the endogenous and the exogenous variables. \( \square \)

References


### Tables and figures

<table>
<thead>
<tr>
<th>T</th>
<th>N</th>
<th>IV-MG</th>
<th>IV-P</th>
<th>TSLS-MG</th>
<th>TSLS-P</th>
<th>OLS</th>
<th>CCEMG</th>
<th>CCEP</th>
<th>TSLS</th>
<th>IV-HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>25</td>
<td>-0.0192</td>
<td>-0.0263</td>
<td>0.0058</td>
<td>0.0036</td>
<td>0.0965</td>
<td>0.0610</td>
<td>0.0262</td>
<td>0.1527</td>
<td>0.0654</td>
</tr>
<tr>
<td>50</td>
<td>0.0375</td>
<td>-0.0329</td>
<td>-0.0264</td>
<td>-0.0212</td>
<td>0.0942</td>
<td>0.0638</td>
<td>0.0151</td>
<td>0.1465</td>
<td>0.0683</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0148</td>
<td>-0.0165</td>
<td>-0.0037</td>
<td>-0.0015</td>
<td>0.0963</td>
<td>0.0730</td>
<td>0.0238</td>
<td>0.1722</td>
<td>0.0733</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-0.0114</td>
<td>-0.0096</td>
<td>-0.0029</td>
<td>-0.0017</td>
<td>0.0893</td>
<td>0.0567</td>
<td>0.0198</td>
<td>0.1456</td>
<td>0.0711</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0233</td>
<td>-0.0070</td>
<td>0.0056</td>
<td>0.0174</td>
<td>0.0936</td>
<td>0.0601</td>
<td>0.0237</td>
<td>0.1509</td>
<td>0.0848</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0285</td>
<td>-0.0273</td>
<td>-0.0173</td>
<td>-0.0150</td>
<td>0.0957</td>
<td>0.0530</td>
<td>0.0203</td>
<td>0.1557</td>
<td>0.0688</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0065</td>
<td>-0.0111</td>
<td>0.0024</td>
<td>0.0087</td>
<td>0.0970</td>
<td>0.0718</td>
<td>0.0238</td>
<td>0.1656</td>
<td>0.0776</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-0.0048</td>
<td>-0.0090</td>
<td>0.0001</td>
<td>-0.0034</td>
<td>0.0938</td>
<td>0.0646</td>
<td>0.0197</td>
<td>0.1594</td>
<td>0.0718</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0268</td>
<td>-0.0164</td>
<td>-0.0018</td>
<td>0.0067</td>
<td>0.0989</td>
<td>0.0718</td>
<td>0.0304</td>
<td>0.1707</td>
<td>0.0754</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0170</td>
<td>-0.0154</td>
<td>-0.0068</td>
<td>-0.0045</td>
<td>0.0934</td>
<td>0.0619</td>
<td>0.0178</td>
<td>0.1550</td>
<td>0.0729</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0132</td>
<td>-0.0111</td>
<td>-0.0070</td>
<td>-0.0039</td>
<td>0.0974</td>
<td>0.0630</td>
<td>0.0214</td>
<td>0.1619</td>
<td>0.0767</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-0.0026</td>
<td>-0.0003</td>
<td>0.0049</td>
<td>0.0050</td>
<td>0.0967</td>
<td>0.0680</td>
<td>0.0216</td>
<td>0.1610</td>
<td>0.0749</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-0.0057</td>
<td>-0.0084</td>
<td>0.0176</td>
<td>0.0152</td>
<td>0.1031</td>
<td>0.0766</td>
<td>0.0293</td>
<td>0.1709</td>
<td>0.0798</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0018</td>
<td>-0.0029</td>
<td>0.0104</td>
<td>0.0096</td>
<td>0.0976</td>
<td>0.0676</td>
<td>0.0216</td>
<td>0.1627</td>
<td>0.0796</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0081</td>
<td>-0.0145</td>
<td>0.0001</td>
<td>-0.0051</td>
<td>0.0957</td>
<td>0.0600</td>
<td>0.0181</td>
<td>0.1600</td>
<td>0.0729</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.0000</td>
<td>-0.0022</td>
<td>0.0055</td>
<td>0.0030</td>
<td>0.0966</td>
<td>0.0648</td>
<td>0.0205</td>
<td>0.1655</td>
<td>0.0803</td>
<td></td>
</tr>
</tbody>
</table>

### Table 1

Bias and MSE for the IV-MG, IV-P, TSLS-MG, TSLS-P, OLS, CCEMG, CCEP, TSLS and IV-HL estimators when both classical and factor endogeneity are small: $c = 0.8$, $\rho_{k,t} = 0.2$, $\rho_{t,r} = 0.2$. 

