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OPTIMAL FISCAL AND MONETARY POLICY, DEBT CRISIS AND MANAGEMENT

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Optimal Fiscal and Monetary Policy, Debt Crisis and Management*

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Abstract

The initial government debt-to-GDP ratio and the government's commitment play a pivotal role in determining the welfare-optimal speed of fiscal consolidation in the management of a debt crisis. If the government cannot commit, quickly consolidating government debt is the optimal policy. If the government can commit, for low or moderate initial government debt-to-GPD ratios, the optimal consolidation is very slow. A faster pace is optimal when the economy starts from a high level of public debt, implying high sovereign risk premia, unless these are suppressed via a bailout by official creditors. Simple monetary-fiscal rules with passive fiscal policy, designed for an environment with "normal shocks", perform reasonably well in mimicking the Ramsey-optimal response to one-off government debt shocks. When the government can issue also long-term bonds – under commitment – the optimal debt consolidation pace is slower than in the case of short-term bonds only, and entails an increase in the ratio between long and short-term bonds.

Keywords: Optimal fiscal-monetary policy, Ramsey policy, time-consistent policy, optimised simple rules, debt consolidation, long-term debt, fiscal limits, sovereign default risk.

JEL Codes: E52, E62, H12, H63

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1 Introduction

The global financial crisis has left many advanced economies with a legacy of high government debt. Despite substantial heterogeneity as regards initial conditions, macroeconomic management and post-crisis outcomes, in most key economies, governments currently face the challenge of fiscal consolidation (see Figure 1). Whilst in policy and academic circles there is agreement on the necessity of reducing the stock of government debt, an interesting and often controversial question is: *how fast should debt be consolidated?*

In this paper we analyse such a macroeconomic policy dilemma from a welfare-optimising viewpoint, by seeking answers to the following four question: (i) What is the welfare-optimal speed of debt consolidations? (ii) How does the picture change if the government cannot commit? (iii) What is the welfare-optimal form of simple monetary and fiscal rules to be used to achieve both one-off fiscal consolidation and to conduct stabilization policy in the face of exogenous uncertainty? (iv) Does the welfare-optimal speed of debt consolidation change if the government can optimally alter the maturity composition of its debt obligations?

We investigate these issues through the lense of a Dynamic Stochastic General Equilibrium (DSGE) model, paying particular attention to the subtle interactions between fiscal and monetary policy. The core of the model is a fairly standard New-Keynesian (NK) model featuring frictions as in Smets and Wouters (2007) with prices and nominal wage rigidity. This basic setup is augmented with a detailed fiscal sector, which is instrumental for our analysis of monetary and fiscal interactions. First, the government finances its expenditures by raising a mix of lump-sum and distortionary taxes and by issuing government bonds. Second, holding government debt is subject to sovereign default risk, modelled along the lines of Corsetti et al. (2013) and Bi and Traum (2014). Third, government expenditures are utility-enhancing and we allow for a versatile private-public consumption aggregator encompassing substitutability or complementarity as in Bouakez and Rebei (2007) and Cantore et al. (2012). Although most of the analysis is conducted in a framework in which the government only issues short-term bonds, we provide an extension allowing the government to also issue long-term bonds. We use US data to calibrate-estimate parameter values in the model, in order to match key stylized facts and minimize a weighted loss function of key volatilities and correlations.

A number of possible interest rate and fiscal policies are compared: first, the welfare-optimal (Ramsey) policy; second, a time-consistent policy; third optimised simple Taylor type rules (of which price-level or superinertial rules are special cases). For the simple rules, both passive and active fiscal policy stances – in the sense of Leeper (1991) – are examined. We study policy rules responding both to continuous future stochastic shocks (policy in "normal times") and to a one-off shock to government debt ("debt crisis management"). This results in what we believe to be the first comprehensive assessment of the optimal timing and optimal combination of instruments – including the maturity composition of government debt – to

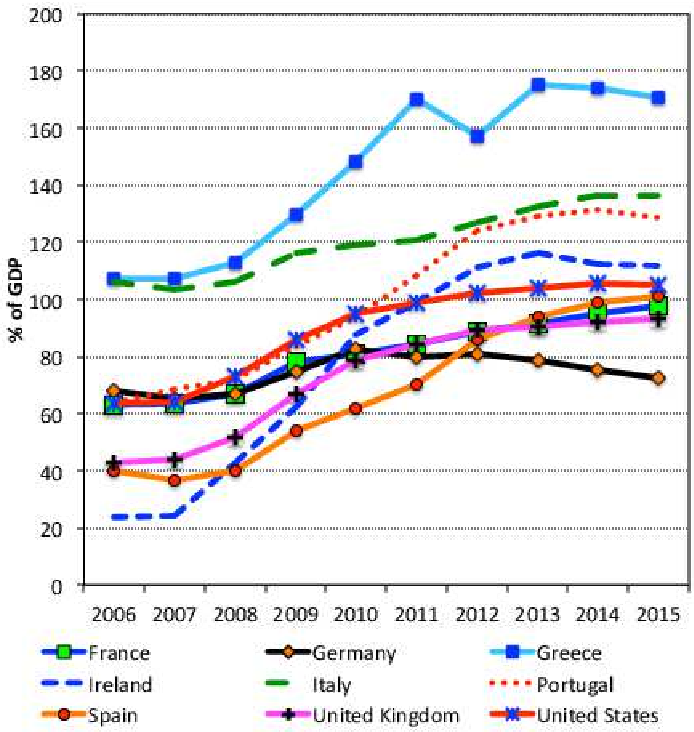

Figure 1: Gross general government debt (% of GDP) in selected advanced economies (Source: Fiscal Monitor, October 2014, International Monetary Fund)

achieve a fiscal consolidation. Results are as follows.

As regards our first and second questions, the initial government debt-to-GDP ratio and the government's commitment play a pivotal role in determining the optimal speed of consolidation in the management of a debt crisis. In fact, if the government is not able to commit, quickly consolidating government debt is the optimal policy. If the government can commit, a greater margin for manoeuvre is possible, and the optimal pace of consolidation is determined by the initial level of government debt. For low or moderate initial government debt-to-GPD ratios, the optimal consolidation is very slow. A faster pace is instead optimal when the economy starts from a high level of public debt (requiring high financing costs in the form of sovereign risk premia). However, if the economy is in a "bailout" regime, in which official creditors grants concessional loans, *de facto* permanently suppressing sovereign risk premia, then again it is optimal to enact a slow debt consolidation.

With reference to our third question, welfare calculations indicate that the ability of the simple rules to closely mimic the Ramsey optimal policy (as observed in the literature with optimal monetary policy alone) is still a feature of optimal policy with a variety of fiscal instruments. This occurs, however, only with "passive" fiscal policy. In addition, simple monetary-fiscal rules with passive fiscal policy, designed for an environment with "normal shocks", perform reasonably well in mimicking the Ramsey-optimal response to one-off government debt shocks.

As far as our fourth question is concerned, when the government has the possibility of issuing long-term bonds, it is optimal to increase the ratio of long-to-short bonds, in the face of a debt shock. Without government's commitment (time-consistent policy), the pace of consolidation is invariably fast. Under commitment, it is still the initial government debt-to-GDP ratio that is the main driver of the optimal consolidation speed. However, if the government also issues long-term bonds, the optimal debt consolidation pace is slower than it is in the case of short-term bonds only.

The implications of these results agree with the findings of a number of recent studies. Batini et al. (2012) show, in the context of regime-switching vector-autoregressions, that smooth and gradual consolidations are preferred to frontloaded consolidations, especially for economies in a recession. Erceg and Linde (2013) obtain similar findings in a DSGE model of a currency union. Denes et al. (2013) highlight limitations of austerity measures, while Bi et al. (2013) show that in the current economic environment, consolidation efforts are more likely to be contractionary rather than expansionary.

In seminal papers on monetary-fiscal interactions, Schmitt-Grohe and Uribe (2004a) and Schmitt-Grohe and Uribe (2007) exploit their own contribution to the computation of the solution of non-linear DSGE models in Schmitt-Grohe and Uribe (2004b) to relax the simplifying assumptions of the earlier workhorse New Keynesian model, such as the absence of capital and linearization around a zero-inflation steady state. They examine optimal commitment and optimised simple rules as do Leith et al. (2012) who extend the model to incorporate deep habits. These papers study policy in what we later call "normal times" and show that government debt optimally follows a near random walk. Adam (2011) and Michal (2011) adopt a similar model but focus on what we term "crisis management", which considers monetary and fiscal policy following a large build-up of government debt. Then, as in our results, optimal debt reduction should proceed at a slightly faster rate.[1]

There are a number of more recent works that address issues of sovereign risk and/or the benefits of commitment as in our paper. Using a standard NK model with government sovereign risk, Corsetti et al. (2013) carry out a comparison of different fiscal consolidation scenarios. Our analysis differs in that we consider optimal or optimised simple rules whereas they study *ad hoc* policies. Bi et al. (2013) move beyond the standard complete information rational expectations framework of our paper and address an important but different issue of fiscal consolidation when the private sector is uncertain about the timing and composition of fiscal policy. Perhaps, the closest papers to ours are Kirsanova and Wren-Lewis (2012) and Leith and Wren-Lewis (2013). Both papers employ a simple core NK model without capital

[1] This result emerges in these papers from a second-order perturbation stochastic solution that captures budget risk considerations. In our model these emerge from sovereign risk and the possibility of default.

and nominal wage rigidity and with a far simpler fiscal dimension that omits government sovereign risk and private-public consumption subsitutability/complementarity. The first of these papers examines different *ad hoc* degrees of fiscal feedback alongside optimal monetary policy and, as in our paper, allows fiscal policy to become "active" and monetary policy "passive", with the price level jumping in order to satisfy the government budget constraint. As in our paper, Leith and Wren-Lewis (2013) compare commitment and discretion, thus drawing conclusions regarding the importance of the former. Apart from our richer estimated model, especially in terms of fiscal policy, our additional main contributions are first, to systematically compare optimal commitment, time-consistent, and optimized simple rules drawing conclusions about the costs of simplicity. Second, we impose a zero lower bound (ZLB) constraint for the monetary policy rate as in Levine et al. (2008a). Finally we extend the analysis on the fiscal side to allow for long-term debt.

The remainder of the paper is organised as follows. Section 2 sets out the model. Section 3 outlines the calibration-estimation. Section 4 carries out the policy experiments. Section 5 provides an extension with long-term debt. Last, Section 6 concludes. More technical details are appended to the paper.

2 The Model

The backbone of the DSGE model is a fairly standard New-Keynesian (NK) model with Rotemberg (1982) prices and nominal wages, featuring frictions as in Smets and Wouters (2007).[2] The real frictions in the model are habit formation, convex investment adjustment costs, and variable capital utilisation, while the nominal frictions are sticky prices and wages. This basic setup is augmented with a more detailed fiscal sector, instrumental for our analysis. First, the government finances its expenditures by raising a mix of lump-sum and distortionary taxes and by issuing government bonds. Second, holding government debt is subject to sovereign default risk. Third, government expenditures are utility-enhancing. Although most of the analysis is conducted in a framework in which the government only issues short-term bonds, Section 5 contains an extension allowing the government to issue also long-term bonds. Moreover, while this section outlines the optimisation problems of all agents in the economy, Appendix A reports the full set of symmetric equilibrium conditions.

2.1 Households

A continuum of identical households $j \in [0,1]$ has preferences over differentiated consumption varieties $i \in [0,1]$ and derive utility from $(X_t)^j = X\left((X_t^c)^j, G_t\right)$, i.e. a habit-adjusted

[2]The only difference is the use of Rotemberg rather than Calvo contracts. It is well-known that for a low steady state inflation rate the differences in the dynamic properties are small up to first order. Moreover the Calvo approach cannot aggregate prices with a time-varying elasticity of demand as in (2) and (8).

composite of differentiated private and public consumption goods, respectively, similar to that in Bouakez and Rebei (2007), Pappa (2009) and Cantore et al. (2012), which allows for $(X_t)^j = (X_t^c)^j$ as a special case and allows both for complementarity and substitutability between the two types of goods, as explained in Section 2.6.

