Study of Tyre Damping Ratio and In-Plane Time Domain Simulation with Modal Parameter Tyre Model (MPTM)

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Outline

1. Introduction to Modal Parameter Tyre Model (MPTM)
2. Modeling of In-Plane Time Domain Simulation
3. Modeling of Damping Ratio
4. Conclusions
Introduction to MPTM

- An analytical tyre model by using experimental modal parameters

- In past two decades, used in a series of publications and experiments to address tyre mechanics models under different conditions

- Modal parameters reflecting tyre mechanics natural behaviors: tyre type, pressure

- Modeling tyre mechanics properties with modal parameters under different external inputs and constraints: e.g., velocity, load, contact surface, etc.

- Experiments for the specific working cases not required after extracting tyre modal parameters
MPTM: Tyre Modal Experiments

- Free-suspending wheel
- 3-D modal experiments and identification under 3-D excitation
MPTM: Tyre Models

- Static vertical properties: SAE 1998
- Steady-state cornering: VSD 2000
- Dynamic cornering: VSD 2002
- In-plane steady-state and dynamic under different conditions: cleat (TMPT), speeds
- Other models

![Graphs and diagrams showing various properties and models of tyres.](Image)
MPTM: Modeling Methodology

- 3-D free-suspending modal experiments
- Tyre physical parameters, and discretization at tyre perimeter

- **Constraint equations**
  - Tyre mechanics constraint equations with modal analysis theory
  - Tyre geometric constraint equations with or without rotation
  - External input constraints such as rolling velocity, acceleration, contact surface, load, etc.

- Solving constraint equations for each time step in real environments to obtain force and deformation distribution in time series
Modeling of In-Plane Time Domain Simulation: Static Case

- Tyre mechanics equations
- Geometric motion constraints
- External inputs and constraints
- Calculation
- Result

\[ D_r = t_r + s_r + c_r \]
\[ D_t = t_t + s_t + c_t \]

\[
\begin{bmatrix}
  c_r \\
  c_t \\
\end{bmatrix} =
\begin{bmatrix}
  H_{rr} & H_{tr} \\
  H_{rt} & H_{tt} \\
\end{bmatrix}
\begin{bmatrix}
  f_r \\
  f_t \\
\end{bmatrix}
\]

\[
H_{rr}^{ji} = \sum_{k=1}^{N} \frac{\varphi_{jk} \varphi_{ik}}{K_k}
\]

\[
x = (R_0 - D_r) \sin \theta + D_t \cos \theta
\]
\[
z = (R_0 - D_r) \cos \theta - D_t \sin \theta
\]

\[
\begin{align*}
  z &= H \\
  F_z &= \sum f_z \\
  F_x &= \sum f_x = 0
\end{align*}
\]

\[ AX = b, \ X = \{f_z, f_x, H, L\}, \ (2n+2) \text{ dimensions} \]
Modeling of In-Plane Time Domain Simulation: Rolling (Dynamic Case)

- Geometric rolling motion constraints

\[
D_t = -R_e \Delta \theta \cos \theta + (R_0 - D_{r0}) \sin \Delta \theta + D_{t0} \cos \Delta \theta
\]

\[
D_r = \frac{-H + R_0 \cos \theta - D_t \sin \theta}{\cos \theta}
\]
Modeling of In-Plane Time Domain Simulation: Rolling

Tyre mechanics equations

\[
\begin{align*}
d_{ji}(t) &= \sum_{k=1}^{N} \int_{0}^{t} \phi_{jk} \varphi_{jk} \cos(k \left( \frac{V\tau}{R_e} + \theta_0 \right)) \frac{1}{\omega_{dk}} e^{-\xi_k \omega_k (t-\tau)} \sin \omega_{dk} (t-\tau) \cdot f_j d\tau \\
d_{ji}(t) &= \sum_{k=1}^{N} \int_{0}^{t} h_k(t-\tau) \cdot \cos(k \left( \frac{V\tau}{R_e} + \theta_0 \right)) f_j d\tau
\end{align*}
\]