20
<table>
<thead>
<tr>
<th>T</th>
<th>N</th>
<th>IV-MG</th>
<th>IV-P</th>
<th>TSLS-MG</th>
<th>TSLS-P</th>
<th>OLS</th>
<th>CCEMG</th>
<th>CCEP</th>
<th>TSLS</th>
<th>IV-HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIAS</td>
<td>25</td>
<td>-0.0319</td>
<td>-0.0286</td>
<td>-0.0086</td>
<td>-0.0082</td>
<td>0.0802</td>
<td>0.0647</td>
<td>0.0289</td>
<td>0.2107</td>
<td>0.2383</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>-0.0319</td>
<td>-0.0395</td>
<td>-0.0242</td>
<td>-0.0254</td>
<td>0.0720</td>
<td>0.0768</td>
<td>0.0287</td>
<td>0.2082</td>
<td>0.2300</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>-0.0101</td>
<td>-0.0031</td>
<td>0.0010</td>
<td>0.0063</td>
<td>0.0768</td>
<td>0.0746</td>
<td>0.0231</td>
<td>0.2261</td>
<td>0.2360</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>-0.0051</td>
<td>-0.0003</td>
<td>0.0043</td>
<td>0.0059</td>
<td>0.0668</td>
<td>0.0641</td>
<td>0.0233</td>
<td>0.2098</td>
<td>0.2287</td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>-0.0345</td>
<td>-0.0278</td>
<td>-0.0036</td>
<td>0.0008</td>
<td>0.0707</td>
<td>0.0601</td>
<td>0.0173</td>
<td>0.2156</td>
<td>0.2400</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>-0.0333</td>
<td>-0.0305</td>
<td>-0.0200</td>
<td>-0.0154</td>
<td>0.0761</td>
<td>0.0611</td>
<td>0.0245</td>
<td>0.2106</td>
<td>0.2404</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>-0.0015</td>
<td>0.0011</td>
<td>0.0070</td>
<td>0.0097</td>
<td>0.0753</td>
<td>0.0750</td>
<td>0.0228</td>
<td>0.2115</td>
<td>0.2413</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>-0.0096</td>
<td>-0.0063</td>
<td>-0.0047</td>
<td>-0.0013</td>
<td>0.0729</td>
<td>0.0660</td>
<td>0.0206</td>
<td>0.2139</td>
<td>0.2372</td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>-0.0117</td>
<td>-0.0164</td>
<td>0.0163</td>
<td>0.0145</td>
<td>0.0754</td>
<td>0.0768</td>
<td>0.0285</td>
<td>0.2294</td>
<td>0.2420</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>-0.0187</td>
<td>-0.0236</td>
<td>-0.0069</td>
<td>-0.0100</td>
<td>0.0729</td>
<td>0.0653</td>
<td>0.0183</td>
<td>0.2177</td>
<td>0.2382</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>-0.0161</td>
<td>-0.0151</td>
<td>-0.0074</td>
<td>-0.0056</td>
<td>0.0744</td>
<td>0.0659</td>
<td>0.0216</td>
<td>0.2164</td>
<td>0.2394</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>-0.0012</td>
<td>0.0021</td>
<td>0.0071</td>
<td>0.0088</td>
<td>0.0739</td>
<td>0.0737</td>
<td>0.0218</td>
<td>0.2230</td>
<td>0.2400</td>
</tr>
<tr>
<td>100</td>
<td>25</td>
<td>-0.0052</td>
<td>-0.0142</td>
<td>0.0204</td>
<td>0.0126</td>
<td>0.0808</td>
<td>0.0833</td>
<td>0.0302</td>
<td>0.2279</td>
<td>0.2471</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>-0.0084</td>
<td>-0.0077</td>
<td>0.0060</td>
<td>0.0059</td>
<td>0.0751</td>
<td>0.0704</td>
<td>0.0215</td>
<td>0.2223</td>
