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Citation: AIP Conf. Proc. 1491, 273 (2012); doi: 10.1063/1.4764255

View online: http://dx.doi.org/10.1063/1.4764255

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Quantifying the $^{12}$C + $^{12}$C Sub-Coulomb Fusion with the Time-Dependent Wave-Packet Method

Alexis Diaz-Torres* and Michael Wiescher†

*ECT*, Strada delle Tabarelle 286, I-38123 Villazzano, Trento, Italy
† JINA and Department of Physics, University of Notre Dame, IN 46656, USA

Abstract. This contribution provides a preliminary study of the $^{12}$C + $^{12}$C sub-Coulomb fusion reaction using the time-dependent wave-packet method within a nuclear molecular picture. The theoretical sub-Coulomb fusion resonances seem to correspond well with observations. The present method might be a more suitable tool for expanding the cross-section predictions towards lower energies than the commonly used potential-model approximation.

Keywords: Molecular collective states, Fusion, S-factor, Time-dependent wave-packet method
PACS: 24.30.-v, 25.70.Ef, 26.30.-k

Understanding the $^{12}$C + $^{12}$C sub-Coulomb fusion is a long-standing issue in heavy-ion physics. This reaction is critical for a number of stellar environments and conditions, and plays a key role in the chemical evolution of the Universe [1]. Of importance is to know the fusion cross section at energies near the Gamow peak (∼ 1.5 MeV). It is usually obtained by extrapolating high-energy fusion data [2, 3], as direct experiments are extremely difficult to carry out at very low incident energies (< 3 MeV). The presence of pronounced resonance structures in the observed fusion excitation curve makes the extrapolation very uncertain [4, 5, 6].

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The resonances may be mainly related to collective excitation modes in the dinuclear system, when the two $^{12}$C nuclei come into contact (Fig. 1). The $^{12}$C intrinsic symmetry axis vibrates and rotates with respect to the internuclear axis. The single-particle molecular shell structure at small internuclear distances is very sensitive to the alignment of the $^{12}$C nuclei [7]. Non-axial symmetric configurations preserve the individuality of the overlapping nuclei, while this is not the case for the axial symmetric configuration. The
former favors re-separation, and the latter fusion. The competition among these configurations, as a function of the incident energy and orbital angular momentum, should result in molecular resonance structures in the fusion excitation curve.

This picture is here quantified using the time-dependent wave-packet (TDWP) method which has not been much exploited in nuclear physics [8] unlike chemical physics [9]. This method involves three steps [9]:

1. the definition of the initial wave function $\Psi(t = 0)$,
2. the propagation $\Psi(0) \rightarrow \Psi(t)$, dictated by the time evolution operator, $\exp(-i\hat{H}t/\hbar)$, where $\hat{H}$ is the total Hamiltonian,
3. the calculation of molecular collective states and the fusion cross section from the time-dependent wave function, $\Psi(t)$.

The wave function and the Hamiltonian are represented in a multi-dimensional numerical grid. In this work, these are considered a function of five collective coordinates (Fig. 1): the internuclear distance $R$, and the $(\theta_1, \phi_1)$ and $(\theta_2, \phi_2)$ spherical angles of the $^{12}$C nuclei symmetry axis, thus reducing the complexity of the quantum many-body reaction problem. Moreover, the wave function is not expanded in any intrinsic basis (e.g., rotational or vibrational states of the individual nuclei), but it is calculated directly. The outgoing-wave-boundary condition at large internuclear distances as well as the irreversible process of fusion at small internuclear distances (usually described with an ingoing-wave-boundary condition) are here treated with the absorbing-boundary-condition [10]. The low-energy collision is described in the rotating center-of-mass frame within the nuclear molecular picture [11].