Steady-state in rolling: equal to a harmonic input

\[
\begin{align*}
d_{ji} &= \sum_{k=1}^{N} X_k \cdot \cos \left( b_k \frac{R_e}{V} \theta_0 - \lambda_k \right) \cdot f_j \\
X_k &= \frac{\varphi_{jk} \varphi_{jk}}{\sqrt{(\omega_k - b_k)^2 + (2\xi_k \omega_k b_k)^2}} \\
b_k &= \frac{kV}{R_e} \\
\lambda_k &= \tan^{-1} \left( \frac{2\xi_k \omega_k b_k}{\omega_k^2 - b_k^2} \right)
\end{align*}
\]
Modeling of In-Plane Time Domain Simulation: Rolling

- Tyre mechanics dynamic transient equations: decay + transient response, e.g.,

\[
d_{ji}(t) = \sum_{k=1}^{N} \int_{0}^{t} \varphi_{jk} \varphi_{jk} \cos(k \left( \frac{V \tau}{R_e} + \theta_0 \right)) \frac{1}{\omega_{dk}} e^{-\xi_k \omega_k (t-\tau)} \sin \omega_{dk} (t - \tau) \cdot f_j d\tau
\]

\[
= \frac{1}{2} f_j \sum_{k} \frac{\varphi_{jk}^2}{\omega_{dk}} \cos k \theta_0 \left\{ \frac{1}{(\xi_k \omega_k)^2 + p_{k1}^2} \left[ p_{k1} - p_{k1} e^{-\xi_k \omega_k t} \cos p_{k1} t - \xi_k \omega_k e^{-\xi_k \omega_k t} \sin p_{k1} t \right] \right\} + \\
\left[ \frac{1}{(\xi_k \omega_k)^2 + p_{k2}^2} \left[ p_{k2} \cos p_{k2} t - \xi_k \omega_k \sin p_{k2} t \right] \right] + \\
\left[ \frac{1}{(\xi_k \omega_k)^2 + p_{k1}^2} \left[ \cos p_{k1} t - \xi_k \omega_k \sin p_{k1} t \right] \right] - \\
\left[ \frac{1}{(\xi_k \omega_k)^2 + p_{k2}^2} \left[ \sin p_{k2} t - \xi_k \omega_k \cos p_{k2} t \right] \right]
\]

\[
p_{k1} = \omega_{dk} + k\omega \quad p_{k2} = \omega_{dk} - k\omega
\]
Modeling of In-Plane Time Domain Simulation: Rolling

- External inputs and constraints, such as:
  - Calculation

\[ \sum (f_z \cdot x + f_x \cdot H) = 0 \]

\[ A \cdot u = b \]

\[
\begin{align*}
  u &= \begin{bmatrix} f_z \\ f_x \\ R_e \end{bmatrix} \\
  b &= \begin{bmatrix} (R_0 - g_r)' \cdot [\cos \theta] - g_t' \cdot [\sin \theta] - H \\
                             D_t \cos \Delta \theta - (R_0 - D_{r0}) \sin \Delta \theta - g_t \\
                             0 \end{bmatrix}
\end{align*}
\]

\[
A = \begin{bmatrix}
    M_t \cdot [\sin \theta] + M_r \cdot [\cos \theta] & 0 \\
    M_t & -\Delta \theta \cdot \cos \theta \\
    (R_0 - D_{r0})' \cdot [\sin \theta] + D_{t0}' \cdot [\cos \theta] & -H \cdot ones(1,n) & 0
\end{bmatrix}
\]

- Rolling speed involved, 2n+1 unknown variables
- Time simulation for each tick: distri. forces and deformations
- Linear equation solving problem
- Under different inputs: velocity/acceleration, contact surface
Modeling of In-Plane Time Domain Simulation: Results

- Convergence of dynamic to steady-state in time simulation
- Dynamic response

<table>
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<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
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<td>123</td>
<td>150</td>
<td>177</td>
<td>206</td>
<td>236</td>
<td>268</td>
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Modeling of Damping Ratio with MPTM: A typical solution example for tough problems

- Question: using viscous or structural damping in modal theory for tyre models, especially longitudinal direction properties

- For example: rolling resistance issue in lower velocities

\[ m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \]

\[ m\ddot{x}(t) + k(1 + j\beta)x(t) = F(\omega)e^{j\omega t} \]