<td>0.2468</td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>-0.0115</td>
<td>-0.0176</td>
<td>-0.0026</td>
<td>-0.0061</td>
<td>0.0743</td>
<td>0.0638</td>
<td>0.0198</td>
<td>0.2164</td>
<td>0.2395</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>-0.0041</td>
<td>-0.0022</td>
<td>-0.0034</td>
<td>-0.0027</td>
<td>0.0743</td>
<td>0.0705</td>
<td>0.0203</td>
<td>0.2235</td>
<td>0.2420</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MSE</th>
<th>T</th>
<th>N</th>
<th>IV-MG</th>
<th>IV-P</th>
<th>TSLS-MG</th>
<th>TSLS-P</th>
<th>OLS</th>
<th>CCEMG</th>
<th>CCEP</th>
<th>TSLS</th>
<th>IV-HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>25</td>
<td>0.8013</td>
<td>0.6713</td>
<td>0.8473</td>
<td>0.6871</td>
<td>0.1686</td>
<td>0.3688</td>
<td>0.2632</td>
<td>0.4706</td>
<td>0.3760</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.5331</td>
<td>0.4498</td>
<td>0.5719</td>
<td>0.4730</td>
<td>0.1268</td>
<td>0.2599</td>
<td>0.2099</td>
<td>0.3609</td>
<td>0.3034</td>
<td></td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>0.4378</td>
<td>0.3675</td>
<td>0.4619</td>
<td>0.3854</td>
<td>0.1176</td>
<td>0.2223</td>
<td>0.1538</td>
<td>0.3278</td>
<td>0.2825</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.3875</td>
<td>0.3130</td>
<td>0.4072</td>
<td>0.3281</td>
<td>0.1019</td>
<td>0.1911</td>
<td>0.1445</td>
<td>0.2919</td>
<td>0.2676</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>25</td>
<td>0.4726</td>
<td>0.4245</td>
<td>0.4864</td>
<td>0.4387</td>
<td>0.1297</td>
<td>0.2381</td>
<td>0.1743</td>
<td>0.3536</td>
<td>0.3254</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.3338</td>
<td>0.2990</td>
<td>0.3410</td>
<td>0.3032</td>
<td>0.1116</td>
<td>0.1783</td>
<td>0.1359</td>
<td>0.2858</td>
<td>0.2874</td>
<td></td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>0.2663</td>
<td>0.2458</td>
<td>0.2747</td>
<td>0.2525</td>
<td>0.1000</td>
<td>0.1552</td>
<td>0.1106</td>
<td>0.2700</td>
<td>0.2701</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.2222</td>
<td>0.1959</td>
<td>0.2275</td>
<td>0.2037</td>
<td>0.0933</td>
<td>0.1341</td>
<td>0.0943</td>
<td>0.2545</td>
<td>0.2607</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>0.3713</td>
<td>0.3270</td>
<td>0.3772</td>
<td>0.3356</td>
<td>0.1197</td>
<td>0.1996</td>
<td>0.1544</td>
<td>0.3242</td>
<td>0.3361</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.2630</td>
<td>0.2393</td>
<td>0.2691</td>
<td>0.2435</td>
<td>0.0995</td>
<td>0.1472</td>
<td>0.1081</td>
<td>0.2686</td>
<td>0.2734</td>
<td></td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>0.2166</td>
<td>0.1951</td>
<td>0.2203</td>
<td>0.1986</td>
<td>0.0927</td>
<td>0.1239</td>
<td>0.0902</td>
<td>0.2535</td>
<td>0.2624</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.1871</td>
<td>0.1691</td>
<td>0.1917</td>
<td>0.1747</td>
<td>0.0892</td>
<td>0.1205</td>
<td>0.0801</td>
