UniS

University of Surrey

Discussion Papers in Economics

THE DYNAMIC WAGE BARGAINING PROBLEM

By

Renuka Metcalfe
(University of Swansea)

DP 11/06

Department of Economics
University of Surrey
Guildford
Surrey GU2 7XH, UK
Telephone +44 (0)1483 689380
Facsimile +44 (0)1483 689548
Web www.econ.surrey.ac.uk
ISSN: 1749-5075

THE DYNAMIC WAGE BARGAINING PROBLEM

Renuka Metcalfe[1]

University of Swansea

This paper considers dynamic equilibria in wage bargaining unifying for the first time the models of Coles and Wright (1998) and Pissarides and producing in contrast to the Coles and Wright model, a non-deficient equilibrium. In sharp contrast to the Pissarides model we analyse a *fully* dynamic model with *non-linear* cost functions and *risk-averse* agents, to provide overall, *saddle-path stability* and *unique* wage and employment outcome which is *devoid* of limit cycles.

KEYWORDS: Wage determination, job matching, unemployment, labour markets, bargaining. J23, J31, J40, J64.

1. INTRODUCTION

A two-person ongoing bargaining situation arises when two individuals have the opportunity to collaborate in a long-term relationship for mutual benefit in various ways. In the simpler case, which is the one considered in this chapter, all aspects of the game are analysed, within a dynamic equilibrium. No aspect of the game is analysed in steady-state (in particular, the terms of trade of the ongoing partnership), as has been done in the past in the labour economics literature. No agent will bargain with his trading partner, myopically, without taking into account what his partner's expected bargaining endowments may be in the future, irrespective of whether these are favourable or adverse.

The ongoing labour economics situations of bargaining between members of trade union workers and firms, as expounded in the trade union models of Monopoly Union, Right to Manage and Efficient Bargaining Models, insiders and firms, can be regarded as dynamic

[1] I am grateful to Professor Paul Levine for his assistance. This paper is extracted from Chapter 5 of my thesis. Corresponding author: r.metcalfe@swan.ac.uk

bargaining problems. The object of this chapter is to provide a theoretical discussion of this problem, to obtain equilibria and to show overall a definite solution to dynamic analysis of wage determination, which could then provide an explanation of unemployment in a imperfectly competitive labour market[2]. Of course the results found here can be generalised to any economic situation where the relationship is ongoing between the two agents.

This is a classical problem of exchange in a labour market model in a dynamic environment, and more specifically of firms and workers, as developed by Shapiro and Stiglitz (1984), Stigler (1961), Lindbeck and Snower (1988c) and others. A different approach is suggested by Pissarides (2000), which permits the identification of this typical exchange in a dynamic labour market model. However, his analysis imposes Nash's axiomatic bargaining solution within a dynamic environment, a feature in common with the large literature on decentralised trade. In addition, he focuses on linear utility with risk-neutral agents and linear cost functions. Agents in general are risk averse. I have generalised the Pissarides model by considering *risk-averse* agents and *non-linear* cost functions.

In general terms, we assume that the two individuals have perfect foresight, are rational, forward-looking, and each can compare the utility he or she derives from various economic situations, each can accurately estimate the other's expected endowments at the time of bargaining, each is fully equipped with astute bargaining skills and each has perfect knowledge of the other's tastes, preferences and time preferences.

In developing our labour market model with random matching, strategic bargaining as in Pissarides (2000) (henceforth, referred to as Pissarides model), our terms of trade, the wage, is a differential function of time, in contrast to the Pissarides model. We assume that agents are forward-looking, and show that this makes a qualitative difference in the types of equilibria that emerge.