- Using structural damping ratio due to hysteresis
Modeling of Damping Ratio: Difficulties

- Structural damping ratio brings more problems: no analytical transient equation
  - How to get the structural damping ratio
  - How to use and define it in steady-state case
  - How to use and define it in transient case

- The model based on MPTM will solve these problems
Modeling of Damping Ratio

- Obtain the structural damping ratio from viscous damping ratio

\[ H_{\text{visco}} = \frac{1}{\omega_k^2 - \omega^2 + j2\xi_k \omega \omega_k} \]

\[ H_{\text{struc}} = \frac{1}{\omega_k^2 - \omega^2 + j\beta_k \omega_k^2} \]

\[ \omega_{\infty} \omega_k \rightarrow \beta_k = 2 \xi_k \]
Modeling of Damping Ratio

- **Steady-state solution with structural damping**
  - steady-state response in rolling is equal to an harmonic input which has the theoretical solution

\[
m\ddot{x} + \frac{\beta k}{\omega} \dot{x} + kx = F \cos \omega t \quad x(t) = \frac{F}{[(k - \omega^2)^2 + (\beta k)^2]^{1/2}} \cos(\omega t - \tan^{-1}\left(\frac{\beta k}{k - \omega^2}\right))
\]

\[
x(\theta) = \sum_{r=1}^{N} f_i \cdot \bar{X}_r \cos(b_r \frac{R}{V} \theta - \lambda_r)
\]

\[
\bar{X}_r = \frac{\varphi_{ir} \varphi_{ir}}{[(\omega^2_r - b_r^2)^2 + (\beta_r \omega_r^2)^2]^{1/2}}, \quad \lambda_r = \tan^{-1}\left(\frac{\beta_r \omega_r^2}{\omega_r^2 - b_r^2}\right), \quad b_k = \frac{kV}{R_e}
\]
Modeling of Damping Ratio

- **Dynamic transient with structural damping**
  - The value of viscous damping used in transient should be changed with velocities.
  - The viscous damping is obtained by comparing its steady-state result with structural damping result.

\[
d_{ji} = \sum_{k=1}^{N} X_k \cdot \cos(b_k \frac{R_e}{V} \theta_0 - \lambda_k) \cdot f_j
\]

\[
X_k = \frac{\varphi_{jk} \phi_{jk}}{\sqrt{(\omega_k - b_k)^2 + (2\xi_k \omega_k b_k)^2}}
\]

\[
b_k = \frac{kV}{R_e}
\]

\[
\lambda_k = \tan^{-1}\left(\frac{2\xi_k \omega_k b_k}{\omega_k^2 - b_k^2}\right)
\]

\[
x(\theta) = \sum_{r=1}^{N} f_i \cdot \overline{X_r} \cos(b_r \frac{R}{V} \theta - \lambda_r)
\]

\[
\overline{X_r} = \frac{\varphi_{kr} \phi_{kr}}{[(\omega_r^2 - b_r^2)^2 + (\beta_r \omega_r^2)^2]^{1/2}}, \quad \lambda_r = \tan^{-1}\left(\frac{\beta_r \omega_r^2}{\omega_r^2 - b_r^2}\right)
\]

\[
b_k = \frac{kV}{R_e}
\]

\[
2\xi_k \omega_k b_k = \beta_k \omega_k^2,
\]

\[
\xi_k = \frac{\beta_k \omega_k}{2\omega_k}
\]
Modeling of Damping Ratio: Results

- Rolling resistance
- Dynamic transient response over a cleat

![Graphs showing rolling resistance and damping ratio vs. velocity and frequency](image-url)
Conclusions

- MPTM is an effective tyre analytical modeling method by using only standard modal experiments almost without requiring any other experiments.

- In-plane tyre static and dynamic simulations based on MPTM shows the complete analytical process with geometric motion, mechanics and modal theory, external inputs, and calculation procedures. It can also be used in virtual proving ground simulation and other conditions.

- Modeling of structural damping ratio shows good agreements in rolling resistance and other calculation results. More importantly, this is the first time the methods of extraction and usage of structural damping in tyre mechanics are proposed.

- MPTM as the analytical model can be used to study and calculate more tyre mechanics properties in real environments.
Thank You for your attention!