<td>0.2507</td>
<td>0.2584</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>25</td>
<td>0.3048</td>
<td>0.2883</td>
<td>0.3145</td>
<td>0.2936</td>
<td>0.1158</td>
<td>0.1807</td>
<td>0.1407</td>
<td>0.3026</td>
<td>0.3208</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.2246</td>
<td>0.2112</td>
<td>0.2302</td>
<td>0.2165</td>
<td>0.0986</td>
<td>0.1327</td>
<td>0.0966</td>
<td>0.2645</td>
<td>0.2791</td>
<td></td>
</tr>
<tr>
<td></td>
<td>75</td>
<td>0.1816</td>
<td>0.1704</td>
<td>0.1837</td>
<td>0.1717</td>
<td>0.0900</td>
<td>0.1137</td>
<td>0.0803</td>
<td>0.2439</td>
<td>0.2604</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.1528</td>
<td>0.1423</td>
<td>0.1551</td>
<td>0.1428</td>
<td>0.0856</td>
<td>0.1076</td>
<td>0.0685</td>
<td>0.2435</td>
<td>0.2571</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Bias and MSE for the IV-MG, IV-P, TSLS-MG, TSLS-P, OLS, CCEMG, CCEP, TSLS and IV-HL estimators when classical endogeneity is small but factor endogeneity is large: $c = 0.8$, $\rho_{s,t} = 0.2$, $\rho_{s,r,t} = 0.8$. 
<table>
<thead>
<tr>
<th>T</th>
<th>N</th>
<th>IV-MG</th>
<th>IV-P</th>
<th>TSLS-MG</th>
<th>TSLS-P</th>
<th>OLS</th>
<th>CCEMG</th>
<th>CCEP</th>
<th>TSLS</th>
<th>IV-HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>25</td>
<td>-0.0145</td>
<td>-0.0232</td>
<td>0.0088</td>
<td>0.0085</td>
<td>0.1036</td>
<td>0.2465</td>
<td>0.0926</td>
<td>0.1513</td>
<td>0.0662</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>-0.0342</td>
<td>-0.0315</td>
<td>-0.0213</td>
<td>-0.0210</td>
<td>0.1014</td>
<td>0.2523</td>
<td>0.0781</td>
<td>0.1482</td>
<td>0.0694</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>-0.0122</td>
<td>-0.0145</td>
<td>-0.0006</td>
<td>0.0006</td>
<td>0.1040</td>
<td>0.2535</td>
<td>0.0843</td>
<td>0.1742</td>
<td>0.0759</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>-0.0109</td>
<td>-0.0093</td>
<td>-0.0013</td>
<td>-0.0020</td>
<td>0.0958</td>
<td>0.2487</td>
<td>0.0820</td>
<td>0.1465</td>
<td>0.0718</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>-0.0204</td>
<td>-0.0080</td>
<td>0.0100</td>
<td>0.0187</td>
<td>0.1005</td>
<td>0.2408</td>
<td>0.0886</td>
<td>0.1513</td>
<td>0.0865</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>-0.0267</td>
<td>-0.0250</td>
<td>-0.0137</td>
<td>-0.0110</td>
<td>0.1028</td>
<td>0.2461</td>
<td>0.0834</td>
<td>0.1573</td>
<td>0.0701</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>-0.0065</td>
<td>-0.0009</td>
<td>0.0035</td>
<td>0.0105</td>
<td>0.1038</td>
<td>0.2637</td>
<td>0.0875</td>
<td>0.1657</td>
<td>0.0776</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>-0.0037</td>
<td>-0.0072</td>
<td>0.0017</td>
<td>-0.0010</td>
<td>0.1006</td>
<td>0.2563</td>
<td>0.0810</td>
<td>0.1604</td>
<td>0.0727</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>-0.0011</td>
<td>0.0004</td>
<td>0.0069</td>
<td>0.0066</td>