The concept of expectation with foresight in an ongoing relationship is crucial in this theory. This concept will be partly explained by way of illustration. Suppose a worker who, having formed a match with a firm, is bargaining with the firm as to the wage at some period t: the worker knows that the firm is expected to make a great deal of revenue in the near future. With knowledge of this, the worker bargains a higher wage. The firm accepts, which is consistent with rent-sharing theories, the sociological model of the efficiency wage

[2] I will leave this for my future research.

theory (Akerlof, 1982), insider-outsider theory and so forth. On the other hand, if the worker knows that the firm is going to perform adversely in the near future, and may even be on the verge of bankruptcy, the worker suppresses his wage demands. There is a body of evidence which is consistent with workers accepting paycuts, see for example, Smith (1994), Smith (2000), Nickell and Quintini (2003), who indicates that wages are significantly downward flexible. Also Brown, Ingram and Wadsworth (2004) and more recently, stewards of American Airlines[3] accepting paycuts to postpone bankruptcy of the airline.

Labour market models in the past, including that of Pissarides's analyses, imposes the Nash bargaining solution (henceforth, referred to as NBS), in a special case of the dynamic model. That is, although the bargainers are in an ongoing relationship, the bargainers bargain wage at its steady-state values, i.e., the values when $t \to \infty$, or they have equal rates of time preference. But the theory developed there makes no attempt to reconcile that rational bargainers in an ongoing relationship will bargain with foresight and will be forward-looking in a dynamic environment in all aspects and hence a dynamic equilibrium should be sought for a given non-stationary environment, that is, to determine what it is worth to each agent to have the opportunity to engage in an ongoing game with a long-term partner[4]. This determination is only accomplished in the case of a game in stationary environment.

In any labour market model, the way in which wage is determined plays a crucial role for the number of job matches taking place per unit of time, and thus for unemployment. Therefore, special attention will be focused on dynamic determination, which has been relatively neglected in the labour economics literature. I state at the outset that I have unified for the first time the Coles and Wright (1998) [henceforth, referred to as CW] and Pissarides (2000) model, to show a stable and unique wage and employment outcome with no limit cycles, that is, I show a new equilibrium concept. Whilst the CW model is equilibrium deficient in an entirely different model and Pissarides does not analyse a fully dynamic labour market model as effectively the steady state Nash solution is imposed in an otherwise dynamic model. In addition, Pissarides only analyses linear cost functions. In contrast, I analyse a *fully* dynamic labour market model with *risk averse* agents and *non-linear* cost functions. The CW model had a limit cycle in their model, whilst Pissarides did not perform *any* simulations and only

[3] See Financial Times, 26/4/03.

[4] I will refrain from discussing the typical difficulty (in terms of both monetary costs and time) of finding long-term partners, although this is an interesting issue. Search theory has covered some elements of this, for example, Mortensen (2002).

analysed a *two-dimensional* non-linear model. The latter derived the stability properties of his system purely by *visually* inspecting the signs of his simple two-dimensional system with only the unemployment and the market tightness variable.

Wages should reflect an equilibrium which is derived as a function of time, and by the construction of the said equilibrium the agreement is immediate. That is, this equilibrium will depend continuously on the set of the value of the labour services and the value of the output of these services over time, constituting the mathematical description of the game and which expresses the utility to each player of the opportunity to engage in the ongoing game. The ensuing equilibrium is non deficient, that is, it has a stable and unique wage outcome, in contrast to the extant literature.

CW study forward-looking bargaining in a *totally* different environment to ours in monetary economics, where of course, agents are different and behave markedly differently to ours. For example, where the two agents adopts the role of the other upon completion of the trade. This phenomenon is *unobserved* in an *ongoing* game in the labour market. However, their general theoretical result can be applied to our model.

The organisation of the chapter is as follows: In Section 2 I review the basic labour market model of matching. In Section 3 I analyse dynamic wage-bargaining and characterise the equilibrium in terms of a simple differential equation. In Section $4-7$ I integrate the dynamic bargaining solution to market equilibria. Section 8 concludes.