Habits are internal (rather than external) in order to avoid counterfactual welfare externalities typical of external habits.[3] As a result, the private component of the habit-adjusted consumption composite is given by

$$(X_t^c)^j = C_t^j - \theta C_{t-1}^j, \tag{1}$$

where C_t^j is the level of consumption and $\theta \in (0,1)$ is the degree of internal habit formation.

Each household j is a monopolistic provider of a differentiated labour service and supplies labour H_t^j to satisfy demand,

$$H_t^j = \left(\frac{w_t^j}{w_t}\right)^{-e_t^W \eta} H_t, \tag{2}$$

where w_t^j is the real wage charged by household j, w_t is the average real wage in the economy, η is the intratemporal elasticity of substitution across labour services, e_t^W is a wage mark-up shock, and H_t is average demand of labour services by firms. Similarly to Zubairy (2014), the households' budget constraint also includes a Rotemberg quadratic cost of adjusting the nominal wage, W_t^j, which is zero at the steady state, and that this is proportional to the average real value of labour services as in Furlanetto (2011),

$$\frac{\xi^W}{2}\left(\frac{W_t^j}{W_{t-1}^j} - \Pi\right)^2 w_t H_t = \frac{\xi^W}{2}\left(\frac{w_t^j}{w_{t-1}^j}\Pi_t - \Pi\right)^2 w_t H_t, \tag{3}$$

where ξ^W is the wage adjustment cost parameter and Π is the steady state value of inflation.

Households hold capital holdings, evolving according to

$$K_{t+1}^j = (1-\delta)K_t^j + e_t^I I_t^j \left[1 - S\left(\frac{I_t^j}{I_{t-1}^j}\right)\right], \tag{4}$$

where K_t^j is the beginning-of-period capital stock, δ is the capital depreciation rate, I_t^j is investment, $S(\cdot)$ represents an investment adjustment cost satisfying $S(1) = S'(1) = 0$ and $S''(1) > 0$, and e_t^I is an investment-specific shock. Households can also control the utilisation rate of capital. In particular using capital at rate u_t^j entails a cost of $a\left(u_t^j\right) K_t^j$ units of composite good, satisfying $a\left(u\right) = 0$, where u is the steady-state utilisation rate.

[3]External habit formation, i.e. on the average level of consumption rather than on own's consumption, creates an externality whereby households supply too much hours of work in the steady state. As a result, policies curbing employment can be welfare enhancing.

Households buy consumption goods, C_t^j; invest in (i) investment goods, I_t^j, (ii) risk-less private bonds that are mutually traded between consumers (which are in zero net supply), B_t^j, (iii) nominal short-term government bond holdings, $\left(B_t^S\right)^j$; bear the wage adjustment cost defined by equation (3) as well as the capital utilisation cost $a\left(u_t^j\right)K_t^j$; pay a mixture of net lump-sum, τ_t^L, and distortionary taxes τ_t^C, τ_t^W, τ_t^K; receive (i) the hourly wage, W_t, (ii) the rental rate R_t^K on utilised capital $u_t^j K_t^j$, (iii) the return on nominal private bond holdings, R_t, (iv) the return on short-term nominal government bond holdings, R_t^S, discounted at the *ex-ante* expected haircut rate, Δ_t^g, (v) firms' profits, $\int_0^1 J_{it}di$, (vii) a transfer made by the government in case of default (the rationale of which is later explained in Section 2.3), $\tilde{\Xi}_t$, (viii) a depreciation allowance for tax purposes, $\delta Q_t \tau_t^K K_t^j$. Therefore, households' budget constraint reads as

$$\left(1+\tau_t^C\right)C_t^j + I_t^j + \tau_t^L + \frac{\xi^W}{2}\left(\frac{w_t^j}{w_{t-1}^j}\Pi_t - \Pi\right)^2 w_t H_t + a\left(u_t^j\right)K_t^j + \frac{B_t^j}{P_t} + \frac{(B_t^S)^j}{P_t}$$
$$\leq \left(1-\tau_t^W\right)\frac{W_t^j}{P_t}H_t^j + \left(1-\tau_t^K\right)R_t^K u_t^j K_t^j + \frac{R_{t-1}B_{t-1}^j}{P_t}$$
$$+(1-\Delta_t^g)\frac{R_{t-1}^S (B_{t-1}^S)^j}{P_t} + \int_0^1 J_{it}di + \tilde{\Xi}_t + \delta Q_t \tau_t^K u_t K_t^j. \tag{5}$$

Household j's optimisation problem is then given by

$$\max_{\left\{C_t^j, S_t^j, B_{t+1}^j, (B_{t+1}^S)^j, K_{t+1}^j, u_t, I_t^j, w_t^j\right\}} E_t \sum_{s=0}^{\infty}\left\{e_{t+s}^B \beta^{t+s} U\left(X_{t+s}^j, H_{t+s}^j\right)\right\}, \tag{6}$$

subject to constraints (1), (2), (4), (5), where $\beta \in (0,1)$ is the discount factor, $U\left(X_t^j, H_t^j\right)$ is the instantaneous utility, and e_t^B is a preference shock.

2.2 Firms

A continuum of monopolistically competitive firms indexed by $i \in [0,1]$ rents capital services, $\tilde{K}_{it} \equiv u_t K_{it}$, and hires labour, H_{it} to produce differentiated goods Y_{it} with concave production function $F\left(e_t^A H_{it}, \tilde{K}_{it}\right)$ – where e_t^A is a labour-augmenting technology shock – which are sold at price P_{it}.

Assuming a standard Dixit-Stiglitz aggregator for consumption goods,

$$Y_t = \left[\int_0^1 (Y_{it})^{1-\frac{1}{e_t^P \zeta}} di\right]^{\frac{1}{1-\frac{1}{e_t^P \zeta}}}, \tag{7}$$

where e_t^P is a price mark-up shock and ζ is the intratemporal elasticity of substitution across varieties of goods, the optimal level of demand for each variety, Y_{it}, for a given composite, is obtained by minimising total expenditure $\int_0^1 P_{it}Y_{it}di$ over Y_{it}. This leads to

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-e_t^P\zeta} Y_t, \tag{8}$$

where $P_t = \left[\int_0^1 P_{it}^{1-e_t^P\zeta} di\right]^{\frac{1}{1-e_t^P\zeta}}$ is the nominal price index.

Firms face quadratic price adjustment costs $\frac{\xi^P}{2}\left(\frac{P_{it}}{P_{it-1}} - \Pi\right)^2 Y_t$, as in Rotemberg (1982) – where parameter ξ^P measures the degree of price stickiness – and solve the following profit maximisation problem:

$$\max_{\{\tilde{K}_{it+s}, H_{it+s}, P_{it+s}\}} J_{it} = E_t \left\{ \sum_{s=0}^{\infty} D_{t,t+s} \left[\begin{array}{c} \frac{P_{it+s}}{P_{t+s}} Y_{it+s} - w_{it}H_{it+s} - R_{t+s}^K \tilde{K}_{it+s} \\ -\frac{\xi^P}{2}\left(\frac{P_{it+s}}{P_{it+s-1}} - \Pi\right)^2 Y_{t+s} \end{array} \right] \right\}, \tag{9}$$

subject to the firm-specific demand (8) and the firm's resource constraint,

$$Y_{it} = F(e_t^A H_{it}, \tilde{K}_{it}) - FC, \tag{10}$$

where fixed production costs, FC, are set to ensure that the free entry condition of long-run zero profits is satisfied.

2.3 Government

The government finances its expenditures, G_t, by levying taxes, T_t, and by issuing one-period bonds, B_t^S. The government promises to repay one-period bonds the next period and the gross nominal interest rate applied is R_t^S. However, in order to introduce a sovereign risk premium, we assume that government bond contracts are not enforceable. As in Bi and Traum (2014), each period a stochastic fiscal limit expressed in terms of government debt-to-GDP ratio and denoted by Γ_t^*, is drawn from a distribution, the cumulative density function (CDF) of which is represented by a logistical function, p_t^*, with parameters η_1 and η_2:

$$p_t^* = P\left(\Gamma_t^* \leq \Gamma_{t-1}\right) = \frac{\exp\left(\eta_1 + \eta_2\Gamma_{t-1}\right)}{1 + \exp\left(\eta_1 + \eta_2\Gamma_{t-1}\right)}, \tag{11}$$

where $\Gamma_t \equiv B_t^S/P_tY_t$. If government-debt-to-GDP exceeds the fiscal limit, i.e. $\Gamma_{t-1} \geq \Gamma_t^*$, then the government defaults. Hence p_t^* represents the probability of default. This occurs in the form of an haircut $\Delta_t^g \in [0, 1]$ applied as a proportion to the outstanding stock of government debt. In order to be able to solve the model with perturbation methods, we follow Corsetti et al. (2013) and assume that agents consider the *ex-ante* expected haircut rate,

$$\Delta_t^g = \begin{cases} 0 & \text{with probability } 1 - p_t^* \\ \bar{\Delta}^g & \text{with probability } p_t^* \end{cases}, \tag{12}$$

where $\bar{\Delta}^g \in (0, 1]$ is the haircut rate applied in the case of default. In other words,

$$\Delta_t^g = p_t^* \bar{\Delta}^g. \tag{13}$$

Let $b_t^S \equiv B_t^S / P_t$ and $\Pi_t \equiv P_t / P_{t-1}$ be the gross inflation rate, then the government budget constraint, in real terms, reads as

$$b_t^S = (1 - \Delta_t^g) \frac{R_{t-1}^S}{\Pi_t} b_{t-1}^S + G_t - T_t + \tilde{\Xi}_t, \tag{14}$$

where $\tilde{\Xi}_t \equiv \Delta_t^g \frac{R_{t-1}^S}{\Pi_t} b_{t-1}^S$ represents a transfer made by the government in a way that sovereign default does not alter the actual debt level.[4]

Total government revenue T_t is given by

$$T_t = \tau_t^C C_t + \tau_t^W w_t h_t + \tau_t^K \left[\left(R_t^K - \delta Q_t \right) u_t K_t \right] + \tau_t^L, \tag{15}$$

where τ_t^C, τ_t^W, τ_t^K are tax rates on aggregate private consumption C_t, labour income, $w_t h_t$, and the net capital rental rate, $\left(R_t^k - \delta Q_t \right) u_t K_t$, respectively, and τ_t^L are lump-sum taxes net of government transfers.

In order to reduce the number of tax instruments to one, we impose that τ_t^C, τ_t^W, τ_t^K and τ_t^L deviate from their respective steady state by the same proportion (i.e. $\tau_t^C = \tau_t \tau^C$, $\tau_t^W = \tau_t \tau^W$, $\tau_t^K = \tau_t \tau^K$,$\tau_t^L = \tau_t \tau^L$) and that the proportional uniform tax change, τ_t, becomes one of our fiscal policy instruments. The other instrument is represented by government spending G_t. We allow the instruments to be adjusted according to the following Taylor-type rules:

$$\log \left(\frac{\tau_t}{\tau} \right) = \rho_\tau \log \left(\frac{\tau_{t-1}}{\tau} \right) + \rho_{\tau B} \log \left(\frac{b_{t-1}^S}{b^S} \right) + \rho_{\tau Y} \log \left(\frac{Y_t}{Y} \right), \tag{16}$$

$$\log \left(\frac{G_t}{G} \right) = \rho_G \log \left(\frac{G_{t-1}}{G} \right) - \rho_{GB} \log \left(\frac{b_{t-1}^S}{b^S} \right) - \rho_{GY} \log \left(\frac{Y_t}{Y} \right), \tag{17}$$

[4]As Corsetti et al. (2013) point out, in the real world, sovereign default cause some redistribution among households, that is from savers to borrowers, however in standard DSGE models risk sharing allows them to perfectly insure themselves against the distributional consequences of sovereign default. Therefore, the absence of such transfers would imply lower risk premia prior to default, as the lower post-default debt stock would already be taken into account. This assumption essentially eliminates this counterintuitive effect.

where ρ_τ implies persistence in the tax instrument, $\rho_{\tau B}$ is the responsiveness of the tax instrument to the deviation of government debt from its steady state, and $\rho_{\tau Y}$ is the responsiveness to output deviations. Parameters ρ_G, ρ_{GB}, and ρ_{GY} are the analogues in the expenditure rule. Notice that these are Taylor-type rules as in Taylor (1993) that respond to deviations of output and debt from their deterministic steady state values and not from their flexi-price outcomes. Such rules have the advantage that they can be implemented using readily available macro-data series rather than from model-based theoretical constructs (see e.g. Schmitt-Grohe and Uribe, 2007). Leeper et al. (2010) also show that such a specification for fiscal rules fits the data reasonably well.

