Tests of Policy Ineffectiveness in Macroeconometrics

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This paper proposes tests of policy ineffectiveness for rational expectations DSGE models, with or without exogenous variables. The policy intervention involves changes in parameters of a policy rule. It is assumed that the policy change is transparent, credible and is fully understood. The tests are based on the differences between the post-intervention realisations of the target variable and a counterfactual outcome, predicted using parameters estimated on pre-intervention data. The Lucas Critique is not a problem since the counterfactual, embodies the pre-intervention parameters while the post-intervention outcomes embody any effect of the intervention, the consequent change in expectations and the change in parameters.
The performance of the test is illustrated with a 3 equation NK model, using Monte Carlo simulations.

We present the usual shock impulse response functions but also policy impulse response functions that show the effect of a change to the policy parameters.

The MC results show that the policy ineffectiveness tests have the correct size.

The tests only have a high probability of detecting policy interventions that change the steady states.
The model

Suppose that the target variable, $y_t$, is affected by a vector of variables, $z_t$, and assume that the $(k_z + 1) \times 1$ vector $q_t = (y_t, z_t)'$ is determined by the RE model of the form

$$A_0 q_t = A_1 E_t(q_{t+1}) + A_2 q_{t-1} + u_t,$$

The structural shocks, $u_t$, have $E(u_t) = 0$, are serially uncorrelated with constant variance matrix, $E(u_t u_t') = \Sigma_u$, typically diagonal.

$E_t(q_{t+1}) = E(q_{t+1} \mid I_t)$, $I_t$ is the information set that includes $u_t$, and the lagged values of the variables, $q_t$.

We assume that $q_t$ are measured as deviations from their steady state values.
Policy intervention occurs at time $t = T_0$, 

pre-intervention sample runs: $t = M, M + 1, \ldots, T_0$, 

post-intervention sample: $t = T_0 + 1, T_0 + 2, \ldots, T_0 + H$. 

post-intervention horizon is $H$ 

sample size for estimation of the pre-intervention parameters is $T = T_0 - M + 1$. 

Solution

Suppose the RE model has the unique solution

\[ q_t = \Phi(\theta)q_{t-1} + \Gamma(\theta)u_t, \]

- Policy change is defined as a change in one or more elements of \( \theta \) (from \( \theta^0 \) to \( \theta^1 \)) will affect the mean outcomes through \( \Phi(\theta) \) and variances through \( \Gamma(\theta) \).
- If the intervention at \( T_0 \) is fully understood and expectations adjust immediately, then the process switches from

\[ q_t = \Phi(\theta^0)q_{t-1} + \Gamma(\theta^0)u_t, \quad t = M, M + 1, M + 2, \ldots, T_0 \]

\[ \text{to} \]

\[ q_t = \Phi(\theta^1)q_{t-1} + \Gamma(\theta^1)u_t, \quad t = T_0 + 1, T_0 + 2, \ldots, T_0 + H. \]

- In addition to the usual "shock impulse response functions", SIRFs, we also use "policy impulse response functions" (PIRF), that measure the effect over time of a policy intervention that changes the parameters of a policy rule, rather than shocks its equation error.
Construction of the test

The estimated policy effects are given by

\[ \hat{d}_{T_0+h}(\hat{\theta}_T^0) = s'q_{T_0+h} - s' \left[ \Phi \left( \hat{\theta}_T^0 \right) \right]^h q_{T_0} , \]

for \( h = 1, 2, \ldots, H \). Calculating \( \hat{d}_{T_0+h}(\hat{\theta}_T^0) \) only requires estimates of \( \theta^0 \) that can be obtained from the pre-intervention sample. The sampling distribution of \( \hat{d}_{T_0+h}(\hat{\theta}_T^0) \), depends on post-intervention parameters only under the alternative that the policy is effective, but not under the null hypothesis of no policy change as defined by

\[ H_0 : \theta^1 = \theta^0. \]

The role of distributional assumptions regarding the shocks, \( u_t \), can be minimized by basing the test on the "mean policy effect":

\[ \bar{d}_H(\hat{\theta}_T^0) = \frac{1}{H} \sum_{h=1}^{H} \hat{d}_{T_0+h}(\hat{\theta}_T^0). \]
The policy ineffectiveness test statistic is given by

\[ T_{d,H} = \frac{\sqrt{Hd_H(\hat{\theta}_T^0)}}{\sqrt{\hat{\omega}^2_{0q}}}, \]

where \( \omega^2_{0q} \) can be estimated using pre-intervention sample.

