Precautionary Savings and Debt Stabilization in the Euro Area

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Report EC (2011, p 25)
the increase in the savings ratio in the Euro Area were due to precautionary savings in account of future income uncertainties

Kerdrain et al (2010)
household saving rates in the developed countries rose from pre-crisis levels on account of tighter credit and rising income uncertainty

Weil (1993)
precautionary savings increases with shocks and uncertainty.

Carroll and Kimball (1996)
liquidity constraints & precautionary motives imply that consumption is concave to income and MPC rises when income falls.
Quarterly savings rates of households and changes in the debt to GDP ratio in the Euro Area (EA 17).

A central tenet in macroeconomics: **Consumption smoothing**

1. Transient variation of income ➔ No effect on consumption

\[
\text{var (cons)} \ll 1
\]
Post-crisis excess consumption volatility vs. the size of fiscal adjustment 2009-2013 as a ratio to GDP.

Note: Excess volatility is the ratio of average consumption variability over that of GDP per capita. Variability in each period is obtained as a 3-year moving average of standard deviations. Variable is per capita consumption and per capita GDP in constant 2005 prices. Source: AMECO.
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<td>GDP growth</td>
<td>‘North’ 1.16 1.09</td>
<td>‘North’ 3.20 3.16</td>
<td>similar</td>
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<td>Consumption Growth</td>
<td>‘South’ 0.69 1.05</td>
<td>‘South’ 1.20 3.17</td>
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<td>Public Debt to GDP</td>
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<td>Public Spending to GDP</td>
<td>‘North’ 0.82 0.82</td>
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<td>Tax Revenues to GDP</td>
<td>‘South’ 0.62 0.90</td>
<td>‘South’ 0.50 1.15</td>
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Modeling income as a Brownian process

\[ dy = (\delta - \theta y) \cdot dt + [\sigma y^m] \cdot dz \]

1. Simple Brownian: \(\theta=m=0\), with drift \((\delta)\) and volatility \((\sigma)\)

2. Geometric Brownian: \(\delta = 0\), \(m=1\), drift rate \((-\theta)\), growth volatility \((\sigma)\).

3. Ornstein–Uhlenbeck process: \(m=0\), mean–reverting at speed \((\theta)\).

4. Square–root mean–reverting: \(\mu = \frac{\delta}{\theta}\), \(m=1/2\), volatility depends on income

prob bounded: For \(\sigma^2 < 2\theta \mu\), \(\Rightarrow\) \(\text{Prob}\{y < 0\} = 0\)

It captures fundamental changes after the crisis: \(\mu, \theta, \sigma\)
Random realizations of a stochastic square process with parameters $\theta=0.5$, $\mu=1$, $\sigma=0.2$, $\Omega=1.50$. Time unit $dt=0.01$. 
Households

\[ \max_{\{c\}} \int_0^{\infty} e^{-rs} u(c_{t+s}) ds \]

Time preferences = real rate of interest = \( r \)

Savings

\[ \dot{a} = ra + y - c \quad \lim_{s \to \infty} e^{-rs} a_{t+s} = 0 \]

Utility quadratic

\[ u(c) = c - \frac{\gamma}{2} c^2 \]

Optimal consumption

\[ c_t = r \cdot (a_t + H_t) \]

Human capital:

\[ H_t = \int_0^{\infty} e^{-rs} E_t \{y_{t+s}\} ds \]

**Constraints:** \( 0 \leq y \leq \Omega \mu, \ 1 < \Omega \) and \( 0 \leq a \)
Banking sector balance sheet: \[ a_t = b_t + z_t \]

\( b_t \) is public debt outstanding and \( z_t \) investment abroad

**Public debt**

Government spends \( \{g_t\} \), or taxes \( \{-g_t\} \), and also collects non-distortionary revenues, \( \{\tau_t\} \), e.g. privatizations, lump-sum levies, etc.

Debt outstanding

\[ \dot{b} = rb_t + g_t - \tau_t \]

Future debt liabilities

\[ d_t = \int_0^\infty e^{-rs} E_t\{g_{t+s}\} ds \]

Revenue-generating capacity

\[ f_t = \int_0^\infty e^{-rs} E_t\{\tau_{t+s}\} ds \]

**Debt sustainability**; see Giammaroli (ECB, 2007)

\[ \lim_{s \to \infty} e^{-rs} b_{t+s} \leq 0 \quad \Rightarrow \quad b_t + d_t \leq f_t \]
Debt crisis when...

\[ b_t + d_t > f_t \]

d rises: Future debt liabilities uncontrollable, (Italy, Portugal)

f falls: Revenue-generating capacity collapses, (Greece)

b surges: If \( \{z_t\} \) gets toxic, i.e. \( \Delta z < 0 \) and the Government issues debt \( \Delta b > 0 \) to rescue the commercial banks (Ireland, Spain)