2.4 Monetary policy

Monetary policy is set according to a Taylor-type interest-rate rule,

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + \rho_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \rho_y \log\left(\frac{Y_t}{Y}\right), \tag{18}$$

where ρ_r is the interest rate smoothing parameter and ρ_π and ρ_y are the monetary responses to inflation and output relative to its steady state. The rationale for using readily observable variables is the same as that concerning the fiscal rules.[5]

2.5 Equilibrium

In equilibrium all markets clear. The model is completed by the output, capital and labour market equilibrium conditions,

$$Y_t = C_t + I_t + G_t + \frac{\xi^P}{2}\left(\Pi_t - \Pi\right)^2 Y_t + \frac{\xi^W}{2}\left(\Pi_t^W - \Pi\right)^2 w_t H_t + a\left(u_t\right) K_t, \tag{19}$$

$$\sum_i K_{it} = \sum_j K_t^j, \qquad \sum_i H_{it} = \sum_j H_t^j, \tag{20}$$

and the following autoregressive processes for exogenous shocks:

$$\log\left(\frac{e_t^\varkappa}{e^\varkappa}\right) = \rho_\varkappa \log\left(\frac{e_t^\varkappa}{e^\varkappa}\right) + \epsilon_t^\varkappa, \tag{21}$$

where $\varkappa = \{B, P, I, A, W\}$, $\rho_\varkappa$ are autoregressive parameter and $\epsilon_t^\varkappa$ are mean zero, i.i.d. random shocks with standard deviation $\sigma^\varkappa$.

[5]In the context of a NK model, Cantore et al. (2012) compare simple interest-rate rules embedding the model-based definition of the output gap to rules employing deviations of output from the steady state. They find that when the two types of rule are designed to be optimal, they result in almost identical real and inflation outcomes.

2.6 Functional forms

The utility function specialises as in Jaimovich and Rebelo (2008),

$$U(X_t, H_t) = \frac{X_t^{1-\sigma_c}}{1-\sigma_c}\left(1 - \psi H^{\vartheta}\right)^{1-\sigma_c}, \tag{22}$$

where $\sigma_c > 0$ is the coefficient of relative risk aversion, ψ is a scaling parameter that determines the relative weight of leisure in utility and ϑ is a preference parameter that determines the Frisch elasticity of labour supply.

In order to allow for complementarity between private and public consumption we specialise the consumption composite as a constant-elasticity-of-substitution (CES) aggregate,

$$X_t = \left\{\nu_x^{\frac{1}{\sigma_x}}\left[(X_t^c)^j\right]^{\frac{\sigma_x-1}{\sigma_x}} + (1-\nu_x)^{\frac{1}{\sigma_x}} G_t^{\frac{\sigma_x-1}{\sigma_x}}\right\}^{\frac{\sigma_x}{\sigma_x-1}}, \tag{23}$$

where ν_x is the weight of private goods in the aggregate and σ_x is the elasticity of substitution between private and public consumption. Such a specification is desirable in that it encompasses (i) the case of perfect substitutability between private and public consumption (when $\sigma_x \to \infty$); (ii) the Cobb-Douglas case of imperfect substitutability (when $\sigma_x \to 1$); (iii) the Leontief case of perfect complementarity (when $\sigma_x \to 0$); and (iv) the standard case in DSGE modelling of non-utility-enhancing public consumption (when $\nu_x = 1$). If $\nu_x < 1$ and $\sigma_x < \infty$, then public consumption affects the marginal utility of private consumption and the marginal disutility of labour, thus influencing consumption/saving decisions and the labour supply.[6]

The remaining functional forms are as in Smets and Wouters (2007). First, investment adjustment costs are quadratic: $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$, $\gamma > 0$, where γ represents the elasticity of the marginal investment adjustment cost to changes in investment. Next, the cost of capital utilisation is $a\left(u_t\right) = \gamma_1\left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2$. Following the literature, the steady-state utilisation rate is normalised to unity, $u = 1$. It follows that $a\left(u\right) = 0$, $a'\left(u\right) = \gamma_1$, $a''\left(u\right) = \gamma_2$ and the elasticity of the marginal utilisation cost to changes in the utilisation rate is $\frac{a''(u)u}{a'(u)} = \frac{\gamma_2}{\gamma_1} \equiv \sigma_u$, which is what we estimate. The production function is a conventional Cobb-Douglas: $F\left(e_t^A H_t, \tilde{K}_t\right) = \left(e_t^A H_t\right)^{\alpha}\tilde{K}_t^{1-\alpha}$, where α represents the labour share of income and $\tilde{K} \equiv u_t K_t$ is capital services. Equilibrium conditions with these functional forms (for the full model including also the features of long-term government bonds described in Section 5) are provided in Appendix A.

[6] In our estimation below, it turns out that $\sigma_x < 1$ indicating empirical support for *complementarity* between private and public consumption (on this see also the Bayesian estimates of Cantore et al. (2014)).

Parameter		Value
Discount factor	β	0.99
Capital depreciation rate	δ	0.025
Production function parameter	α	0.67
Steady-state gross inflation rate	Π	1.0075
Steady-state government spending share of output	$\frac{G}{Y}$	0.19
Steady-state consumption tax rate	τ^C	0.05
Steady-state labor income tax rate	τ^W	0.24
Steady-state capital tax rate	τ^K	0.32
Steady-state government debt-to-GDP ratio	Γ	4×0.70
Scaling factor in utility function	ϑ	2.8343
Elasticity of substitution in goods market	η	6
Elasticity of substitution in labor market	ζ	21
Haircut rate	$\bar{\Delta}$	0.0246
Scaling factor in default probability	η_1	-9.7480
Slope parameter in default probability	η_2	1.9921
Labor preference parameter	ψ	4.0728
Relative risk aversion	σ_c	1.9380
Habit formation	θ	0.7110
Investment adjustment cost elasticity	γ	4.8653
Capital variable utilisation elasticity	σ_u	4.9118
Preference for private goods	ν_x	0.6815
Elast. of subst. between private/public goods	σ_x	0.6349
Rotemberg wage stickiness	ξ^W	99.9851
Rotemberg price stickiness	ξ^P	29.9947
Interest rate smoothing	ρ_r	0.7879
Monetary policy response to inflation	ρ_π	0.4445
Monetary policy response to output	ρ_y	0.0342
Persistence of tax rates	ρ_τ	0.6252
Tax response to government debt	$\rho_{\tau b}$	0.0777
Tax response to output	$\rho_{\tau y}$	0.1478
Persistence of government spending	ρ_g	0.7662
Government spending response to government debt	ρ_{gb}	0.1197
Government spending response to output	ρ_{gy}	0.1624
Persistence of technology shocks	ρ_A	0.8868
Persistence of preference shocks	ρ_B	0.4149
Persistence of investment-specific shocks	ρ_I	0.6456
Persistence of price mark-up shocks	ρ_P	0.3308
Persistence of wage mark-up shocks	ρ_W	0.5714
Standard deviation of technology shocks	σ^A	0.0101
Standard deviation of preference shocks	σ^B	0.0163
Standard deviation of investment-specific shocks	σ^I	0.0591
Standard deviation of price mark-up shocks	σ^P	0.0422
Standard deviation of wage mark-up shocks	σ^W	0.0264

Table 1: Parameter values

3 Calibration-estimation

We assign numerical values to the parameters to match a number of stylised facts and moments of key macroeconomic variables of the US economy during the Great Moderation. The time period in our model corresponds to one quarter in the data. Table 1 reports all parameter values.

Three parameter values are conventional in the DSGE literature: the subjective discount factor $\beta = 0.99$ implies an annual real rate of interest of 4 percent; the capital depreciation rate $\delta = 0.025$ corresponds to an annual depreciation of 10 percent; while the Cobb-Douglas production function parameter $\alpha = 0.67$ entails a labor share of income of 2/3.

At the steady state we set a government spending share of output of 20 percent ($G/Y = 0.20$) and a gross inflation rate $\Pi = 1.0075$, corresponding to a net annual rate of inflation of 3 percent. The steady-state values of the tax rates are as in Christiano et al. (2014), i.e. $\tau^C = 0.05$, $\tau^W = 0.24$, and $\tau^K = 0.32$, while the baseline government debt is 70 percent of annual output ($\Gamma = 4 \times 0.70$).[7]

The scaling factor in the utility function $\vartheta = 2.8343$ is set to match a labor supply of 1/3 of available time at the steady state ($H = 0.33$), while the intratemporal elasticities of substitution in the goods and labor market ($\eta = 6$ and $\zeta = 21$, respectively) are set as in Zubairy (2014) to match average mark-ups of 20% and around 40% (given the labor income tax rate), respectively.

To calibrate the CDF of the fiscal limit, depicted in Figure 2, we fix two points on the function in a way consistent with empirical evidence. Given two points (Γ_1, p_1^*) and (Γ_2, p_2^*), with $\Gamma_2 > \Gamma_1$, parameters η_1 and η_2 are uniquely determined by

$$\eta_2 = \frac{1}{\Gamma_1 - \Gamma_2} \log \left(\frac{p_1^*}{p_2^*} \frac{1 - p_2^*}{1 - p_1^*} \right), \tag{24}$$

$$\eta_1 = \log \left(\frac{p_1^*}{1 - p_1^*} \right) - \eta_2 \Gamma_1. \tag{25}$$

Let us assume that when the ratio of government debt to GDP is Γ_2, the probability of exceeding the fiscal limit is almost unity, i.e. $p_2^* = 0.99$. We set the fiscal limit at $\Gamma_2 = 4 \times 1.8$, broadly in line with the Greek experience. Let us fix $\Gamma_1 = 4 \times 0.7$, the average public-debt-to-GDP ratio in US post-WWII experience. Given that, for most of this period, the US sovereign risk premium was very low – around 15 annual basis points (ABP) – we assume that for $\Gamma_1 = 4 \times 0.7$, $ABP_1 = 15$. At the onset of the sovereign debt crisis, the Greek sovereign risk premium skyrocketed to an order of magnitude of around 1,000 annual basis points, hence we fix $ABP_2 = 1,000$. From equation (13), the haircut rate, $\bar{\Delta}^g$, consistent with ABP_2 and p_2^* is

$$\bar{\Delta}^g = \frac{1 - \frac{1}{\frac{ABP_2}{40000} + 1}}{p_2^*},$$

[7] Later we examine the effects of high debt levels.

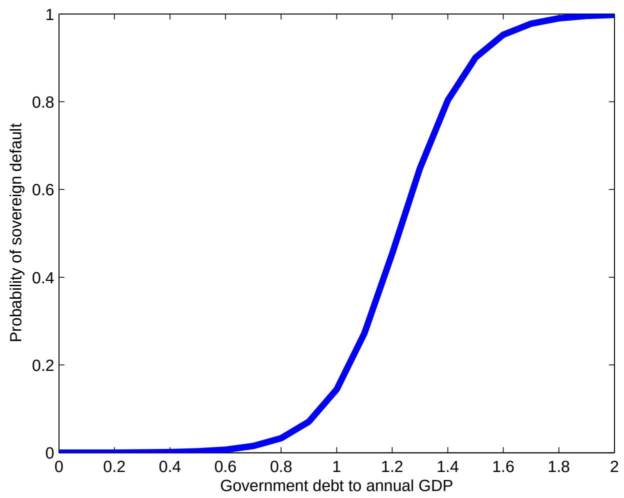

Figure 2: Cumulative density function of the fiscal limit

which implies $\bar{\Delta}^g = 0.0246$.[8] At this point, again from equation (13), we can recover the probability of default when $\Gamma = \Gamma_1$,

$$p_1^* = \frac{1 - \frac{1}{\frac{ABP_1}{40000}+1}}{\bar{\Delta}^g},$$

which is $p_1^* = 0.0152$, and parameters η_1 and η_2 of the fiscal limit CDF can be recovered by using equations (24) and (25), i.e. $\eta_1 = -9.7480$ and $\eta_2 = 1.9921$. This parameterisation implies that the sovereign risk premium rises from 15 annual basis points when government debt is 70% of annual GDP to, e.g., 143 and 452 annual basis points when the government debt ratio increases to 100%, and 120%, respectively. This captures the fact that problems related to sovereign default premia may mount at a very fast pace as public debt accumulates.