- Under the null hypothesis of policy ineffectiveness, and assuming that the errors \( u_{T_0+h} \) for \( h = 1, 2, \ldots, H \) are normally distributed, then as \( T \to \infty \), we have \( T_{d,H} \to_d N(0, 1) \).
- We abstract from the pre-intervention sampling uncertainty by assuming that \( T \) is sufficiently large and \( H/T \) sufficiently small.
Power of the policy ineffectiveness test: standard case

- The test is consistent if its power exceeds its size in finite samples, and if the power tends to unity as $H \to \infty$.

$$\sqrt{H\hat{d}_H} = H^{-1/2} \sum_{h=1}^{H} \hat{\epsilon}_{T_0,h} + H^{-1/2} \sum_{h=1}^{H} \nu_{T_0,h}.$$  

- $H^{-1/2} \sum_{h=1}^{H} \nu_{T_0,h}$, has a limiting distribution with mean zero and a finite variance both under the null and the alternative hypotheses.
- Therefore, for the test to be consistent the mean component of $\sqrt{H\hat{d}_H}$ must diverge to infinity with $H$, but a stationary DSGE returns to steady state.
- The internal dynamics of the RE model do not contribute to the power of the policy ineffectiveness test for $T$ and $H$ large.
- Thus tests based on the average policy effects, $\hat{d}_H$, will not be consistent in the case of stationary DSGE models.
Model with exogenous variables

Again let \( q_t = (y_t, z_t')' \), be a \((k_z + 1) \times 1\) vector, but add \( s_t = (x_t, w_t')' \), a \((1 + k_w) \times 1\) vector, where \( x_t \) are policy and \( w_t \) non-policy exogenous variables with

\[
A_0 q_t = A_1 E_t (q_{t+1}) + A_2 q_{t-1} + A_3 s_t + u_t,
\]

Assume

\[
s_t = R s_{t-1} + \eta_t,
\]

\[
R = \begin{pmatrix} \rho & 0 \\ 0 & R_w \end{pmatrix}, \quad \eta_t = \begin{pmatrix} \eta_{xt} \\ \eta_{wt} \end{pmatrix},
\]

so that \( w_t \) is invariant to changes in \( x_t \). \( u_t \) and \( \eta_t \) are serially and cross sectionally uncorrelated, with zero means and constant variances, \( \Sigma_u \), and \( \Sigma_{\eta} \).

- Whereas with standard stationary DSGE models the tests based on the average policy effects, \( \tilde{d}_H \), were not consistent, when there are exogenous variables or changes in the steady state the tests are consistent.
Simulated Policy analysis using a New Keynesian model

For interest rates, $R_t$, output, $y_t$, and inflation, $\pi_t$, measured as deviations from steady states, the model is

$$R_t = \delta_R R_{t-1} + (1 - \delta_R) (\psi_\pi \pi_t + \psi_y y_t) + u_{Rt},$$

$$y_t = \delta_y y_{t-1} + \kappa E(y_{t+1} | \mathcal{I}_t) - \sigma [R_t - E(\pi_{t+1} | \mathcal{I}_t)] + u_{yt},$$

$$\pi_t = \delta_\pi \pi_{t-1} + \beta E(\pi_{t+1} | \mathcal{I}_t) + \gamma y_t + u_{\pi t}.$$

Table 1. Pre-intervention parameter values, $\theta^0$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
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<td>$\kappa$</td>
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<td>$\delta_y$</td>
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<td>$\delta_\pi$</td>
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<td>$\psi_\pi$</td>
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</tr>
<tr>
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<tr>
<td>$\sigma_{uy}$</td>
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<td>$\sigma_{uy}$</td>
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<tr>
<td>$\sigma_{uR}$</td>
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</table>
We consider four separate policy interventions, in which each of the parameters of the Taylor rule are changed one at a time, leaving the other parameters unchanged.