<td>0.1028</td>
<td>0.2588</td>
<td>0.0817</td>
<td>0.1613</td>
<td>0.0747</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>-0.0027</td>
<td>-0.0048</td>
<td>0.0221</td>
<td>0.0208</td>
<td>0.1098</td>
<td>0.2635</td>
<td>0.0931</td>
<td>0.1729</td>
<td>0.0805</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>0.0009</td>
<td>0.0017</td>
<td>0.0150</td>
<td>0.0132</td>
<td>0.1040</td>
<td>0.2592</td>
<td>0.0860</td>
<td>0.1625</td>
<td>0.0806</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>-0.0063</td>
<td>-0.0127</td>
<td>0.0023</td>
<td>-0.0024</td>
<td>0.1020</td>
<td>0.2538</td>
<td>0.0809</td>
<td>0.1603</td>
<td>0.0728</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>-0.0001</td>
<td>-0.0016</td>
<td>0.0059</td>
<td>0.0035</td>
<td>0.1030</td>
<td>0.2555</td>
<td>0.0812</td>
<td>0.1665</td>
<td>0.0814</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T</th>
<th>N</th>
<th>IV-MG</th>
<th>IV-P</th>
<th>TSLS-MG</th>
<th>TSLS-P</th>
<th>OLS</th>
<th>CCEMG</th>
<th>CCEP</th>
<th>TSLS</th>
<th>IV-HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>25</td>
<td>0.7624</td>
<td>0.6375</td>
<td>0.7911</td>
<td>0.6652</td>
<td>0.1660</td>
<td>0.4261</td>
<td>0.2677</td>
<td>0.4379</td>
<td>0.2801</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>0.5208</td>
<td>0.4249</td>
<td>0.5507</td>
<td>0.4474</td>
<td>0.1392</td>
<td>0.3462</td>
<td>0.2065</td>
<td>0.3263</td>
<td>0.1951</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>0.4056</td>
<td>0.3378</td>
<td>0.4214</td>
<td>0.3541</td>
<td>0.1289</td>
<td>0.3202</td>
<td>0.1693</td>
<td>0.2873</td>
<td>0.1567</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>0.3694</td>
<td>0.2996</td>
<td>0.3863</td>
<td>0.3147</td>
<td>0.1181</td>
<td>0.3048</td>
<td>0.1610</td>
<td>0.2467</td>
<td>0.1476</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>0.4593</td>
<td>0.4035</td>
<td>0.4759</td>
<td>0.4208</td>
<td>0.1417</td>
<td>0.3338</td>
<td>0.1956</td>
<td>0.3194</td>
<td>0.2178</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>0.3154</td>
<td>0.2758</td>
<td>0.3233</td>
<td>0.2782</td>
<td>0.1255</td>
<td>0.2958</td>
<td>0.1572</td>
<td>0.2455</td>
<td>0.1569</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>0.2550</td>
<td>0.2255</td>
<td>0.2611</td>
<td>0.2301</td>
<td>0.1193</td>
<td>0.2963</td>
<td>0.1409</td>
<td>0.2279</td>
<td>0.1366</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>0.3728</td>
<td>0.3253</td>
<td>0.3820</td>
<td>0.3422</td>
<td>0.1359</td>
<td>0.3198</td>
<td>0.1781</td>
<td>0.2977</td>
<td>0.1958</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>0.2560</td>
<td>0.2276</td>
<td>0.2599</td>
<td>0.2323</td>
<td>0.1165</td>
<td>0.2878</td>
<td>0.1337</td>
<td>0.2243</td>
<td>0.1442</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>0.2095</td>
<td>0.1852</td>
<td>0.2130</td>
<td>0.1873</td>
<td>0.1151</td>