Last, we estimate (i) the remaining 18 structural parameters, (ii) the standard deviations, and (iii) the persistences of the five structural shocks via moment-matching of (a) the empirical standard deviations and (b) the persistences of real output, private consumption, investment, inflation, the real wage, hours worked, government spending, government revenues, and the federal funds rate; (c) the cross correlations between each macroeconomic variable and real output; and (d) the cross-correlation between private and public consumption, and between

[8] To see this, note that in the absence of long-term government bonds, equations (A.7) and (A.9) imply the following steady-state sovereign risk premium:

$$\frac{R^S}{R} = \frac{1}{(1-\Delta^g)} = 1 + \frac{ABP}{40000},$$

using which Δ^g can be written as a function of a chosen premium expressed in annual basis points, $\Delta^g = 1 - \frac{1}{1+\frac{ABP}{40000}}$. Finally, from equation (13) $\bar{\Delta}^g = \Delta^g / p^*$.

inflation and the federal funds rate. Details on data sources and transformations are provided in Appendix C.

Given the difficulty in matching exactly all moments, we construct a quadratic loss function $L = \sum_{j=1}^{28} \omega_j \left(x_j^m - x_j^d\right)^2$, where x_j^m is the j-th moment in the model, x_j^m is its analogue in the data while ω_j are weights. Given that matching volatilities is key for optimal policy exercises, we numerically search for those parameters that minimise L, assigning double weights to deviations of volatilities and splitting the remaining weights uniformly across the remaining targets.

This calibration-estimation procedure is similar to a more general method of moments estimation and delivers plausible parameter values, as well as volatilities, persistences and correlations of key macroeconomic variables that reasonably match the data, as it can be seen in Table D.1.[9]

4 Optimal monetary and fiscal stabilisation policy

We consider two aspects of monetary and fiscal optimal stabilisation policy. The first is stabilisation policy for *normal times*. Rules are designed to minimise an expected conditional welfare loss starting at some steady state. In this case the optimal policy problem is purely stochastic: optimal policy is in response to all future stochastic shocks hitting the economy. By contrast, *crisis management* starts with the economy (the debt-GDP ratio in particular) off the steady state (for whatever reason) so that policy is required both for the economy to return to the steady state (a deterministic problem) and for it to deal with future stochastic shocks (a stochastic problem). For both problems we adopt a linear-quadratic (LQ) set-up which, for a given set of observed policy instruments, considers a model linearised around a steady state, with a welfare function that is quadratic in deviations about the steady state.

4.1 The Ramsey problem and the LQ approximation

In the LQ approximation, the steady state about which the linearisation takes place corresponds to the steady state of the solution to the non-linear deterministic Ramsey problem, modified as described below, where the welfare to be maximised is that of the representative agent. From Levine et al. (2008a) and Benigno and Woodford (2012) we know that the quadratic approximation to the welfare is then given by the second-order approximation to the Lagrangian of the Ramsey problem, together with a linear term dependent on the initial value of the forward-looking variables of the system. When the latter is omitted (as is done below), the quadratic welfare approximation corresponds to the *timeless* approach to

[9]We prefer a moment-matching approach to a Bayesian estimation approach as the latter produces counterfactual volatilities for macroeconomic variables such as consumption and hours that are crucial for the policy analysis.

the solution. The timeless approach implies that the deterministic part of the solution has been followed for a long time, so that any deviations from steady state that are observed are purely due to the shocks hitting the system. The advantage of this LQ approximation is that the normal and crises components of policy conveniently decompose, and one optimal policy emerges conditional on the initial point.[10]

To be more explicit, Appendix B shows that the problem to be solved in the LQ context boils down to

$$\min_{\mathbf{x}_0,\mathbf{y}_t,\mathbf{w}_t} : -\mathbf{a}_1^T\mathbf{z}_0 - \mathbf{a}_2^T\mathbf{x}_0 + \frac{1}{2}\sum_{t=0}^{\infty}(\mathbf{y}_t^T\mathbf{Q}\mathbf{y}_t + \mathbf{w}_t^T\mathbf{R}\mathbf{w}_t), \qquad \mathbf{y}_t^T = [\mathbf{z}_t^T \;\; \mathbf{x}_t^T], \tag{26}$$

given $\mathbf{z}_0$, where $\mathbf{a}_1$ and $\mathbf{a}_2$ are constant row vectors, subject to linear dynamic constraints, where $\mathbf{z}_t$ represents the predetermined, $\mathbf{x}_t$ the non-predetermined variables, $\mathbf{w}_t$ a vector of policy instruments, while $\mathbf{Q}$ and $\mathbf{R}$ are conformable matrices. The first stage of the fully optimal solution to this is obtained by ignoring the forward-looking nature of $\mathbf{x}_t$, and the remainder of the problem can be formally represented as

$$\min_{x_0} : -\mathbf{a}_1^T\mathbf{z}_0 - \mathbf{a}_2^T\mathbf{x}_0 + \frac{1}{2}\mathbf{y}_0^T\mathbf{S}\mathbf{y}_0, \tag{27}$$

where $\mathbf{S}$ is the solution to a Riccati equation. Writing $\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix}$, it is easy to show that the jump in the initial value $\mathbf{x}_0$ is given by $\mathbf{x}_0 = \mathbf{S}_{22}^{-1}\mathbf{a}_2 - \mathbf{S}_{22}^{-1}\mathbf{S}_{21}\mathbf{z}_0$. The sub-optimal timeless solution is arrived at by removing the jump $\mathbf{S}_{22}^{-1}\mathbf{a}_2$. In our simulations that follow, we focus on a shock to debt in the initial period, and this is subsumed within the term $-\mathbf{S}_{22}^{-1}\mathbf{S}_{21}\mathbf{z}_0$, so that omitting $\mathbf{S}_{22}^{-1}\mathbf{a}_2$ does not have an impact with regard to the effect of a debt shock.

Our paper focuses on stabilization policy and not on the optimal tax structure. The steady state about which we approximate the model and the loss function is not the Ramsey optimum but rather a modified one that corresponds to observed fiscal variables (tax and debt). To achieve such a steady state in the LQ approximation procedure, we make an underlying assumption that fiscal policymakers assign a small cost to quadratic deviations of the fiscal variables about some target values corresponding to historically observed values, so that the steady state solution to the first order conditions are forced to these values. As a result of the small cost of these quadratic deviations, we can ignore their contribution to the

[10]Of the three policy regimes compared in the paper, optimal timeless commitment, optimized simple rules and optimal time-consistent policy (discretion), the first two can be computed without an LQ approximation of the non-linear set-up using perturbation methods for the stochastic solution, although the zero lower bound constraint still requires the penalty function approach of our paper. A Markov-perfect time-consistent solution requires global methods which are, as yet, not feasible for a medium-sized NK model with many state variables. For a small RBC model however see Dennis and Kirsanova (2015).

LQ approximation to the welfare. In addition, we pin down the government debt to GDP ratio by appropriately changing (non-distortionary) lump sum taxes. The advantage of this approach is that the steady states of the key macroeconomic variables are the same across (i) the three policy regimes, (ii) for the case of short bonds only versus that of long and short bonds; and (iii) steady state debt/income ratios of 70%, 90% and 120%.

4.2 Optimal monetary-fiscal rules for normal times

In this section we examine optimal policy using both monetary and fiscal instruments. As in Cantore et al. (2012) "optimality" can mean the welfare-optimal (Ramsey) policy, or time-consistent policy or optimised Taylor-type interest rate and fiscal rules.[11] Fiscal rules use (16) for the taxation instrument τ_t and (17) for government spending G_t.[12] Monetary policy is conducted according to (18).

One can think of this choice of rules as assigning responsibility for stabilising inflation and debt to the monetary authority and fiscal authorities respectively.[13] With both the interest rate and the fiscal instruments responding to fluctuations of output, the two authorities are sharing responsibility for output fluctuations.

The assignment issue arises in a different form in Leeper (1991), who provides the original characterisation of policy rules as being "active" or "passive". An active monetary policy rule is one in which the monetary authority satisfies the Taylor principle in that they adjust nominal interest rates such that real interest rates rise in response to excess inflation. Conversely, a passive monetary rule is one which fails to satisfy this principle. In Leeper's terminology a passive fiscal policy is one in which the fiscal instrument is adjusted to stabilise the government's debt stock, while an active fiscal policy fails to do this. Our simple rules allow for both these possibilities.[14]

For simple rules we impose two 'feasibility' constraints (Schmitt-Grohe and Uribe, 2007): $\rho_{r\pi} \leq 5$ and $\rho_{\tau B}, \rho_{GB} \leq 1$ to avoid the threat of excessive changes in the interest rate, tax

[11]The LQ solutions for these three policy regimes are now standard - see, for example, Levine et al. (2008a) for details. Regarding discretionary policy, recent important contributions by Blake and Kirsanova (2012) and Dennis and Kirsanova (2013) raise the possibility of multiple discretionary equilibria. These are of two types: "point-in-time", which give multiple responses of the private sector to a given policy rule and those arising from more than one discretionary policy. The iterative algorithm we use rules out the former. The latter can in principle be found by experimenting with different initialisations; however for the model and loss function employed in this paper we have not been able to find more than one equilibrium.

[12]A previous version of this paper compared the use of one or the other, but we found this of little of importance with regard to the main thrust of the paper.

[13]For a recent discussion of the assignment issue see Kirsanova et al. (2009).

[14]Cochrane (2011) proposes passive fiscal rules to avoid the arbitrary assumption of a non-explosive path for the price level needed in the standard Blanchard-Kahn rational expectations solution. But Sims (2013) points out that introducing a very small feedback from inflation to the tax-rate, together with ZLB constraint on the nominal interest rate and an upper bound on government asset accumulation, are sufficient to rule out such explosive paths.

rate and government spending.[15]

We shall examine the effects of these different policies after modifying our model by the introduction of long-term debt in Section 5.

4.2.1 Interest rate zero lower bound considerations

The aggressive nature of the optimal, time-consistent or passive simple rules often leads to high interest rate variances resulting in a ZLB problem for the interest rate for each of them. One way of getting round this problem is by a penalty function approach that adds a cost to the welfare function of $-w_r(R_t - R^*)^2$. The role of w_r is to lower the variance of the interest rate, thereby narrowing its probability density function, whereas the role of $R^* > 1/\beta$ is to shift the probability density function to the right. Both of these have the effect of lowering the probability of the ZLB being violated (see Levine et al., 2008a). In our results we focus solely on choosing the value of w_r in each regime such that the probability of hitting the ZLB is 0.0025, i.e. a very stringent ZLB requirement that the probability of hitting the zero lower bound is only once every 400 quarters or 100 years. Once the optimum has been found, subject to the choice of w_r, the representative agent's utility approximation is then corrected by adding back the term $w_r var(R_t)$. In welfare loss terms, as shown in the tables, this is done by subtracting the latter term. In practice, we have found that $R^* > 1/\beta$ is only of importance in the time-consistent case, which is nevertheless considerably inferior to the benchmark timeless optimal rules and optimal simple rules with passive fiscal policy; as a consequence, we have ignored this aspect of the ZLB rules in the tables below.

4.2.2 Second-order conditions

For the Ramsey problem that we are addressing here we find that the second order conditions at the steady state are always violated. This implies that the correct optimal solution is one that even in a deterministic setup does not converge to a steady state. Such a solution is beyond the scope of this paper, but it turns out that we can always ensure that the second-order conditions are satisfied by introducing a cost $-w_r(R_t - R^*)^2$ into the utility function, as required anyway by the ZLB constraint. Note that even when we examine the full Ramsey case, when we allow for the steady state to emerge directly from the statement of the problem, second-order conditions are also violated. Once again we can usually correct for this by introducing the cost above. When this cost on the interest rate is insufficient,

[15] In fact $\rho_{\tau B}, \rho_{GB} \leq 0.25$ is the minimal feedback for either instrument separately to stabilise the government debt-income ratio when there is no risk premium - effectively the case for a debt to income ratio of 70%. However, for the higher debt to income ratios of this paper, interest payments on debt are higher, so this minimal feedback must be higher. For consistency throughout, we have therefore imposed an upper bound of 1, but robustness exercises have shown that changing this bound has very little effect on the performance of optimal passive rules for debt to income ratios of 70%.

