Exotic shapes and exotic clusterization

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Content

I. Superdeformed (SD) states in N=Z nuclei
II. Clusterization of $^{36}$Ar: select reactions
III. $^{24}$Mg+$^{12}$C scattering: HD band
IV. Quasi-dynamical SU(3) symmetry
V. HD state from Nilsson-model
VI. Alpha-emitting reactions
VII. Summary and outlook
I. SD states in N = Z nuclei

Recently.
E.g. in $^{36}$Ar: Svensson et al, PRL 85 (2000) 2693.
Many theoretical studies concentrating on SD.
Clusterization is also important for the understanding of their structure.

HD state?
Alpha-cluster-model prediction. (Rae, Merchant, PLB 279 (1992) 207.)
Clusterization: which reaction channel can populate the shape isomers.
II. Binary clusterizations

of the GS, SD and HD in $^{36}$Ar
(J. Cseh, A. Algora, J. Darai, P.O. Hess, PRC 70 (2004) 0334311.)

GS and SD are experimentally known.
HD? Alpha-cluster-model, or $\beta = 0.86$.
Method:
a) Microscopic structure:
   U(3) selection rule + Harvey-prescription
b) Energetic preference.
Conclusion:
GS: asymmetric, HD: symmetric, SD: in between $^{24}$Mg+$^{12}$C in each of them.

HD state: $^{24}$Mg+$^{12}$C and $^{20}$Ne+$^{16}$O clusterizations.
Selection rules and Harvey’s prescription

Harvey’s prescription:

Only the number of quanta along the molecular axis (z) is effected (can change).

U(3) selection rule:

In general: \[ n_1, n_2, n_3 = n_1^{CI}, n_2^{CI}, n_3^{CI} \otimes n_1^{C2}, n_2^{C2}, n_3^{C2} \otimes n_1^r, 0, 0 \]

E.g.: \[ [4, 4, 0] \otimes [4, 4, 0] \otimes [12, 0, 0] = [16, 8, 4] \]

Similarly for the \( U^{ST}(4) \) spin-isospin group.
Harvey’s prescription

Only the number of quanta along the molecular axis (z) can change, while the numbers along the two other directions remain unchanged.

\[
\begin{array}{ccc}
0 & 0 & 3 \\
0 & 2 & 0 \\
1 & 1 & 0 \\
2 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
0 & 0 & 2 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{ccc}
12C & + & 12C \\
16O & + & 16O \\
\end{array}
\]
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<th>st</th>
<th>$I_{ex}$</th>
<th>$I_{sm}$</th>
<th>$U(3)$</th>
<th>$a : b : c$</th>
<th>$^{24}\text{Mg} + ^{12}\text{C}$</th>
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<td>$I_t$</td>
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<td>0.92</td>
<td>2.00</td>
<td>[20,20,12]</td>
<td>1.3:1.3:1</td>
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<tr>
<td></td>
<td></td>
<td>1.97</td>
<td>[20,18,14]</td>
<td>1.2:1.1:1</td>
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<td></td>
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<td>2.09</td>
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<td>[32,12,12]</td>
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<td>[32,16,8]</td>
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<td>4.21</td>
<td>[48,8,8]</td>
<td>2.5:1:1</td>
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HD state: $^{24}\text{Mg}+^{12}\text{C}$ and $^{20}\text{Ne}+^{16}\text{O}$ clusterizations, therefore, the HD state can be populated in the $^{24}\text{Mg}+^{12}\text{C}$ and $^{20}\text{Ne}+^{16}\text{O}$ reactions.
III. $^{24}\text{Mg}(^{12}\text{C},^{12}\text{C})^{24}\text{Mg}$ S. Paolo experiment

No satisfactory description with potentials.

2007-2008: potential + resonances:

GS, SD and HD states in $^{36}$Ar

![Graph showing the excitation energy $E^{*}(^{36}\text{Ar})$ (MeV) versus $J(J+1)$ for different bands in $^{36}$Ar. The bands include the HD-band, the GS-band, and the SD-band, indicated by different markers and colors.](image_url)
Structure of the shape isomers

SD state:
Nilsson, large-scale sm, Skyrme-HF, HF-BCS, AMD, core+alpha,…
4 hw excited, prolate shape with small triaxiality (uncertainty)

HD state:
algebra-cluster.
IV. Quasi-dynamical \textit{SU(3)} symmetry

from Nilsson-model + quasidynamical symmetry

Quasidynamical symmetry (Rowe et al, JMP 29 (1988) 572)

<table>
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<th>eigenvector</th>
<th>example</th>
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<td>+</td>
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<tr>
<td>q-dyn.sym.</td>
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Energy-eigenstate: linear combination of basis states belonging to different irreps, but a very special one.
Logic of the calculation

Quadrupole shape >
    Nilsson-model >
        q-dyn. SU(3) symmetry >
            quadrupole shape.

Shape-isomers: stability of the shape with respect to the input parameter.
V. HD state from Nilsson-model

SD: 4hw; joined conclusion of many theoretical studies.

HD: 12hw; alpha-cluster + Nilsson-q-SU(3) symmetry + binary clusterizations + experiment.
VI. Alpha-emitting reactions

So far experimental indications from scattering or transfer reactions: channels match with shape isomers.

How about alpha-emitting reactions (with possible gamma-decay), like e.g. that used for SD?

One-alpha-emitting reactions of alpha-like nuclei are also possible from the structural viewpoint:

\[ ^{20}\text{Ne}(^{20}\text{Ne},\alpha)^{36}\text{Ar}(\text{HD});
^{24}\text{Mg}(^{16}\text{O},\alpha)^{36}\text{Ar}(\text{HD});
^{28}\text{Si}(^{12}\text{C},\alpha)^{36}\text{Ar}(\text{HD}). \]

Two- or three-alpha emitting is also right (from structural viewpoint):

\[ ^{24}\text{Ne}(^{20}\text{Ne},2\alpha)^{36}\text{Ar}(\text{HD});
^{24}\text{Mg}(^{24}\text{Mg},3\alpha)^{36}\text{Ar}(\text{HD}). \]
VII. Summary and outlook

Symmetry-considerations can be helpful in studying the shape isomers, cluster configurations and their interrelations in light nuclei.

E.g. in $^{36}$Ar HD state (in addition to GS and SD states).
HD state: alpha-cluster + Nilsson-q. calculations
$^{24}$Mg+$^{12}$C and $^{20}$Ne+$^{16}$O prediction + observation.
Further experimental verification?! E.g. alpha-emitting react.

Heavy nuclei: exotic clusterization, very exotic clusterization.
Energetics + Pauli-principle.
Phenomenological models (microscopically very difficult).
Quasidynamical SU(3) can be helpful.
Thank you for your attention!