2. TRADE IN THE LABOUR MARKET MODEL

The main idea of the model is that trade in the labour market is typically a decentralised economic activity by agents in a dynamic environment, which is reflected more importantly in the price the agents trade as well. Trade is uncoordinated, time-consuming and costly for both firms and workers. That is, firms and workers expend resources prior to job creation and production, and existing jobs command rents in equilibrium, unlike the Walrasian labour markets. We use a simple modeling device to capture the implications of trade in a market equilibrium where I ensure that every aspect of the model is dynamic, including wage determination.

Pissarides develops a theory of unemployment which includes as a special case the two-person static wage-bargaining problem within an otherwise dynamic model, that is, where the expected values of employed workers, unemployed workers, the expected value of an occupied job and vacancies to the firm are all dynamic. But the theory there developed makes no endeavour to find a value for a given ongoing wage-bargaining game, that is, to determine what it is worth to each agent to have the opportunity to engage in the ongoing bargaining game, which is crucial to a proper development of any unemployment theory. The determination is accomplished only in the case of the two-person static bargaining game, within an otherwise dynamic environment. In other words, the special dynamic model using steady-state wages developed by Pissarides, makes no endeavour to characterise the wage as a differential function of time, in an ongoing relationship where in all other respects the model is dynamic. The model in this chapter is dynamic in all respects as it should be.

3. THE MATCHING FUNCTION AND THE BEVERIDGE CURVE

We use the matching function as a modelling device that captures the outcome of the investment of resources by firms and workers in the trading process as a function of inputs. In order to provide a theoretical treatment of dynamic wage-bargaining situations in the labour market, I abstract from the intricacies of the matching function, for example, with heterogeneous workers, jobs, skills, search intensity of both workers and firms, geographical areas and so forth, to form a mathematical model in terms of which to develop the theory.

Vacant jobs and unemployed workers become matched and move from trading to production activities, in accordance with the prevailing matching technology. Unemployment persists in the steady-state, due to the fact that during the matching process, and prior to all job-worker pairs matching, some of the existing jobs break up, due to shocks such as demand and technology, providing a flow into unemployment. Firms and workers search for the other agent, with full knowledge of the job-matching and job-separation process, but make no attempt to coordinate their actions.

The equilibrium developed in this chapter is a dynamic equilibrium. The aggregate equilibrium is where both agents maximise their respective objective functions, subject to the matching and separation technology and where both the inflows into and outflows out of unemployment are equal. We assume, that there is no on-the-job search. It has been claimed (see, for example, Pissarides (2000)), that it makes no qualitative difference to the theory of unemployment, whether the assumption of no on-the-job or on-the-job search is introduced. Time is considered as a sequence of discrete periods of length $\Delta t > 0$.

Then the number of job matches per unit time is given by the matching function

$$mL = m(uL, vL) \tag{1}$$

where L is the workers in the labour force, u is the unemployment rate, v is the number of vacant jobs as a proportion of the labour force and (1) is assumed to be increasing in both its arguments, concave and homogenous of degree 1. Job matches at any point in time are randomly selected from the sets vL and uL. Thus, a typical matching function is

$$m\left(u, v\right) = v\left[1 - e^{-\frac{u}{v}}\right] \tag{1a}$$

It follows, the Poisson process that fills vacant jobs vL has a rate

$$\frac{mL}{vL} = \frac{m(uL, vL)}{vL} = m(\frac{u}{v}, 1) = q(\theta) \tag{2}$$

or following $(1a)$

$$q\left(\theta\right) = \frac{m\left(u, v\right)}{v} = 1 - e^{-\frac{1}{\theta}} \tag{2a}$$

where, $\theta = \frac{v}{u}$, is a measure of labour market *tightness*, which one can interpret as the balance between the demand and supply of labour as in Brigden and Thomas (2003). Thus, $q\left(\theta\right)$ is the rate at which vacancies become filled. Note that $q\prime\left(\theta\right) < 0$. The unemployed workers move into employment according to a related Poisson process with rate

$$\frac{m(uL, vL)}{uL} = \frac{vL}{uL}\frac{m\left(uL, vL\right)}{vL} = \frac{v}{u}m\left(\frac{u}{v}, 1\right) = \theta q\left(\theta\right) = \theta(1 - e^{-\frac{1}{\theta}}) \tag{3}$$