Table 2: Policy interventions

<table>
<thead>
<tr>
<th>Interventions*</th>
<th>$\theta^0$</th>
<th>$\theta^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1_A$</td>
<td>$\delta_R = 0.7$</td>
<td>$\delta_R = 0.9$</td>
</tr>
<tr>
<td>$1_B$</td>
<td>$\delta_R = 0.7$</td>
<td>$\delta_R = 0.25$</td>
</tr>
<tr>
<td>$1_C$</td>
<td>$\psi_\pi = 1.5$</td>
<td>$\psi_\pi = 2.5$</td>
</tr>
<tr>
<td>$1_D$</td>
<td>$\psi_y = 0.5$</td>
<td>$\psi_y = 1.0$</td>
</tr>
</tbody>
</table>

* The other elements of $\theta^1$ are kept at their pre-intervention values.
In terms of the SIRF analysis the behaviour of the model is very standard.

- A contractionary monetary policy shock raises interest rates and reduces output and inflation, with output falling by more.
- A positive demand shock increases all three variables; output by the most, then interest rates then inflation.
- A negative supply shock, increases inflation and reduces output, the interest rate rises to offset the higher inflation.
- The impact effects of the monetary policy shock are given by the first column of $\Gamma(\theta^0)$, while the impact effects of the demand and supply shocks are given by its second and third columns.
The PIRF, gives the response of the system to a permanent change in the policy parameter(s) and is given by:

\[ \text{PIRF}(h, \theta^1, \theta^0, q_{T_0}) = s' \left[ \Phi \left( \theta^1 \right) \right]^h \left[ \Phi \left( \theta^0 \right) \right]^h q_{T_0}. \]

The PIRF requires knowledge of the parameters before and after the intervention and determines the power of the test.

Unlike the SIRF, the PIRF depends on, \( q_{T_0} \). If the economy is in equilibrium at \( T_0, q_{T_0} = 0 \), the test will have no power.

Of course, if the system was in equilibrium, there would be no reason to change policy. Policy interventions, changes in policy parameters, tend to be provoked by crises, when large absolute values of \( q_{T_0} \) occur.
We choose \( q_{T_0} \) values equal to the impact effects of SIRFs. The graphs below use, \( q_{R,T_0} = \sigma_u R \Gamma(\theta^0) e_R \), \( e_R = (1, 0, 0)' \) which is the impact effect of a monetary policy shock at \( h = 0 \).

Similarly, for the demand and supply shocks \( q_{y,T_0} = \sigma_u y \Gamma(\theta^0) e_y \) and \( q_{\pi,T_0} = \sigma_u \pi \Gamma(\theta^0) e_{\pi} \), respectively, where \( e_y = (0, 1, 0)' \) and \( e_{\pi} = (0, 0, 1)' \).

Considering values of the initial states, \( q_{T_0} \), that correspond to impact effects of structural shocks seems sensible given the focus of the literature on SIRFs.

Below we also consider multiples of the effects of such shocks as representing different degrees of deviations from equilibrium.

The power of the policy ineffectiveness test is an increasing function of the degree of deviation from equilibrium at the time of the policy change.
Figure 2a: Policy Impulse Response Functions: $q_{R,T_0} = \sigma_u \Gamma(\theta^0) e_R$.

2a. Intervention $1_A: \delta_R = 0.7$, to $\delta_R = 0.9$
Intervention $1_A$ increases the degree of persistence from $\delta_R = 0.7$, to $\delta_R = 0.9$. This causes the interest rate to rise and output and inflation to fall initially, with a maximum effect after about three periods before returning to zero.

Intervention $1_B$ reduces the degree of persistence from from $\delta_R = 0.7$, to $\delta_R = 0.25$. This has the opposite effect causing the interest rate to fall and output and inflation to rise.

The initial effects are the same as the values of $\left[ \Phi \left( \theta^1 \right) - \Phi \left( \theta^0 \right) \right]$ for the two cases.

When the degree of persistence is low as in intervention $1_B$, the variables return to zero much faster, making the mean effect of policy much smaller.

This is reflected in the power of the policy ineffectiveness tests.
Table 3: Size, $\theta^0$, and power of policy ineffectiveness tests against alternatives $\theta^{1A}, \theta^{1B}$; horizons $H = 8, 24$; 3 initial states

<table>
<thead>
<tr>
<th></th>
<th>Size ($\theta^0$)</th>
<th>Power ($\theta^{1A}$)</th>
<th>Power ($\theta^{1B}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>$y$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$H = 8$</td>
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</tr>
<tr>
<td>$q_{R,T_0}$</td>
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<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$q_{y,T_0}$</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$q_{\pi,T_0}$</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>$H = 24$</td>
<td></td>
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<tr>
<td>$q_{R,T_0}$</td>
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<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Notes: The rows labelled $q_{R,T_0}$ set the initial state $q_{T_0} = \sigma_{ur}\Gamma(\theta^0)e_R$. Similarly for $q_{y,T_0} = \sigma_{uy}\Gamma(\theta^0)e_y$, and $q_{\pi,T_0} = \sigma_{u\pi}\Gamma(\theta^0)e_{\pi}$. The alternative hypotheses are set out in Table 2.
The size seems very well controlled.