Stabilization policies are then implemented by tax hikes, \( g = -k, k > 0 \) for a period \( (T) \) long enough as to ensure:

\[
\Delta b_t = \int_0^T e^{-rs} E_t \{ k_{t+s} dw_{t+s} \} ds = \frac{\lambda k}{r} [1 - e^{-rT}]
\]

Policy choices to achieve a certain \( \Delta b \): \( \lambda, k, T \)
Stochastic household income:

Three components

- Systematic, affecting the “permanent income”
- Random, affecting the “transient income”
- Government policy: transfers/taxation

\[
dy = \theta (\mu - y) \cdot dt + \left[ \sigma \sqrt{y} \right] \cdot dz - k \cdot dw
\]

\(\mu\) the mean to which income is reverting at a speed \((\theta)\),
\(\{dz\}\) random process normally distributed with mean=0 and var = \(dt\),
\(\{k\}\) tax hikes imposed on households
\(\{dw\}\) policy process of implementing austerity measures.

Poisson process:

\[
dw = \begin{cases} 
1 & \text{with probability } (\lambda dt) \\
0 & \text{with probability } (1 - \lambda dt)
\end{cases}
\]
Human capital dynamics

\[ rH = y + \frac{1}{dt} E_t(dH) \]

**Ito’s Lemma**

\[ \frac{1}{dt} E_t(dH) = \frac{\partial H}{\partial t} + \mu \frac{\partial H}{\partial y} + \frac{1}{2} \sigma^2 \frac{\partial^2 H}{\partial y^2} + E_w(\lambda \cdot [H(y - k) - H(y)]) \]

Taylor’s approx of \( \{dw\} \) effect

\[ H(y - k) - H(y) \approx -k \frac{\partial H}{\partial y} + \frac{1}{2} k^2 \frac{\partial^2 H}{\partial y^2} \]

DE for human capital

\[ \frac{1}{2} (\sigma^2 y + \lambda k^2) H'' + [\theta (\mu - y) - \lambda k] H' - rH + y = 0 \]

**General solution of DE**

\[ H = \frac{\theta \mu - \lambda k}{r(r + \theta)} + \frac{y}{r + \theta} + \xi \cdot F[\alpha; \beta; q(y)] \]

\( F(\alpha; \beta; q) \) is the Cofluent Hyperbolic Function, CHF

Parameters:

\[ \alpha = \frac{r}{\theta} \quad \beta(k) := \frac{2 \theta}{\sigma^2} \cdot [\mu - \frac{\lambda k}{\theta} + \frac{\lambda k^2}{\sigma^2}] \quad q(y) := \frac{2 \theta}{\sigma^2} \cdot [y + \frac{\lambda k^2}{\sigma^2}] \]

\( \xi \) determined from the smooth-pasting condition \( c'(y) \mid_{y=\mu \Omega} = 0 \)
Closed–form solutions for consumption and savings

Consumption rule
\[ c = r a + \frac{\theta \mu - \lambda k}{r + \theta} + \frac{r}{r + \theta} y - S(k) \]

Precautionary savings
\[ S(k) = \frac{1}{2} \cdot \frac{\sigma^2}{r + \theta} \cdot \frac{\beta \cdot F[\alpha; \beta; q(y)]}{F[\alpha + 1; \beta + 1; q(\mu \Omega)]} \]

\( F(\alpha; \beta; q) \) is the Cofluent Hyperbolic Function, CHF

To obtain an approximation set:
\[ \varepsilon = \varepsilon(k) = \frac{q(\mu \Omega)}{\beta} - 1 \]
\[ \varphi := \frac{\varepsilon \varepsilon}{2 \left(\frac{\varepsilon}{2} + 1\right)} \]
Approximation of the precautionary savings function

\[ S(k) \approx \frac{\sigma^2 r}{2(r + \theta)\theta \varphi} \cdot \exp \left[ -\frac{2\theta \varphi}{\sigma^2} (\mu \Omega - y) \right] \]

Marginal propensity to consume:

\[ MPC = \frac{\partial c}{\partial y} \approx \frac{r}{r + \theta} \cdot \frac{2\theta \varphi}{\sigma^2} (\mu \Omega - y) \]

Marginal propensity to precautionary save:

\[ MPPS = \frac{\partial s}{\partial y} = \frac{r}{r + \theta} \cdot \exp \left[ -\frac{2\theta \varphi}{\sigma^2} (\mu \Omega - y) \right] \]
Proposition 1. Precautionary savings rise with:
inequation[\text{income and income variance,}]
inequation[\text{as the income margin gets narrower}]
iniquation[\text{or the mean-reverting process becomes more persisting,}]
inequation[\text{i.e. speed parameter (}\theta\text{) falls.}]

Proposition 2. Precautionary savings rise with interest rates.

