(TP12.2)

A Study of Transient Cornering Property by Use of an Analytical Tyre Model

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1. Purpose of This Study (1)

The side force $F_y$ of a cornering tyre shows some delayed responses to steering inputs.

- Vehicle dynamics / Ride quality

- Side force: $F_y$
- Cornering force: $CF \approx F_y \cos a$
- Self-aligning torque: $M_z$ (SAT)
1. Purpose of This Study (2)

- Transient cornering property is characterized by ...
  - (1) Steady-state gain
  - (2) Time constant

- How can the part stiffness (tread, belt and sidewall) affect the response parameter (1) and (2)?
1. Purpose of This Study (3)

- Analytical descriptions of the steady and transient cornering, which is applicable to both tyre design and vehicle simulation.

- How can the tread, belt and sidewall stiffness change the step response of side force $F_y$ and self-aligning torque $M_z$?

Simplified modeling
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2. Steady Cornering Model

Load dependence of side force $F_y$ and self-aligning torque $M_z$ at a small slip angle $\alpha$.

- **Fiala Model**
  - E. Fiala (1954)
  - For bias tyres.
  - Poor description for $M_z$.

- **Neo-FIALA Model**
  - Miyashita & Kabe (2003)
  - For radial tyres.
  - Good description for $F_y$ and $M_z$. 
2. Steady Cornering Model

Origin of Side Force $F_y$

Shear deformation of tread rubber by slip angle $\alpha$

- Side force $F_y$, Self-aligning torque $M_z$
2. Steady Cornering Model

Neo-FIALA Model (1)

The cornering-tyre deformation is approximated by a combination of (i) Shear, (ii) Deflection and (iii) Torsion.

(i) Shear deformation of tread rubber

(ii) In-plane belt deflection ($F_y$ feedback)

(iii) Out-plane sidewall torsion ($M_z$ feedback)

Tyre illustrations are the extremely stretched view to help understanding.
2. Steady Cornering Model

Neo-FIALA Model (2)

[Steady Model] (For a small slip angle)

- Contact length \( l \)
- Contact width \( w \)
- Lateral spring constant of tread element \( C_y \)
- Deflection compliance of belt \( \epsilon \)
- Torsional stiffness of sidewall \( G_{mz} \)

\[
\begin{align*}
\alpha_e &= \alpha - \frac{M_z}{G_{mz}} \\
F_y &= K_{y0} \left( \tan \alpha_e - \frac{\epsilon l F_y}{3} \right), \quad K_{y0} = \frac{C_y w l^2}{2} \\
M_z &= A_{s0} \tan \alpha_e, \quad A_{s0} = \frac{C_y w l^3}{12}
\end{align*}
\]
2. Steady Cornering Model

Neo-FIALA Model (3)

The model can describe the steady load-dependence of $F_y$ and $M_z$ with the deformation stiffness of (i) Shear, (ii) Deflection and (iii) Torsion.

Comparison between measurements and model
(Load dependence at 200kPa, 10km/h, $\alpha=1\text{deg}$ for a 195/65R15 tyre)
The time differential term $\frac{dF_y}{dt}$ and $\frac{dM_z}{dt}$ are added to the $F_y$ and $M_z$ feedback loop, respectively.
3. Transient Cornering Model

Further Simplified Assumption

(a) The dynamic characteristic lengths, such as the relaxation length $\sigma$, are assumed to be roughly proportional to the contact length $l$. And the differential term $\frac{dF_y}{dt}$, $\frac{dM_z}{dt}$ contribute proportionally to $l/v$.

(b) The contribution of initial torsional torque to SAT $M_z$ is neglected.

(a) Bottom view of a cornering tyre (stretched)

(b) An neglected torque
3. Transient Cornering Model (2)

[Transient Model]

\[
\frac{F_y(s)}{a(s)} = \frac{K_y}{(1 + T_1s)(1 + T_2s)} \quad \text{(Transfer function of } F_y; \text{ 2nd-order lag)}
\]

\[
\frac{M_z(s)}{a(s)} = \frac{A_s}{1 + T_2s} \quad \text{(Simplified transfer function of } M_z; \text{ 1st-order lag)}
\]

\[
T_1 = \frac{K_{y0} \varepsilon \kappa l^2}{v (3 + K_{y0} \varepsilon \ell)} = \frac{K_y}{K_{dr}v} \quad \text{(Time constant of belt deflection)}
\]

\[
T_2 = \frac{A_{s0} \kappa l}{(A_{s0} + G_{mz})v} = \frac{A_s}{K_{rr}v} \quad \text{(Time constant of sidewall torsion)}
\]

\[
K_{dr} = \frac{3 \varepsilon \kappa l^2}{v (1 + A_{s0}/G_{mz})} \quad \text{(Deflection stiffness)}
\]

\[
K_{rr} = \frac{G_{mz}}{\kappa l} \quad \text{(Torsional stiffness)}
\]
3. Transient Cornering Model

Block Diagram
3. Transient Cornering Model

Prediction of Step Response

(a) Load dependence of steady cornering

(b) Step response of $F_y$ and $M_z$ at a fixed load
4. Numerical Simulations

How can the tread, belt and sidewall stiffness change the step response of side force $F_y$ and self-aligning torque $M_z$?

**Deformation Stiffness**  
(Neo-FIALA model)

**Part Stiffness**  
(Spring Bedded Ring)

[Conditions]

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>tyre size</td>
<td>195/65R15</td>
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<tr>
<td>inflation</td>
<td>200 kPa</td>
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<tr>
<td>load</td>
<td>4 kN</td>
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<tr>
<td>velocity</td>
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</table>
4. Numerical Simulations

Effect of 'Tread Ctr'

Side force: Down (earlier times)
Up (later times)

SAT: Up (later times)
4. Numerical Simulations

Effect of 'Belt EIz'

- Side force: Up (all times)
- SAT: No change
4. Numerical Simulations

Effect of 'Sidewall ky'

- Side force: Up (all times)
- SAT: Up (all times)
5. Model Validation

Model-parameter estimation by the response against 'Input A'  

[Input A] Quasi-step Input @40 km/h

Prediction of the response against 'Input B'

[Input B] Sinusoidal Input @5, 40, 120 km/h

[Measurement Conditions]

<table>
<thead>
<tr>
<th>Test Tyre</th>
<th>205/55ZR16</th>
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<tbody>
<tr>
<td>Inflation</td>
<td>200 kPa</td>
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<tr>
<td>Load</td>
<td>4.22 kN</td>
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<tr>
<td>Apparatus</td>
<td>MTS Flat-Trac III</td>
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</tbody>
</table>
5. Model Validation

Prediction of Frequency Response

- **Gain** — The estimated model is good agreement with measurements.
  (Input A)
- **Phase** — The model deviates from measurements at higher frequencies.
  (Input B)
  (that may come from the neglected viscoelasticity of rubber parts)
6. Conclusion

- The cornering-tyre deformation is approximately described with a combination of fundamental deformation (i)~(iii).

  (i) Shear deformation of tread rubber block
  (ii) In-plane belt deflection (time constant $T_1$)
  (iii) Out-plane sidewall torsion (time constant $T_2$)

- Deformation stiffness (i)~(iii) lead to the transient cornering model.

  \[
  \text{Side force: } \frac{F_y(s)}{a(s)} = \frac{K_y}{(1 + T_1 s)(1 + T_2 s)} \quad (2\text{nd-order lag})
  \]

  \[
  \text{SAT: } \frac{M_z(s)}{a(s)} = \frac{A_s}{1 + T_2 s} \quad (\text{Simplified, 1st-order lag})
  \]
Thank You for Your Attention.