<td>0.2793</td>
<td>0.1220</td>
<td>0.2086</td>
<td>0.1235</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>0.1822</td>
<td>0.1612</td>
<td>0.1867</td>
<td>0.1673</td>
<td>0.1120</td>
<td>0.2777</td>
<td>0.1147</td>
<td>0.1976</td>
<td>0.1128</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>0.3025</td>
<td>0.2731</td>
<td>0.3112</td>
<td>0.2830</td>
<td>0.1341</td>
<td>0.3109</td>
<td>0.1658</td>
<td>0.2716</td>
<td>0.1890</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>0.2150</td>
<td>0.1983</td>
<td>0.2188</td>
<td>0.2046</td>
<td>0.1189</td>
<td>0.2859</td>
<td>0.1310</td>
<td>0.2168</td>
<td>0.1401</td>
</tr>
<tr>
<td>75</td>
<td>75</td>
<td>0.1762</td>
<td>0.1561</td>
<td>0.1803</td>
<td>0.1599</td>
<td>0.1116</td>
<td>0.2725</td>
<td>0.1165</td>
<td>0.1957</td>
<td>0.1147</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>0.1476</td>
<td>0.1342</td>
<td>0.1486</td>
<td>0.1345</td>
<td>0.1100</td>
<td>0.2707</td>
<td>0.1098</td>
<td>0.1917</td>
<td>0.1129</td>
</tr>
<tr>
<td>T</td>
<td>N</td>
<td>IV-MG</td>
<td>IV-P</td>
<td>TSLS-MG</td>
<td>TSLS-P</td>
<td>OLS</td>
<td>CCEMG</td>
<td>CCEP</td>
<td>TSLS</td>
<td>IV-HL</td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>-------</td>
<td>------</td>
<td>---------</td>
<td>--------</td>
<td>------</td>
<td>-------</td>
<td>------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>BIAS</td>
<td>25</td>
<td>-0.0346</td>
<td>-0.0252</td>
<td>-0.0055</td>
<td>-0.0034</td>
<td>0.0867</td>
<td>0.2607</td>
<td>0.1008</td>
<td>0.2070</td>
<td>0.2362</td>
</tr>
<tr>
<td>50</td>
<td>-0.0319</td>
<td>-0.0384</td>
<td>-0.0197</td>
<td>-0.0203</td>
<td>0.0799</td>
<td>0.2715</td>
<td>0.0931</td>
<td>0.2081</td>
<td>0.2322</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0083</td>
<td>-0.0037</td>
<td>0.0046</td>
<td>0.0068</td>
<td>0.0841</td>
<td>0.2671</td>
<td>0.0867</td>
<td>0.2254</td>
<td>0.2361</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-0.0129</td>
<td>-0.0047</td>
<td>-0.0049</td>
<td>-0.0026</td>
<td>0.0746</td>
<td>0.2707</td>
<td>0.0923</td>
<td>0.2070</td>
<td>0.2287</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0301</td>
<td>-0.0246</td>
<td>0.0015</td>
<td>0.0064</td>
<td>0.0792</td>
<td>0.2603</td>
<td>0.0886</td>
<td>0.2178</td>
<td>0.2429</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0355</td>
<td>-0.0322</td>
<td>-0.0200</td>
<td>-0.0151</td>
<td>0.0830</td>
<td>0.2672</td>
<td>0.0928</td>
<td>0.2105</td>
<td>0.2398</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0024</td>
<td>0.0010</td>
<td>0.0076</td>
<td>0.0108</td>
<td>0.0829</td>
<td>0.2782</td>
<td>0.0896</td>
<td>0.2216</td>
<td>0.2411</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-0.0064</td>
<td>-0.0045</td>
<td>-0.0024</td>
<td>0.0015</td>
<td>0.0800</td>
<td>0.2713</td>
<td>0.0859</td>
<td>0.2157</td>
<td>0.2386</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0105</td>