Proposition 3. MPC is decreasing in income
inequation[\text{and is concave w.r.t. the tax hike (}\text{k}\text{).}]
Rightarrow if y falls, MPC rises \Rightarrow multipliers rise \Rightarrow more recession

Proposition 4. Precautionary savings,
iniquation[\text{the marginal propensity to precautionary savings (MPPS)}]
iniquation[\text{and the variability of savings}]
iniquation[\text{are convex w.r.t. the tax hike (}\text{k}\text{).}]

Use this approximation to obtain:
Stylized Facts of austerity in the Euro Area economies

The intensity of austerity (k)


Source: KPMG Tax rates in the Euro Area.

Recessionary impact is defined as

\[
R_j = \left[1 + \sum_{t=2009}^{T=2014} \frac{\hat{y}_{j,t} - y_{j,t}}{y_{j,2008}} \right]^{1/6} - 1
\]

\(\hat{y}_t\) a simple time–trend projection of per capita GDP based on period 2000–2007. Hodrick–Prescott filter works as well.
**Stylized fact 1:** The recessionary impact is convex to the intensity of austerity imposed in the Euro Area economies.

Average output losses during 2009-2014 vs. total changes in the tax rates 2009-2013. Losses are expressed as % of GDP in 2008.

Source: AMECO; tax rates from KPMG.
Another measure of recession, is the suppression of consumption growth rates by households. **(Now concave effect)**

Changes in per capita consumption growth rates 2009-2013 over the pre-crisis period vs. total changes in tax rates 2009-2013.

Source: AMECO; tax rates from KPMG.
**Stylized fact 2:** Savings rate volatility is convex to the austerity of intensity imposed in the Euro Area economies.

Savings rate volatility 2009-2013 vs. total changes in the tax rates.

Note: Volatility is measured by the standard deviation of percentage rates. Data for Greece not reported. Source: Eurostat; tax rates from KPMG.
Optimal austerity

Given the convexity of precautionary savings, an optimal level of tax hikes \((k^*)\) can be found so as to minimize it, i.e. maximize consumption

\[
k^* = \frac{-\theta \mu}{\lambda} (\Omega - 1) + \left[ \frac{\theta^2 \mu^2}{\lambda^2} (\Omega - 1)^2 + \frac{\sigma^2 \mu}{\lambda} \Omega \right]^{1/2}
\]

Calibration:
Obtain income deviations by a Hodrick–Prescott filter 2006–2013

Then estimate mean-reverting process:

\[
\hat{y}_t = (1 - \theta)\hat{y}_{t-1} + \theta \mu + \epsilon
\]

Results: \(\theta = 0.13\) and \(\sigma = 8.20\%\) (long term variance)

Set: \(\mu = 1, r = 3\%, \Omega = 1.10\), austerity \(T = 5\) years, \(\lambda = 1/2\) (twice a year)

The optimal tax hike is calculated to be \(k^* = 10\%\),
and the expected value of the tax consolidation is \(\lambda k^* = 5\%\).
Precautionary savings and the log-linear approximation. Parameter values set at \( \theta=0.13, \mu=1, \sigma=0.082, \lambda=1/2, r=3\%, \Omega=1.10 \).
The mechanism would have worked by keeping precautionary savings as low as possible, thus letting consumption to be less depressed and recession to be milder.

- consumption could have been 2-3% higher per year
- output less contracted,
- Debt/GDP ratio would improve by -23% of GDP for the Euro Area
- uncertainty about future incomes could have been dissipated

Instead, public debt in EA12 amid fears of a prolonged recession rose from \textbf{80.66\% in 2009} to \textbf{96.66\% in 2013}
The analysis explains why the early assumptions of low fiscal multipliers went off the mark. Since aggregate output multipliers move in the same direction with MPC, it follows that with post-crisis incomes depressed, MPC and consequently the output multipliers increased substantially from their pre-crisis levels.

This made the recessionary effects of fiscal consolidation more pronounced.

By now, it is widely recognized that the severity of front-loaded austerity programs in the Euro Area has actually accentuated recession and indebtedness rather than dissipating them.

Batini et al (2012) report that “front loaded consolidations tend to be more contractionary and, hence, delay the reduction in the debt-to-GDP ratio relative to smoother consolidations”.

Blanchard and Leigh (2013) found that the depressing effect of fiscal consolidation was seriously underestimated and the size of the fiscal multiplier is likely to be three times the level assumed by the early estimates.
Some conclusions:

1. After the crisis, EA countries behave differently
   ➔ Fiscal cuts affected incomes,
   ➔ uncertainty multiplied
   and precautionary savings increased.

3. Inflexible austerity programs fuelled fluctuations, and caused deep recession & high unemployment

4. A gradual adjustment program could have had milder effects on recession