B/4Y=0.7						
Rule	$[\rho_r,\ \rho_{r\pi},\ \rho_{ry}]$	$[\rho_\tau,\ \rho_{\tau B},\ \rho_{\tau y}]$	$[\rho_G,\ \rho_{GB},\ \rho_{Gy}]$	w_r	Adjusted Loss	cons eq %
Optimal	n/a	n/a	n/a	371	-0.0038	
TCT	n/a	n/a	n/a	135	0.0083	0.45
Simple (PF)	1, 0.232, 0	0.051, 0.25, 0	0.789, 0.018, 0	385	0.0000	0.14
Simple (AF)	0,0,0	0,0,0	0,0,0	135	0.0069	0.40
B/4Y=0.9						
Rule	$[\rho_r,\ \rho_{r\pi},\ \rho_{ry}]$	$[\rho_\tau,\ \rho_{\tau B},\ \rho_{\tau y}]$	$[\rho_G,\ \rho_{GB},\ \rho_{Gy}]$	w_r	Adjusted Loss	cons eq %
Optimal	n/a	n/a	n/a	296	-0.0046	
TCT	n/a	n/a	n/a	102	0.0088	0.49
Simple (PF)	1, 0.236, 0	0.504, 0.25, 0	0.733, 0.046, 0	381	0.0000	0.17
Simple (AF)	0,0,0	0,0,0	0,0,0	79	0.0066	0.41
B/4Y=1.2						
Rule	$[\rho_r,\ \rho_{r\pi},\ \rho_{ry}]$	$[\rho_\tau,\ \rho_{\tau B},\ \rho_{\tau y}]$	$[\rho_G,\ \rho_{GB},\ \rho_{Gy}]$	w_r	Adjusted Loss	cons eq %
Optimal	n/a	n/a	n/a	221	-0.0021	
TCT	n/a	n/a	n/a	63	0.0032	0.20
Simple (PF)	1,0.261,0	0.579,1,0	0.161, 0.749,0	291	0.0015	0.13
Simple (AF)	0,0,0	0,0,0	0,0,0	60	0.0054	0.28
B/4Y=1.2	**Bailout**					
Rule	$[\rho_r,\ \rho_{r\pi},\ \rho_{ry}]$	$[\rho_\tau,\ \rho_{\tau B},\ \rho_{\tau y}]$	$[\rho_G,\ \rho_{GB},\ \rho_{Gy}]$	w_r	Adjusted Loss	cons eq %
Optimal	n/a	n/a	n/a	310	-0.0047	
TCT	n/a	n/a	n/a	244	0.011	0.58
Simple (PF)	1,0.232,0	0,0.25,0	0,0,0	382	0.0002	0.18
Simple (AF)	0,0,0	0,0,0	0,0,0	60	0.0054	0.38

Table 2: Optimal policy results for alternative government debt/GDP ratios

then we need to introduce further costs to the other instruments, so this is a further reason to compare our various cases with the steady state fixed at the historical average.

4.2.3 Results

Table 2 shows the consumption-equivalent welfare losses for the various regimes - time consistent, optimal simple passive (PF) and active fiscal (AF) policy, coupled with active and passive monetary policy, relative to optimal timeless policy. These simulations are performed for three cases of steady state state debt/GDP ratio - 70%, 90% and 120%. As anticipated above, in each of these cases, the only additional calibrations that have to be performed are the steady-state levels of lump sum taxes (τ^L). In addition, when the steady state state debt/GDP ratio is 120%, because the effect of the risk premium is very large (see earlier discussion), we also run simulations for this case, under the assumption that the country never defaults - denoted as the “No Sovereign Risk” or “Bailout” case.

As we can see from the table, the optimal simple passive fiscal, active monetary rules generate an average welfare loss equivalent on average to 0.14% in consumption-equivalent terms, whereas the consumption equivalent welfare losses under time consistent or optimal simple active fiscal, passive monetary rules are considerably larger. Note too that in the latter case the optimal rules conform consistently to the the fiscal theory of the price level (FTPL),

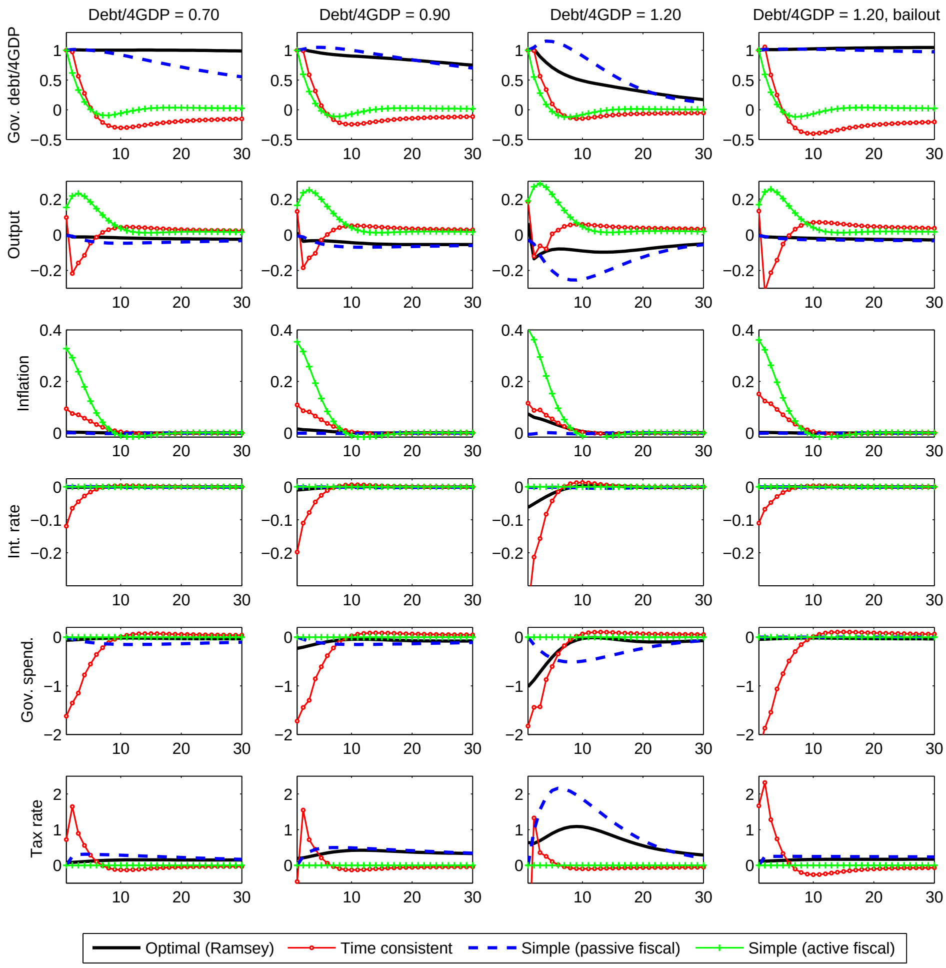

Figure 3: Effects of a shock to the level of debt under each of the four regimes, and under each of the four debt scenarios

with no movement at all in any of interest rates, government spending and taxation, and as we shall see shortly, the main reaction to shocks being an initial jump in the price level.

In all cases with passive fiscal policy, the optimised simple monetary policy rule is a price level rule, and both monetary and fiscal instruments do not react to output fluctuations. In addition, a low steady-state government debt/GDP ratio (70%) implies a stronger responsiveness of taxes relative to government spending (the latter displaying a high degree of inertia and a low elasticity to government debt). With high debt (120% of GDP) government spending becomes much more reactive to debt, unless the sovereign risk premium is suppressed.

B/4Y=0.7						
Rule	$[\rho_r,\ \rho_{r\pi},\ \rho_{ry}]$	$[\rho_\tau,\ \rho_{\tau B},\ \rho_{\tau y}]$	$[\rho_G,\ \rho_{GB},\ \rho_{Gy}]$	w_r	Adjusted Loss	cons eq %
Optimal	n/a	n/a	n/a	371	-0.0038	
Simple (PF)	1, 0.232, 0	0.051, 0.25, 0	0.789, 0.018, 0	385	0.0000	0.14
Superinertial	1.59,0.215,0.034	0,0.15,0	0.39,0.18,0	370	0.0025	0.05

Table 3: Optimal policy results for B/4Y=0.7 allowing for superinertial monetary policy

4.3 Crisis management of high debt: how fast, how deep?

We now turn to the case in which policy is used to stabilise the economy in the face of a sudden shock to government debt. In Figure 3 we depict the effects of a shock that raises the government debt/GDP ratio by 1% under the four policy regimes and each of the four debt scenarios. For the optimised simple rules we use the coeffiecients computed under "normal shocks" and reported in Table 2.

A number of noteworthy results emerge from the inspection of this figure. First, for the cases of debt/GDP ratio of 70% and 120% with no sovereign risk, optimal policy yields a level of debt that barely shifts at all over time. This is the standard "random walk" result that arises for debt under optimal policy when the inverse of the discount rate matches the gross real interest rate. Second, optimal simple passive fiscal rules produce trajectories very similar to those of the optimal timeless rules. Third, under optimal simple active fiscal rules, there is a large jump in prices as we would expect to see under the FTPL, and this jump is two or three times the size of that under optimal time consistent rules. Fourth, under time-consistent policy, debt consolidation is always very fast.

As a result, if the government can commit, the optimal rules and optimal passive simple rules display slow consolidation of debt levels unless the level of steady state debt is at 120%. Therefore, in a case similar to the recent Greek experience, the risk premium that must be paid would force debt levels to drop quickly, unless there is a bailout, which entails not charging this risk premium.

From this figure we see that optimal simple passive rules do not always closely mimic the path of debt, e.g. for the case of a debt/GDP ratio of 70%. A better simple rule can be designed in a straightforward way via *super-inertial* monetary policy i.e. allowing the value of ρ_r to be greater than 1.[16] For this case, the optimal simple rules lead to a very similar path for debt as the optimal rule (not reported in the figure), and we show the results on welfare for this case in Table 3, with a consumption-equivalent welfare compared with optimal policy now reduced to 0.05%. Thus, simple monetary-fiscal rules with passive fiscal policy, designed for an environment with "normal shocks", turn out to perform well in mimicking the Ramsey-optimal response to one-off government debt shocks.

We finally turn to the case of how to manage sudden larger jumps in debt. In particular, do

[16] For example, see Woodford (2003), Chapter 2.

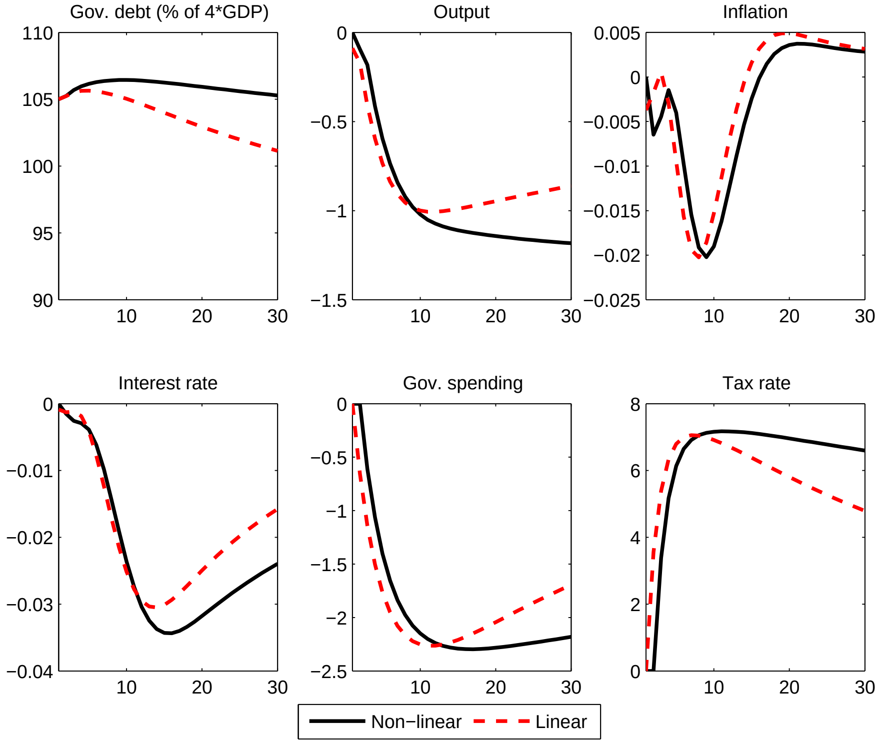

Figure 4: Comparison of responses of linear and nonlinear models for a jump from debt/GDP of 90% to 105%

the lessons drawn from studying the effects of a 1% shock to the government debt/GDP ratio apply to cases of larger shocks? In fact, given the nonlinear nature of the model, one needs to check whether the linear approximation of the LQ methodology provides a good enough guide to policy design for larger debt shocks. Therefore, in Figure 4, we plot the impulses responses from the linear and nonlinear versions of the model with the optimal simple passive fiscal, active monetary rule to a shock bringing the debt/annual GDP ratio from 90% to 105% .[17] Although there are some quantitative differences, notably that for the nonlinear simulations output takes longer to return to its base level, qualitatively there is not too much difference between the impulse responses of the two cases. If anything government debt is consolidated at an even slower pace in the nonlinear model, thus reinforcing the slow fiscal consolidation lesson drawn by using a linear model.