The power is highest for intervention, $1_A$, where the degree of persistence of the Taylor rule increases from $\delta_R = 0.7$, to $\delta_R = 0.9$, confirming what was apparent from the PIRFs. However, even in this case the power is not high, the highest is 25% for $H = 24$ testing the effect on $y_t$.

The test has little power against the other three types of interventions.

Whereas the test has power against the increase in persistence of the Taylor rule, it has less power against the reduction in the persistence of the Taylor rule for output and inflation, because the variables return to zero quickly.

Figure 3 shows the power increases with the size of the deviation from equilibrium at the time of the policy change.
Figure 3. Rejection frequencies for intervention $1_A$ (increasing $\delta_R$ from 0.7 to 0.9) with the initial states at $k$ standard deviations of $q_{R,T_0}$, and $H = 8$ quarters
Changes to the inflation target

Re-write the inflation and interest rate deviations in terms of their realized values $\hat{\pi}_t = \pi_t + \pi_*$ and $\hat{R}_t = R_t - (r + \pi_*)$, where $\pi_*$ is the target rate of inflation, and $r$ denotes the steady state value of the real interest rate.

In terms of the realized values of inflation and interest rates, $\hat{\pi}_t$ and $\hat{R}_t$, and deviations $y_t$, for the output gap, the model is

$$\hat{R}_t = (1 - \delta_R) [r + (1 - \psi_\pi)\pi_*] + \delta_R \hat{R}_{t-1} + (1 - \delta_R)(\psi_\pi \hat{\pi}_t + \psi_y y_t) + u_{Rt}$$

$$y_t = -\sigma r + \delta_y y_{t-1} + \kappa E(y_{t+1} | \mathcal{J}_t) - \sigma \left[ \hat{R}_t - E(\hat{\pi}_{t+1} | \mathcal{J}_t) \right] + u_{yt}$$

$$\hat{\pi}_t = (1 - \delta_\pi - \beta) \pi_* + \delta_\pi \hat{\pi}_{t-1} + \beta E(\hat{\pi}_{t+1} | \mathcal{J}_t) + \gamma y_t + u_{\pi t}.$$ 

The power tends to unity for inflation and the nominal interest rate but the test has no power as $H \to \infty$, for real output, $y_t$. The change in the inflation target has short run effects on real output, but not as $H \to \infty$. 
Two scenarios are considered: a reduction of $\pi_*$ from 2% to 1% per quarter and an increase of $\pi_*$ from 1% to 2% per quarter. Other policy parameters are fixed at baseline values.

In the case of a reduction, inflation falls more than the interest rate, raising the real interest rate on impact to 0.44% and thus depressing output.

The real interest rate and output return to zero, leaving the nominal interest rate and inflation rate at the new target, 1% lower, after about seven quarters.

When the target rate of inflation is increased the effects are reversed: inflation jumps more than interest rates, the real interest rate falls on impact to -0.44%, temporarily raising output.

Although the two cases are symmetrical numerically, they are not symmetrical in welfare terms, since the output loss associated with the reduction in inflation is something that one would wish to avoid.
Figure 5a shows the effects of both reducing the inflation target from 2% to 1% and increasing $\delta_R$, from 0.7 to 0.9, with the initial state $q_{R,T_0}$.

Inflation drops sharply, overshooting its steady state of 1%, hitting 1.55% after about 4 quarters. The real interest rate rises to 1.25%, depressing output, before the variables return to steady state.

Figure 5b shows that increasing the target rate of inflation has similar but the opposite effects.

Comparing the reduction in the target rate of inflation in Figure 5a with that in Figure 4a, the increased interest rate smoothing has resulted in a much larger loss of output.