![Figure 2](image-url)

**FIGURE 2.** Cuts of the $^{12}$C + $^{12}$C collective PES, $V(R, \theta_1, \phi_1 = 0, \theta_2, \phi_2 = 0)$, as a function of the nuclei separation and alignment. The 90-90 alignment (dashed line) facilitates the access by tunneling to the potential pockets (3 – 6 fm). These are explored by the system, guided by the kinetic-energy operator [13]. The Coriolis force drives non-axial symmetric configurations towards the potential pocket of the axial symmetric configuration (solid line), where fusion occurs by a strong absorption.

Figure 2 shows cuts of the collective potential energy surface (PES) which is calculated using the finite-range liquid drop model [12] with nuclear shapes of the two-center shell model [7]. Fusion is here determined by two processes treated simultaneously:
(i) the tunneling through the many Coulomb barriers in Fig. 2, and (ii) the $^{12}$C nuclei re-alignment in the potential pockets. The 90-90 alignment (dashed line) dominates the Coulomb barriers penetrability, but the 0-0 alignment (solid line) is critical for fusion. Non-zero $J$ partial waves substantially contribute to the fusion cross section, as the centrifugal contribution to the 90-90 Coulomb barrier is small for low $J \leq 6\hbar$ and the $J$-dependent Coriolis interaction, whose strength increases with $J$ and the system compactness, strongly drives the non-axial symmetric configurations towards the potential pocket of the axial symmetric one (solid line), where fusion occurs.

The molecular resonance states with a total width of $\sim 100-180$ keV are presented in Fig. 3, which represent doorway states that irreversibly decay into more complex compound-nucleus states (simulated by the strong absorption providing the inclusive fusion probability) and the $^{12}$C + $^{12}$C continuum scattering states.

![Energy spectrum of the $^{12}$C + $^{12}$C system in the potential pockets of Fig. 2. The peaks are molecular resonances with a given spin and total width of $\sim 100-180$ keV. The widths relate to the total time interval spent by the dinuclear system in the potential pockets. These resonances are doorway states for fusion, which decay into more complex compound-nucleus states and the $^{12}$C + $^{12}$C scattering states. Various resonances exist near the Gamow peak energy (1.5 MeV).](image)

These molecular collective states are reflected in the total fusion excitation curve, as presented in Fig. 4 through the S-factor that includes $J \leq 8\hbar$. The fusion cross section, $\sigma_{\text{fus}}(E)$, is calculated taken into account the identity of the interacting nuclei and the parity of the radial wave function (only even partial waves $J$ are included), i.e., $\sigma_{\text{fus}}(E) = \pi\hbar^2 / (2\mu E) \sum_J (2J + 1)(1 + \delta_{1,2})P_J(E)$, where $\mu$ is the reduced mass, $E$ is the incident energy in the center-of-mass frame and $P_J$ is the partial fusion probability. The S-factor is $S(E) = \sigma_{\text{fus}}(E)E \exp(2\pi\eta)$, where the Sommerfeld parameter $\eta = (\mu / 2)^{1/2}Z_1Z_2e^2 / (\hbar E^{1/2})$ and $Z_i$ is the charge number of the $^{12}$C nuclei. The features resulting from the preliminary TDWP calculations (solid line) are consistent with those observed in the experimental data [2, 3] (squares). While the widths of the resonances show good agreement, the amplitudes are noticeably different: the theoretical predictions are too low in the higher energy range and too high in the lower energy range. This issue could be clarified using a bigger numerical grid (for checking convergence) and incorporating shell and pairing corrections into the collective mass and PES [14].
FIGURE 4. S-factor excitation function. There is a good qualitative agreement between measurements [2, 3] and preliminary TDWP calculations, highlighting the symphysis of molecular structure and fusion.

Using time-dependent wave-packet dynamics within a nuclear molecular picture, a quantitative study of the $^{12}$C + $^{12}$C sub-Coulomb fusion has been presented. Preliminary calculations are very promising, indicating a close correlation between molecular collective states and fusion. The present method might be a more suitable tool for expanding the cross section predictions towards lower energies than the usually employed potential-model approach.

ACKNOWLEDGMENTS

Support through the Joint Institute for Nuclear Astrophysics JINA NSF Grant Phys-0822648 is acknowledged.

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