5 Introducing long-term government debt

To introduce long-term government debt in a straightforward way, we follow Harrison (2012) and assume that the government issues not only one-period bonds, B_t^S, but also consols, B_t^C. These yield one unit of currency each period for the infinite future and the value of a consol is denoted by V_t.[18]

The representation of government debt in terms of short and long bonds arises from the government budget constraint,

$$B_t^S + V_t B_t^C = (1 - \Delta_t^g)\left[R_{t-1}^S B_{t-1}^S + (1 + V_t)\, B_{t-1}^C\right] + P_t(G_t - T_t) + \Xi_t, \qquad (28)$$

where $\Xi_t \equiv \Delta_t^g \left[R_{t-1}^S B_{t-1}^S + (1 + V_t)\, B_{t-1}^C\right]$ represents the nominal transfer made by the government in case of default. The total value of long bonds is defined as $B_t^L = V_t B_t^C$ so that equation (28) may be rewritten as

$$B_t^S + B_t^L = (1 - \Delta_t^g)\left[R_{t-1}^S B_{t-1}^S + R_t^L B_{t-1}^L\right] + P_t(G_t - T_t) + \Xi_t, \qquad (29)$$

where $R_t^L \equiv (1 + V_t)/V_{t-1}$ is the *ex post* one-period return on consols.

Let $b_t^L \equiv B_t^S/P_t$ and $\tilde{\Xi}_t \equiv \Xi_t/P_t$, then the government budget constraint in real terms (14) is replaced by

$$b_t^S + b_t^L = (1 - \Delta_t^g)\left[\frac{R_{t-1}^S}{\Pi_t} b_{t-1}^S + \frac{R_t^L}{\Pi_t} b_{t-1}^L\right] + G_t - T_t + \tilde{\Xi}_t. \qquad (30)$$

Harrison (2012) assumes the real stock of consols to be held fixed at b^C so that the value of long-term bonds is given by

$$b_t^L = b^C V_t,$$

but here we assume that b_t^C *is a policy instrument*, and that the simple rule implemented for it is given by

$$\log\left(\frac{b_t^C}{b^C}\right) = \rho_{bc} \log\left(\frac{b_{t-1}^C}{b^C}\right) + \rho_{bcB} \log\left(\frac{b_t^g}{b^g}\right) - \rho_{bcY} \log\left(\frac{Y_t}{Y}\right) \qquad (31)$$

where ρ_{bc} is a smoothing parameter and $\rho_{bc,bg}$, $\rho_{bc,Y}$ are the feedback parameters on total real government debt $b_t^g \equiv (B_t^S + V_t B_t^C)/P_t$ and output. When we allow for the presence of

[17] The nonlinear simulations were obtained using the perfect-foresight fully nonlinear solver available in Dynare.

[18] Modelling long-term bonds as consols is a useful alternative to assuming that the long-term bond is a zero-coupon fixed-maturity bond and that there is no secondary market for long-term bonds. In fact the use of consols allows assuming that they can indeed be traded each period and that the optimal long-bond holdings depend on the one-period return on consols.

long-term debt, real short-term government debt in the fiscal rules (16) and (17) needs to be replaced by *total* real government debt.

In order for the presence of long-term government bonds to matter in the model, there must be impediments to arbitrage behaviour that equalises asset returns. We introduce these impediments as in Andrés et al. (2004) and Harrison (2012), i.e. by assuming that households perceive long-term bonds as less liquid and hence demand additional holdings of short-term government bonds when their holdings of long-term bonds increase. This assumption captures Tobin's claim that relative returns of different assets are affected by their relative supplies. To operationalise this mechanism, households are assumed to have a preference for keeping the ratio of short-to-long-term bond holdings constant and that departures from the preferred portfolio composition causes a welfare cost. This assumption translates into adding a convex portfolio adjustment cost, $-\frac{\nu^B}{2}\left(\delta^B \frac{(B^S_{t+s})^j}{(B^L_{t+s})^j} - 1\right)^2$, in households' utility function, which will affect the first-order conditions on $(B^S_t)^j$ and $(B^L_t)^j$. Parameter δ^B is set equal to the steady-state ratio of long-term bonds to short-term bonds, rendering the cost equal to zero at the steady state, while ν^B represents the elasticity of the long-term bond rate with respect to the portfolio mix. Following Harrison (2012), we set $\delta^B = 3$ and $\nu^B = 0.1$, in accordance with empirical evidence for the US.

5.1 Results

If we use the simple rule of Harrison (2012) that the number of long bonds is fixed ($b^C_t = b^C$, as in (A.35)), then although the results for the various regimes with regard to consumption equivalent welfare losses are very similar, the impulse response functions are different (results available upon request). This is clearly the result of not allowing for sufficient instruments given that there is a new variable that can affect the standard optimal policy result on random walk in debt. If we allow for the number of long bonds b^C_t to be an instrument, then we obtain qualitatively similar results to the case of short bonds only, as far as welfare rankings and optimised simple rules are concerned (see Table 4).

To appreciate the interesting differences arising from the addition of long-term government bonds, it is convenient to plot the impulse responses of key macroeconomic variables to a sudden increase in government debt, under the various debt/GDP scenarios, comparing optimal (Ramsey) and time-consistent policies across the baseline (short-term bonds only) versus the extended model (short and long-term bonds combined), as we do in Figure 5. When the government has the possibility of issuing long-term bonds, it is always optimal to increase the ratio of long-to-short bonds, in the face of a positive debt shock. Without government's commitment (time-consistent policy), the pace of consolidation is invariably fast, as it is in the model with short-term bonds only. Under commitment, it is still the initial government debt-to-GDP ratio to be the main driver of the optimal consolidation speed. However, if the

B/4Y=0.7							
Rule	$[\rho_r, \rho_{r\pi}, \rho_{ry}]$	$[\rho_\tau, \rho_{\tau B}, \rho_{\tau y}]$	$[\rho_G, \rho_{GB}, \rho_{Gy}]$	$[\rho_{bc}, \rho_{bcB}, \rho_{bcY}]$	w_r	Adj Loss	c eq %
Optimal	n/a	n/a	n/a	n/a	331	-0.0051	
TCT	n/a	n/a	n/a	n/a	87	0.0059	0.40
Simple (PF)	1.46,0.37,0.03	0,0.16,0	0.87,0,0.10	0.11,0.76,0.82	233	-0.0010	0.15
Simple (AF)	0,0,0	0,0,0	0,0,0	0,0	n/a	0.0061	0.41
B/4Y=0.9							
Optimal	n/a	n/a	n/a	n/a	329	-0.0050	
TCT	n/a	n/a	n/a	n/a	86	0.0051	0.37
Simple (PF)	1.35,0.36,0.02	0,0.50,0	0.86,0.01,0.12	0.01 0.86,0.98	233	-0.0008	0.15
Simple (AF)	0,0,0	0,0,0	0,0,0	0,0	n/a	0.0063	0.41
B/4Y=1.2							
Optimal	n/a	n/a	n/a	n/a	323	-0.0046	
TCT	n/a	n/a	n/a	n/a	88	0.0038	0.31
Simple (PF)	1.23,0.35,0.02	0.47,1,0	0.71,0.13,0.41	0.49,0.34,1	232	0.0006	0.19
Simple (AF)	0,0,0	0,0,0	0,0,0	0,0,0	n/a	0.0061	0.39
B/4Y=1.2	**no sov risk**						
Optimal	n/a	n/a	n/a	n/a	650	-0.0082	
TCT	n/a	n/a	n/a	n/a	88	0.0054	0.50
Simple (PF)	1.42,0.37,0.02	0, 0.21,0	0.88,0,0.10	0.882,1	240	-0.0010	0.27
Simple (AF)	0,0,0	0,0,0	0,0,0	0,0,0	n/a	0.0061	0.53

Table 4: Optimal policy results for alternative government debt/GDP ratios allowing for long-term government debt

government also issues long-term bonds, the optimal debt consolidation pace is slower than it is in the case of short-term bonds only, with the difference being more visible at higher debt levels implying greater sovereign risk premia.

6 Conclusions

Our paper contributes to a large recent literature of both an empirical and DSGE-modelling nature that has studied the optimal speed of fiscal consoldation and the monetary-fiscal rules that should be used to achieve this. It has added to a remarkable consensus on the subject, in part documented in our introduction, that – with some important caveats – optimal consolidation should be slow. Consider our non-linear simulation of the optimised fiscal-monetary simple passive fiscal, active monetary commitment rule (designed in a LQ framework). Even with an initial moderately high steady-state debt/GDP ratio of 90% that jumps to 105% (for whatever reason), sovereign risk implies a debt reduction policy, but one that allows the debt-GDP to rise for about 10 quarters, but then to slowly fall back to 105% after about a further 20 quarters. This 7-8 year adjustment is clearly a big departure from fiscal consolidation programmes we have seen implemented in the post-financial-crisis era.

Our contribution is, first, to employ an *estimated* DSGE model with both a full range of frictions and a rich fiscal component. Second, our comprehensive treatment of ZLB considerations, commitment, timelessness, costs of simplicity with simple passive fiscal, active

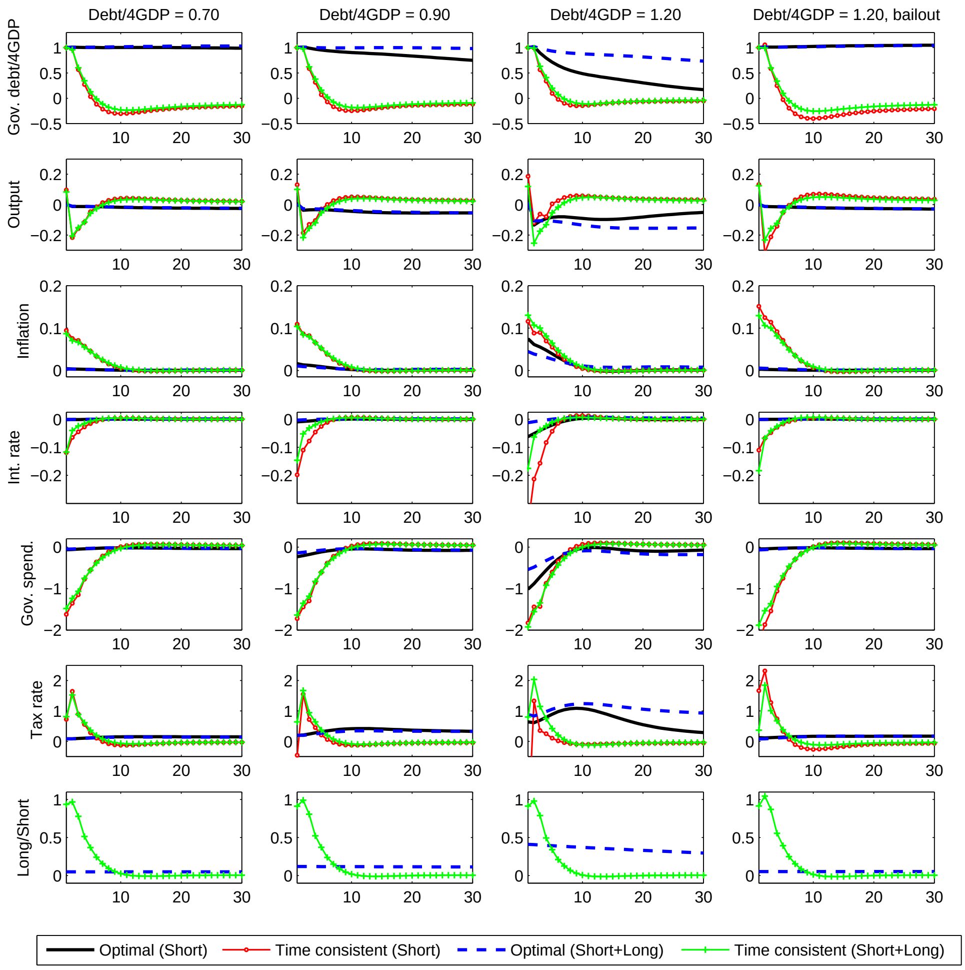

Figure 5: Effects of a shock to the level of debt under optimal and time-consistent policy, and under each of the four debt scenarios, allowing for long-term government debt

monetary and active fiscal, passive monetary rules is – we believe – unique in this literature.