Whereas in Figure 4a the maximum loss of output is 0.3% per quarter, in figure 5a the maximum loss is 1.1%, in both cases around quarter 3.
Figure 5a: Policy impulse response functions for reduction in target rates of inflation plus increased interest rate smoothing

Intervention $1_A: \delta_R$ from 0.7 to 0.9, initial state $q_{R,T_0}$
Figure 6 shows the results when the change in inflation target is combined with reduced interest rate smoothing.

For a credible reduction in the inflation target and very little interest rate smoothing, the interest rate and the inflation rate reduce by almost exactly the same amount and output hardly falls.

With a credible increase in the inflation target and reduced interest rate smoothing, inflation increases more than interest rates and the lower real interest rates provides a boost to output.

While the results are specific to this parameterisation and the assumption of credibility, it seems likely that less interest rate smoothing is optimal when reducing the target rate of inflation, as in Figure 6a, since this causes less output loss, and more interest rate smoothing seems more appropriate when increasing the target rate of inflation, as in the Japanese case, since this provides a bigger boost to output.
Table 4 gives size and power when the inflation target is reduced from 2% to 1% per quarter, the results for an increase were almost identical.

In $\theta^{1E}$, inflation target reduced and $\delta_R = 0.7$, in $\theta^{1F}$, $\delta_R$ is also increased to 0.9.

The power for interest rates and inflation rate are quite high and rise $H$ rises from 8 to 24 quarters.

The test has little power for output, since the effect on output is small and transitory and the power tends to fall with $H$.

Under $\theta^{1F}$, the power of the test on inflation is increased, but on interest rates reduced relative to $\theta^{1E}$, since more smoothing means that interest rates do not change as much.

More smoothing causes a larger movement in real interest rates, with a greater effect on output hence higher power of the test in detecting the effects of the policy change on realized values of output deviations, though power falls with $H$. 
Table 4: Size and power of policy ineffectiveness tests against reducing the inflation target only ($\theta^{1E}$) and when inflation target reduction is accompanied by a rise in interest rate smoothing ($\theta^{1F}$) - Horizons $H = 8, 24$; 3 initial states ($q_{T_0}$)

<table>
<thead>
<tr>
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<th>Size ($\theta^0$)</th>
<th>Power ($\theta^{1E}$)</th>
<th>Power ($\theta^{1F}$)</th>
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<tbody>
<tr>
<td></td>
<td>$R$</td>
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<tr>
<td>$q_{R,T_0}$</td>
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<td>$q_{y,T_0}$</td>
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<tr>
<td>$q_{\pi,T_0}$</td>
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<td>$q_{R,T_0}$</td>
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</table>

Notes: $\theta^{1E}$ assumes that the inflation target is reduced from $\pi^0_{*} = 2\%$ to $\pi^1_{*} = 1\%$ per quarter. $\theta^{1F}$ combines the reduction of the inflation target with a higher degree of interest rate smoothing, raising $\delta_R$ from 0.7 to 0.9.
We consider a policy intervention, defined as a change in one or more parameters of a policy rule.

Tests are based on the differences, over horizon, $H$, between the post-intervention realizations and the associated counterfactuals based on the parameters estimated using pre-intervention data, avoiding the Lucas Critique.

We derive tests for the null hypothesis of policy ineffectiveness using both structural models where all variables including the policy variable are endogenous and ones augmented with exogenous policy or non-policy variables.
Simulations confirms that the size of the test is correct.

The power of the proposed tests depends on

- the size of the parameter change,
- the dynamics of the system,
- the state of the economy at the time of the intervention,
- the size of the policy evaluation horizon and
- whether the model contains policy invariant exogenous variables or changes the steady state.

If there are no dynamics, no backward looking behaviour, policy interventions have no mean effects, though they may change the variances.
• For a standard model with dynamics where all the variables are endogenous, the test has no power if the system is at steady state at the time of the intervention.

• This is unlikely to be a problem in practice since major policy changes are unlikely to be adopted when the economy is at its steady state.

• However, the power of the policy ineffectiveness tests are likely to be low unless the underlying DSGE model contains exogenous variables or changes the steady states.

• The Monte Carlo analysis using a standard three equation NK DSGE model confirm the theoretical results. The tests have power against increases in the persistence of the Taylor rule, but little power against increases in the responses of interest rates to inflation and output.

• The focus of this paper has been on the mean effects of policy changes. But, the volatility effects of policy change are also of interest.