In yet another Poisson process, suppose that the employed workforce of size $(1-u)L$ loses jobs at an exogenous rate λ per unit of time. Thus, the outflows from employment are $\lambda(1-u)$, per unit of time. The latter can be expressed as $u\theta q\left(\theta\right)L$, from (3). Hence the evolution of unemployment is

$$\dot{u} = \lambda\left(1-u\right) - \theta q\left(\theta\right)u \tag{4}$$

Then, in steady-state

$$\lambda\left(1-u\right)=\theta q\left(\theta\right)u \tag{5}$$

which implies,

$$u=\frac{\lambda}{\lambda+\theta q\left(\theta\right)} \tag{6}$$

This equation can be represented in vacancy-unemployment space, by a downward-sloping convex to the origin curve. The curve is referred to as the Beveridge curve. If we restate (5) in terms of the job flows, then clearly the key driving force of this model is job creation. This is because the empirical literature on job flows defines the job creation rate as $m(u,v)/(1-u)$, where $m(u,v)$ is the number of jobs created and $(1-u)$ is employment. In addition, the job destruction rate is also defined as the ratio of $\lambda\left(1-u\right)$ to $(1-u)$. Equating the constant λ to $\theta q\left(\theta\right)u/(1-u)$ yields (5), thereby demonstrating that it is job creation that is the main driving force of the model.

4. THE FIRM: THE VALUE OF A JOB AND A VACANCY

Each firm has one job when entering the market which it desires to fill. When the job is filled the firm obtains revenue by selling its output. The value of the output is some constant $p>0$. While the job is unfilled, the search costs are fixed at $h>0$ each time, which is a proportion of productivity. The rate at which jobs are filled is $q\left(\theta\right)$. These preliminaries are the same as in the Pissarides model.

The number of jobs available at any given time is determined by profit maximisation. All firms can open a vacancy and engage in searching. Thus, profit maximisation requires that the profit from a marginal vacancy is zero. In the environment of this model, with each firm having one vacancy only, profit maximisation is equivalent to a zero profit condition for firm entry. Let $J\left(t\right)$ be the present discounted value of expected profits from a filled job and $V\left(t\right)$ be the corresponding value of a vacant job, both evaluated at the beginning of the period $[t, t+\Delta t]$. Assume the rate at which vacant jobs become filled is $q\left(\theta\right)$ and the rate at which unemployed workers attain jobs is $\theta q\left(\theta\right)$. Assume that at each date t agents who can match complete negotiations immediately at $w\left(t\right)$, which could possibly be random. Then the job generates a profit $\left(p-c(w)\right)\Delta t$, during a small time interval and $c\left(w\right)$ is the cost of labour.

Assume $0 < c'(0) < 1$, and $c''(w) > 0$ for all $w \geq 0$, in sharp contrast to the Pissarides model. The latter implies that there are diminishing returns to scale. The reason for the former assumption will be seen later. The Pissarides model is restricted to *linear* utility only, that is, for instance, $c(w) = cw$. The job also dies with probability $\lambda\Delta t$ and survives with a probability $1 - \lambda\Delta t$. Hence, the standard dynamic programming (D.P.) equations for filled jobs and vacancies gives us

$$J(t) = \frac{1}{1 + r\Delta t}[(p - c(w))\Delta t + (1 - \lambda\Delta t) J(t + \Delta t)$$
$$+\lambda\Delta t V(t + \Delta t) + o(\Delta)] \tag{7}$$

$$V(t) = \frac{1}{1 + r\Delta t}[-h\Delta t + (1 - q(\theta)\Delta t) V(t + \Delta t)$$
$$+q(\theta)\Delta t J(t + \Delta t) + o(\Delta)] \tag{8}$$

where r is the rate of time preference and the term $o(\Delta)$ appears due to the Poisson process, which satisfies $\frac{o(\Delta)}{\Delta}$ as $\Delta t \to 0$[5].