We conclude by commenting on a technical but important question for policy analysis of this type. Our framework adopts a LQ approach. It is, therefore, important to assess how sensitive our results are to neglected non-linearities in the model and those arising from occasionally binding constraints for both the interest rate and from fiscal limits. A resolution of this question requires progress in computational solutions for medium-sized models.[19]

[19]Global solution methods are unlikely to escape from the curse of dimensionality for medium-sized NK models, especially if one insists (as we do) on employing system estimation for the quantitative assessment of policy options. A possible direction is a higher order pruned perturbation method formulated by Holden (2015) and implemented in his dynare toolkit: https://github.com/tholden/OBCToolkit.

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Appendix

A Equilibrium conditions

A.1 Utility function and marginal utilities

$$U(X_t, H_t) = \frac{X_t^{1-\sigma_c}}{1-\sigma_c}\left(1 - \psi H^{\vartheta}\right)^{1-\sigma_c} - \frac{\nu^B}{2}\left(\delta^B \frac{b_t^S}{b_t^L} - 1\right)^2 \tag{A.1}$$

$$X_t = \left\{\nu_x^{\frac{1}{\sigma_x}}\left[(X_t^c)\right]^{\frac{\sigma_x-1}{\sigma_x}} + (1-\nu_x)^{\frac{1}{\sigma_x}} G_t^{\frac{\sigma_x-1}{\sigma_x}}\right\}^{\frac{\sigma_x}{\sigma_x-1}} \tag{A.2}$$

$$U_{X^c,t} = e_t^B \nu_x^{\frac{1}{\sigma_x}} X_t^{-\sigma_c}\left(1 - \psi H_t^{\vartheta}\right)^{1-\sigma_c}\left(\frac{X_t}{X_t^c}\right)^{\frac{1}{\sigma_x}} \tag{A.3}$$

$$U_{H,t} = -e_t^B \vartheta \psi X_t^{1-\sigma_c}\left(1 - \psi H_t^{\vartheta}\right)^{-\sigma_c} H_t^{\vartheta-1} \tag{A.4}$$

A.2 Consumption/saving

$$X_t^c = C_t - \theta C_{t-1} \tag{A.5}$$

$$U_{X_t^c} - (1+\tau_t^C)\lambda_t = \theta\beta E_t\left[U_{X_{t+1}^c}\right] \tag{A.6}$$

$$1 = E_t\left[D_{t,t+1}\frac{R_t}{\Pi_{t+1}}\right] \tag{A.7}$$

$$D_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t} \tag{A.8}$$

$$1 = E_t \left[(1-\Delta_t^g)\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^S}{\Pi_{t+1}} \right] - \frac{\nu^B}{\lambda_t} \frac{\delta^B}{b_t^L} \left(\delta^B \frac{b_t^S}{b_t^L} - 1 \right) \tag{A.9}$$

$$1 = E_t \left[(1-\Delta_t^g)\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_{t+1}^L}{\Pi_{t+1}} \right] + \frac{\nu^B}{\lambda_t} \frac{\delta^B b_t^S}{(b_t^L)^2} \left(\delta^B \frac{b_t^S}{b_t^L} - 1 \right) \tag{A.10}$$

A.3 Investment

$$K_{t+1} = (1-\delta)K_t + e_t^I I_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] \tag{A.11}$$

$$Q_t = E_t \left\{ D_{t,t+1} \left[\left(1 - \tau_{t+1}^K\right) u_{t+1} R_{t+1}^K + \delta Q_{t+1} \tau_{t+1}^K u_{t+1} - a\left(u_{t+1}\right) + (1-\delta) Q_{t+1} \right] \right\} \tag{A.12}$$

$$e_t^I Q_t \left(1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + E_t \left(e_{t+1}^I D_{t,t+1} Q_{t+1} S'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right) = 1 \tag{A.13}$$

$$a'\left(u_t\right) = (1-\tau_t^K) R_t^K + \delta Q_t \tau_t^K \tag{A.14}$$

$$S\left(\frac{I_t}{I_{t-1}} \right) = \frac{\gamma}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \tag{A.15}$$

$$S'\left(\frac{I_t}{I_{t-1}} \right) = \gamma \left(\frac{I_t}{I_{t-1}} - 1 \right) \tag{A.16}$$

$$a\left(u_t\right) = \gamma_1 \left(u_t - 1\right) + \frac{\gamma_2}{2} \left(u_t - 1\right)^2 \tag{A.17}$$

$$a'\left(u_t\right) = \gamma_1 + \gamma_2 \left(u_t - 1\right) \tag{A.18}$$

A.4 Wage setting

$$(1-\tau_t^W) \left(e_t^W \eta - 1 \right) w_t - e_t^W \eta \frac{w_t}{\tilde{\mu}_t} + \xi^W \left(\Pi_t^W - \Pi \right) w_t \Pi_t^W = E_t \left[D_{t,t+1} \xi^W \left(\Pi_{t+1}^W - \Pi \right) w_{t+1} \Pi_{t+1}^W \frac{H_{t+1}}{H_t} \right] \tag{A.19}$$

$$\tilde{\mu}_t = w_t / MRS_t \tag{A.20}$$

$$MRS_t = -U_{H,t} / \lambda_t \tag{A.21}$$

$$\Pi_t^W = \frac{w_t}{w_{t-1}} \Pi_t \tag{A.22}$$

A.5 Production

$$F\left(e_t^A H_t, \tilde{K}_t\right) = \left(e_t^A H_t\right)^{\alpha} \left(u_t K_t\right)^{1-\alpha} \tag{A.23}$$

$$F_{H,t} = \alpha \frac{F(e_t^A H_t, \tilde{K}_t)}{H_t} \tag{A.24}$$

$$F_{K,t} = (1-\alpha) \frac{F(e_t^A H_t, \tilde{K}_t)}{u_t K_t} \tag{A.25}$$

$$Y_t = F(e_t^A H_t, \tilde{K}_t) - FC \tag{A.26}$$

$$R_t^K = MC_t F_{K,t} \tag{A.27}$$

$$w_t = MC_t F_{H,t} \tag{A.28}$$

$$(1 - e_t^P \zeta) + e_t^P \zeta MC_t - \xi^P \left(\Pi_t - \Pi\right) \Pi_t + \xi^P E_t \left[D_{t,t+1} \left(\Pi_{t+1} - \Pi\right) \Pi_{t+1}\right] Y_{t+1}/Y_t = 0 \tag{A.29}$$

A.6 Government

$$p_t^* = P\left(\Gamma_t^* \leq \Gamma_{t-1}\right) = \frac{\exp\left(\eta_1 + \eta_2 \Gamma_{t-1}\right)}{1 + \exp\left(\eta_1 + \eta_2 \Gamma_{t-1}\right)} \tag{A.30}$$

$$\Delta_t = p_t^* \Delta \tag{A.31}$$

$$\Gamma_t = b_t^g / Y_t \tag{A.32}$$

$$b_t^g = b_t^S + b_t^L \tag{A.33}$$

$$\log\left(\frac{b_t^C}{b^C}\right) = \rho_{bc} \log\left(\frac{b_{t-1}^C}{b^C}\right) + \rho_{bcB} \log\left(\frac{b_t^g}{b^g}\right) - \rho_{bcY} \log\left(\frac{Y_t}{Y}\right) \tag{A.34}$$

$$R_t^L = \frac{1 + V_t}{V_{t-1}} \tag{A.35}$$

$$b_t^S + b_t^L = (1 - \Delta_t^g) \left[\frac{R_{t-1}^S}{\Pi_t} b_{t-1}^S + \frac{R_t^L}{\Pi_t} b_{t-1}^L\right] + G_t - T_t + \tilde{\Xi}_t \tag{A.36}$$

$$\tilde{\Xi}_t = \Delta_t^g \left[\frac{R_{t-1}^S}{\Pi_t} b_{t-1}^S + \frac{R_t^L}{\Pi_t} b_{t-1}^L\right] \tag{A.37}$$

$$T_t = \tau_t^C C_t + \tau_t^W w_t h_t + \tau_t^K \left[\left(R_t^K - \delta Q_t\right) u_t K_t\right] + \tau_t^L \tag{A.38}$$

$$\tau_t^C = \tau_t \tau^C \tag{A.39}$$

$$\tau_t^W = \tau_t \tau^W \tag{A.40}$$

$$\tau_t^K = \tau_t \tau^K \tag{A.41}$$

$$\tau_t^L = \tau_t \tau^L \tag{A.42}$$

$$\log\left(\frac{\tau_t}{\tau}\right) = \rho_\tau \log\left(\frac{\tau_{t-1}}{\tau}\right) + \rho_{\tau B}\log\left(\frac{b_{t-1}^g}{b^g}\right) + \rho_{\tau Y}\log\left(\frac{Y_t}{Y}\right) \tag{A.43}$$

$$\log\left(\frac{G_t}{G}\right) = \rho_G \log\left(\frac{G_{t-1}}{G}\right) - \rho_{GB}\log\left(\frac{b_{t-1}^g}{b^g}\right) - \rho_{GY}\log\left(\frac{Y_t}{Y}\right) \tag{A.44}$$

A.7 Monetary policy

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + \rho_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \rho_y \log\left(\frac{Y_t}{Y}\right) \tag{A.45}$$

A.8 Resource constraint

$$Y_t = C_t + I_t + G_t + \frac{\xi^P}{2}\left(\Pi_t - \Pi\right)^2 Y_t + \frac{\xi^W}{2}\left(\Pi_t^W - \Pi\right)^2 w_t H_t + a\left(u_t\right) K_t \tag{A.46}$$

A.9 Autoregressive processes

$$\log\left(\frac{e_t^B}{e^B}\right) = \rho_\varkappa \log\left(\frac{e_t^B}{e^B}\right) + \epsilon_t^B \tag{A.47}$$

$$\log\left(\frac{e_t^P}{e^P}\right) = \rho_P \log\left(\frac{e_t^P}{e^P}\right) + \epsilon_t^P \tag{A.48}$$

$$\log\left(\frac{e_t^I}{e^I}\right) = \rho_I \log\left(\frac{e_t^I}{e^I}\right) + \epsilon_t^I \tag{A.49}$$

$$\log\left(\frac{e_t^A}{e^A}\right) = \rho_A \log\left(\frac{e_t^A}{e^A}\right) + \epsilon_t^A \tag{A.50}$$

$$\log\left(\frac{e_t^W}{e^W}\right) = \rho_W \log\left(\frac{e_t^W}{e^W}\right) + \epsilon_t^W \tag{A.51}$$

B The Ramsey problem and the LQ approximation

The problem is to maximise $E_0 \sum_{t=0}^{\infty} \beta^t u(\mathbf{y}_t, \mathbf{w}_t)$ such that

$$E_t f(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{y}_{t-1}, \mathbf{w}_t, \boldsymbol{\varepsilon}_t) = 0, \tag{B.1}$$

where $\mathbf{y}_t^T \equiv [\mathbf{z}_t^T \mathbf{x}_t^T]$, $\mathbf{z}_t$ is a vector of predetermined variables, $\mathbf{x}_t$ that of non-predetermined, 'jump' variables, $\mathbf{w}_t$ is a vector of instruments and $\boldsymbol{\varepsilon}_t$ is a vector of exogenous shocks. For convenience, assume that there are no higher order leads or lags greater than $\mathbf{y}_{t+1}$ and $\mathbf{y}_{t-1}$.[20]

Now write the Lagrangian for the problem as

$$L = \sum_{t=0}^{\infty} \beta^t [u(\mathbf{y}_t, \mathbf{w}_t) + \boldsymbol{\lambda}_{t+1}^T f(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{y}_{t-1}, \mathbf{w}_t, \varepsilon_t)], \tag{B.2}$$

From Levine et al. (2008b) and Benigno and Woodford (2012) we know that, for a purely backward-looking system, an approximate solution for this problem is obtained by solving for the deterministic steady state of the optimum, and then solving the stabilisation problem obtained by maximising the second order approximation to the Lagrangian, subject to the linearised constraints about this steady state.