<td>-0.0163</td>
<td>0.0201</td>
<td>0.0161</td>
<td>0.0838</td>
<td>0.2783</td>
<td>0.1014</td>
<td>0.2288</td>
<td>0.2418</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0201</td>
<td>-0.0234</td>
<td>-0.0069</td>
<td>-0.0094</td>
<td>0.0795</td>
<td>0.2717</td>
<td>0.0862</td>
<td>0.2188</td>
<td>0.2378</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0140</td>
<td>-0.0134</td>
<td>-0.0046</td>
<td>-0.0039</td>
<td>0.0813</td>
<td>0.2716</td>
<td>0.0888</td>
<td>0.2169</td>
<td>0.2395</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0003</td>
<td>0.0033</td>
<td>0.0084</td>
<td>0.0098</td>
<td>0.0806</td>
<td>0.2799</td>
<td>0.0890</td>
<td>0.2229</td>
<td>0.2399</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-0.0029</td>
<td>-0.0128</td>
<td>0.0249</td>
<td>0.0156</td>
<td>0.0888</td>
<td>0.2812</td>
<td>0.1006</td>
<td>0.2271</td>
<td>0.2502</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>-0.0063</td>
<td>-0.0063</td>
<td>0.0103</td>
<td>0.0098</td>
<td>0.0820</td>
<td>0.2750</td>
<td>0.0899</td>
<td>0.2233</td>
<td>0.2469</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>-0.0109</td>
<td>-0.0162</td>
<td>-0.0017</td>
<td>-0.0046</td>
<td>0.0819</td>
<td>0.2696</td>
<td>0.0871</td>
<td>0.2167</td>
<td>0.2404</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-0.0043</td>
<td>-0.0025</td>
<td>0.0044</td>
<td>0.0035</td>
<td>0.0817</td>
<td>0.2740</td>
<td>0.0876</td>
<td>0.2227</td>
<td>0.2421</td>
<td></td>
</tr>
<tr>
<td>MSE</td>
<td>25</td>
<td>0.7993</td>
<td>0.6734</td>
<td>0.8435</td>
<td>0.6914</td>
<td>0.1736</td>
<td>0.4482</td>
<td>0.2864</td>
<td>0.4712</td>
<td>0.3746</td>
</tr>
<tr>
<td>50</td>
<td>0.5369</td>
<td>0.4525</td>
<td>0.5725</td>
<td>0.4703</td>
<td>0.1310</td>
<td>0.3689</td>
<td>0.2280</td>
<td>0.3595</td>
<td>0.3055</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.4394</td>
<td>0.3697</td>
<td>0.4589</td>
<td>0.3865</td>
<td>0.1226</td>
<td>0.3412</td>
<td>0.1763</td>
<td>0.3262</td>
<td>0.2825</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.3859</td>
<td>0.3111</td>
<td>0.4037</td>
<td>0.3275</td>
<td>0.1072</td>
<td>0.3261</td>
<td>0.1736</td>
<td>0.2894</td>
<td>0.2672</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.4786</td>
<td>0.4302</td>
<td>0.4948</td>
<td>0.4448</td>
<td>0.1348</td>
<td>0.3510</td>
<td>0.1980</td>
<td>0.3548</td>
<td>0.3287</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.3338</td>
<td>0.2985</td>
<td>0.3396</td>
<td>0.3027</td>
<td>0.1168</td>
<td>0.3190</td>
<td>0.1673</td>
<td>0.2865</td>
<td>0.2870</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.2634</td>
<td>0.2446</td>
<td>0.2714</td>
<td>0.2522</td>
<td>0.1059</td>
<td>0.3113</td>
<td>0.1447</td>
<td>0.2683</td>
<td>0.2695</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.2233</td>
<td>0.1963</td>
<td>0.2286</td>
<td>0.2044</td>
<td>0.0990</td>
<td>0.2977</td>
<td>0.1300</td>
<td>0.2559</td>
<td>0.2623</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.3740</td>
<td>0.3311</td>
<td>0.3800</td>
<td>0.3402</td>
<td>0.1253</td>
<td>0.3369</td>
<td>0.1867</td>
<td>0.3242</td>
<td>0.3322</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.2645</td>
<td>0.2396</td>
<td>0.2714</td>
<td>0.2438</td>
<td>0.1047</td>
<td>0.3058</td>
<td>0.1403</td>
<td>0.2690</td>
<td>0.2725</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.2176</td>
<td>0.1963</td>
<td>0.2215</td>
<td>0.2001</td>
<td>0.0983</td>
<td>0.2934</td>
<td>0.1291</td>
<td>0.2542</td>
<td>0.2626</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.1874</td>
<td>0.1695</td>
<td>0.1916</td>
<td>0.1752</td>
<td>0.0948</td>
<td>0.2982</td>
<td>0.1234</td>
<td>0.2504</td>
<td>0.2582</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.3056</td>
<td>0.2991</td>
<td>0.3160</td>
<td>0.2947</td>
<td>0.1216</td>
<td>0.3272</td>
<td>0.1752</td>
<td>0.3020</td>
<td>0.3248</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.2255</td>
<td>0.2118</td>
<td>0.2310</td>
<td>0.2168</td>
<td>0.1039</td>
<td>0.3006</td>
<td>0.1344</td>
<td>0.2657</td>
<td>0.2794</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.1812</td>
<td>0.1691</td>
<td>0.1830</td>
<td>0.1707</td>
<td>0.0966</td>
<td>0.2876</td>
<td>0.1206</td>
<td>0.2442</td>
<td>0.2612</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.1523</td>
<td>0.1422</td>
<td>0.1544</td>
<td>0.1426</td>
<td>0.0920</td>
<td>0.2882</td>
<td>0.1140</td>
<td>0.2428</td>
<td>0.2571</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Bias and MSE for the IV-MG, IV-P, TSLS-MG, TSLS-P, OLS, CCEMG, CCEP, TSLS and IV-HL estimators when both classical and factor endogeneity are large: \( c = 0.8 \), \( \rho_{k,s} = 0.8 \), \( \rho_{y,x} = 0.8 \).
Figure 1: Densities of the IV-MG, IV-P, TSLS-MG, TSLS-P, OLS, CCEMG, CCEP, TSLS and IV-HL estimators when both classical and factor endogeneity are small: $c = 0.8$, $\rho_{x, z} = 0.2$, $\rho_{x, f} = 0.2$, $N = T = 100$ based on 10000 replications.
Figure 2: Densities of IV-MG, IV-P, TSLS-MG, TSLS-P, OLS, CCEMG, CCEP, TSLS and IV-HL estimators when classical endogeneity is small but factor endogeneity is large: $c = 0.8$, $\rho_{\delta_1,\delta_2} = 0.2$, $\rho_{\gamma,x} = 0.8$, $N = T = 100$ based on 10000 replications.
Figure 3: Densities of the IV-MG, IV-P, TSLS-MG, TSLS-P, OLS, CCEMG, CCEP, TSLS and IV-HL estimators when classical endogeneity is large but factor endogeneity is small: $c = 0.8$, $\rho_{z_1z_2} = 0.8$, $\rho_{\gamma_{t}y_{t}} = 0.2$, $N = T = 100$ based on 10000 replications.
Figure 4: Densities of the IV-MG, IV-P, TSLS-MG, TSLS-P, OLS, CCEMG, CCEP, TSLS and IV-HL estimators when both classical and factor endogeneity are large: $c = 0.8$, $\rho_{s_1,x_2} = 0.8$, $\rho_{y,x} = 0.8$, $N = T = 100$ based on 10000 replications.