First-order conditions are given by

$$\frac{\partial L}{\partial \mathbf{w}_t} = u_2 + \boldsymbol{\lambda}_{t+1}^T f_4(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{y}_{t-1}, \mathbf{w}_t, \varepsilon_t), \tag{B.3}$$

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{y}_t} &= u_1 + \boldsymbol{\lambda}_{t+1}^T f_1(\mathbf{y}_t, \mathbf{y}_{t+1}, \mathbf{y}_{t-1}, \mathbf{w}_t, \varepsilon_t) + \frac{1}{\beta}\boldsymbol{\lambda}_t^T f_2(\mathbf{y}_{t-1}, \mathbf{y}_t, \mathbf{y}_{t-2}, \mathbf{w}_{t-1}, \varepsilon_{t-1}) \\ &\quad + \beta \boldsymbol{\lambda}_{t+2}^T f_3(\mathbf{y}_{t+1}, \mathbf{y}_{t+2}, \mathbf{y}_t, \mathbf{w}_{t+1}, \varepsilon_{t+1}), \end{aligned} \tag{B.4}$$

where the subscripts in $\{u_i, f_j\}$ refer to the partial derivatives of the ith, jth variable in u, f.

Now partition $\boldsymbol{\lambda}_t = [\boldsymbol{\lambda}_{1,t}\, \boldsymbol{\lambda}_{2,t}]$ so that $\boldsymbol{\lambda}_{1,t}$, the co-state vector associated with the backward-looking component of (B.1) is of dimension $(n-m)\times 1$ and $\boldsymbol{\lambda}_{2,t}$, the co-state vector associated with the forward-looking component is of dimension $m \times 1$.[21]

An important optimality condition is:

$$\boldsymbol{\lambda}_{2,0} = 0; \text{ (ex ante optimal)} \tag{B.5}$$

$$\boldsymbol{\lambda}_{2,0} = \bar{\boldsymbol{\lambda}}_2; \text{ (`timeless' solution)} \tag{B.6}$$

where $\bar{\boldsymbol{\lambda}}_2$ is the deterministic steady state of $\boldsymbol{\lambda}_{2,t}$. To complete our solution we require $2n$ boundary conditions. Then together with (B.5) or (B.6), Z_0 given are n of these. The 'transversality condition' $\lim_{t\to\infty} \boldsymbol{\lambda}_t = \boldsymbol{\lambda}$ gives us the remaining n.

To arrive at the expression (27) in the main text and the contribution of initial values to the problem with rational expectations, write the second-order approximation to the Lagrangian

[20] If there are then just add to the vector $\mathbf{y}_t$ another variable that includes one of the lagged variables.

[21] In practice, assigning equations or variables as forward or backward looking is a non-trivial issue, since some variables that appear as forward looking may be functions of other forward looking variables. For full details of how to handle this for the LQ approximation see Levine and Pearlman (2011).

(for the deterministic case) as a deviation about its steady state value $\bar{L}$:

$$\begin{aligned} L-\bar{L} &= \sum \beta^t[\bar{u}_1\tilde{\mathbf{y}}_t+\bar{u}_2\tilde{\mathbf{w}}_t+\bar{\boldsymbol{\lambda}}^T(\bar{f}_1\tilde{\mathbf{y}}_t+\bar{f}_2\tilde{\mathbf{y}}_{t+1}+\bar{f}_3\tilde{\mathbf{y}}_{t-1}+\bar{f}_4\tilde{\mathbf{w}}_t) \\ &\quad +\tilde{\boldsymbol{\lambda}}_{t+1}^T(\bar{f}_1\tilde{\mathbf{y}}_t+\bar{f}_2\tilde{\mathbf{y}}_{t+1}+\bar{f}_3\tilde{\mathbf{y}}_{t-1}+\bar{f}_4\tilde{\mathbf{w}}_t)+\frac{1}{2}\sum_{i,j}(\bar{u}+\bar{\boldsymbol{\lambda}}^T f)_{ij}\tilde{\mathbf{x}}_{it}\tilde{\mathbf{x}}_{jt}], \end{aligned} \tag{B.7}$$

where $\bar{\boldsymbol{\lambda}}$ is the steady state of the Lagrange multiplier and $\tilde{\mathbf{y}}_t, \tilde{\mathbf{w}}_t$, represents deviations about $\bar{\mathbf{y}}, \bar{\mathbf{w}}$, where the latter are the steady states of the optimal problem, subscripts on f, u refer to partial derivatives and $\tilde{\mathbf{x}}_{it}$ refers to the various elements of $\tilde{\mathbf{y}}_t, \tilde{\mathbf{y}}_{t+1}, \tilde{\mathbf{y}}_{t-1}, \tilde{\mathbf{w}}_t$.

The problem approximates to maximisation of the discounted sum of the second-order terms in (B.7) subject to the linearised constraints of $f=0$. This is because the linear deviations cancel out with one another because of the first order conditions.

Note however that there are two terms in the first line of (B.7) that remain uncancelled:

$$-\frac{1}{\beta}\bar{\boldsymbol{\lambda}}^T f_2\tilde{\mathbf{y}}_0+\bar{\boldsymbol{\lambda}}^T f_3\tilde{\mathbf{y}}_{-1}. \tag{B.8}$$

This can be seen by expanding the first line of (B.7):

$$\begin{aligned} (L-\bar{L})_{1st\ order} &= \sum \beta^t[u_1\tilde{\mathbf{y}}_t+u_2\tilde{\mathbf{w}}_t+\bar{\boldsymbol{\lambda}}^T(f_1\tilde{\mathbf{y}}_t+f_2\tilde{\mathbf{y}}_{t+1}+f_3\tilde{\mathbf{y}}_{t-1}+f_4\tilde{\mathbf{w}}_t) \\ &= u_1\tilde{\mathbf{y}}_0+u_2\tilde{\mathbf{w}}_0+\bar{\lambda}^T(f_1\tilde{\mathbf{y}}_0+f_2\tilde{\mathbf{y}}_1+f_3\tilde{\mathbf{y}}_{-1}+f_4\tilde{\mathbf{w}}_0) \\ &+ \beta(u_1\tilde{\mathbf{y}}_1+u_2\tilde{\mathbf{w}}_1+\bar{\boldsymbol{\lambda}}^T(f_1\tilde{\mathbf{y}}_1+f_2\tilde{\mathbf{y}}_2+f_3\tilde{\mathbf{y}}_0+f_4\tilde{\mathbf{w}}_1)) \\ &+ \beta^2(u_1\tilde{\mathbf{y}}_2+u_2\tilde{\mathbf{w}}_2+\bar{\boldsymbol{\lambda}}^T(f_1\tilde{\mathbf{y}}_2+f_2\tilde{\mathbf{y}}_3+f_3\tilde{\mathbf{y}}_1+f_4\tilde{\mathbf{w}}_2))+...), \end{aligned} \tag{B.9}$$

and recalling from (B.4) for the $\tilde{\mathbf{y}}_0$ term that

$$u_1+\bar{\boldsymbol{\lambda}}^T(f_1+\frac{1}{\beta}f_2+\beta f_3)=0. \tag{B.10}$$

Clearly there is no maximisation with respect to $\tilde{\mathbf{y}}_{-1}$, but one is free to set the forward-looking variables in $\tilde{\mathbf{y}}_0$. Thus one must account for the forward-looking terms in $-\frac{1}{\beta}\bar{\boldsymbol{\lambda}}^T f_2\tilde{\mathbf{y}}_0$ when optimising policy.

C Data

In this section we describe the data sources and how we constructed the observables to be used in the estimation/calibration. In Table C we present the original dataset and the data sources.

From these sources data we constructed the 9 observables[22] and we considered the sub-

[22] Note that the resulting series of hours (as in table C) is then demeaned before it is used for the estima-

Label	Description	Source	Frequency
GDP	Nominal GDP	BEA NIPA Table 1.1.5	Q
PCE	Personal Consumption expenditure (total)	BEA NIPA Table 1.1.5	Q
PFI	Private Fixed Investment	BEA NIPA Table 5.3.5	Q
GCE	Government consumption expenditure and gross investment	BEA NIPA Table 1.1.5	Q
GR	Government revenues	BEA NIPA Table 3.1	Q
RGDP	Real GDP (base year 2005)	BEA NIPA Table 1.1.6	Q
CNP16OV	Civilian non-institutional population, over 16	BLS	Q
CE16OV	Civilian Employment sixteen years and over	BLS	Q
LBMNU	Non-farm business hours worked	BLS	Q
LBCPU	Hourly non-farm business compensation	BLS	Q
FFR	Federal Funds Rate	St. Louis FRED	Q

Table C.1: Data sources

sample 1984:Q1-2008:Q2 for the matching moment procedure.

All real variables were filtered using an HP(1600) filter.

Variable	Description	Construction
GDP_Deflator	GDP deflator	$\frac{GDP}{RGDP}100$
index	Population index	$\frac{CNP16OV}{CNP16OV_{2005:2}}$
CE16OV_index	Employment index	$\frac{CE16OV}{CE16OV_{2005:2}}100$
Observables	Description	Construction
R_GDP	Real per capita gross domestic product	$LN(\frac{\frac{GDP}{index}}{GDP_{Deflator}})100$
GOV_SP	Real per capita government spending	$LN(\frac{\frac{GCE}{index}}{GDP_{Deflator}})100$
GOV_RV	Real per capita government revenues	$LN(\frac{\frac{GR}{index}}{GDP_{Deflator}})100$
$HOURS$	Per capita hours worked	$LN(\frac{\frac{LBMNU*CE16OV_{index}}{100}}{index})100$
$WAGE$	Real wage	$LN(\frac{LBCPU}{GDP_{Deflator}})100$
FFF	Quarterly Federal Funds rate	$\frac{FFR}{4}$
Π	Inflation	$\Delta GDP_{Deflator}100$
CON	Real per capita consumption	$LN(\frac{\frac{PCE}{index}}{GDP_{Deflator}})100$
INV	Real per capita investment	$LN(\frac{\frac{PFI}{index}}{GDP_{Deflator}})100$

Table C.2: Data transformations - observables

tion/calibration.

D Moments

Moment	Data	Model
Standard deviations (in %)		
Real output	1.09	1.89
Private consumption	0.88	1.40
Private investment	4.06	6.94
Inflation	0.23	0.91
Real wage	1.10	1.82
Hours worked	2.20	1.96
Government spending	1.07	1.84
Government revenue	3.18	3.93
Interest rate	0.61	0.84
Autocorrelations		
Real output	0.8516	0.8824
Private consumption	0.8387	0.9742
Private investment	0.9148	0.9551
Inflation	0.5960	0.3791
Real wage	0.8197	0.8975
Hours worked	0.9113	0.6480
Government spending	0.7105	0.9811
Government revenue	0.8649	0.7606
Interest rate	0.9560	0.8731
Cross-correlations with output		
Private consumption	0.8723	0.6414
Private investment	0.9201	0.8779
Inflation	0.1795	0.1260
Real wage	-0.0539	0.7476
Hours worked	0.3714	0.4113
Government spending	-0.1550	-0.1008
Government revenue	0.7603	0.6920
Interest rate	0.3460	0.0129
Cross-correlations		
Inflation/Interest rate	0.4111	0.6540
Private/public consumption	-0.0334	-0.0938

Table D.1: Moments of key macroeconomic variables