Taking the limit as $\Delta t \to 0$ we obtain the standard continuous time equations

$$\dot{J} = rJ + \lambda(J - V) - (p - c(w)) \tag{9}$$

$$\dot{V} = rV + q(\theta)(V - J) + h \tag{10}$$

In equilibrium, with free entry of firms, all profit opportunities from new jobs are exploited, driving rents from vacant jobs to zero. Thus, in equilibrium, the supply of a vacancy is $V = 0$, which implies from (10)

$$J = \frac{h}{q(\theta)} \tag{11}$$

For an individual firm, $\frac{1}{q(\theta)}$ is the expected duration of an unfilled vacancy. The interpretation of (11) is that, in equilibrium, market tightness is such that the expected profit from a new job is equal to the expected cost of hiring. The latter is due to the competition for vacant

[5] For example, the interpretation of (7) is that between t and $t+\Delta t$ a firm meets a worker, with whom it can form a productive match, which yields payoff $(p - c(w))$ and with the probability $(1 - \lambda\Delta t)$ the job-specific shock not occuring and with probability $\lambda\Delta t V(t + \Delta t)$ the shock occuring.

jobs. Given (11), (9) will now be

$$\dot{J} = \frac{(r+\lambda)\,h}{q\,(\theta)} - (p - c\,(w)) = 0 \tag{9a}$$

Next, we consider the behaviour of workers.

5. THE WORKER: THE VALUE OF EMPLOYMENT AND UNEMPLOYMENT

Workers receive a wage w per unit of time if employed and an unemployed insurance and income z if unemployed. Assume the worker derives current utility of $\mu\,(w)$ from the wage. Assume $\mu'\,(w) > 0$, $\mu''\,(w) < 0$. That is, agents' behaviour are *risk-averse*, as mentioned above. This is another contrast to the Pissarides model. Let W and U be the corresponding value functions of employment and unemployment. Then following the same reasoning as before the D.P. equations are:

$$\begin{aligned} W\,(t) = \frac{1}{1+r\Delta t}[&\mu(w)\Delta t + \lambda\Delta t U(t+\Delta t) \\ &+(1-\lambda\Delta t)W\,(t+\Delta t) + o\,(\Delta)] \end{aligned} \tag{12}$$

$$\begin{aligned} U(t) = \frac{1}{1+r\Delta t}[&z\Delta t + (1-\theta q\,(\theta)\,\Delta t)\,U\,(t+\Delta t) \\ &+\theta q\,(\theta)\,\Delta t W\,(t+\Delta t) + z + o\,(\Delta)] \end{aligned} \tag{13}$$

where z is the unemployment benefit and letting $\Delta t \to 0$, we obtain

$$\dot{W} = rW - \mu(w) + \lambda\,(W - U) \tag{14}$$

$$\dot{U} = rU + \theta q(\theta)\,(U - W) - z \tag{15}$$

Since $q\,(\theta) = 1 - e^{-\frac{1}{\theta}}$, this implies, $\theta q\,(\theta) = \theta(1 - e^{-\frac{1}{\theta}})$. Considering both the discrete and continuous time value functions of both the firms and workers, it is abundantly clear, that it is the determination of the wage that plays a very crucial role in this model and indeed, in imperfectly-competitive theories of unemployment that have been put forward over the years. The wage, $w\,(t)$ remains to be determined. In the next section, I analyse explicit strategic bargaining games between the firm and worker. Prior to this, for illustrative purposes, I will consider the implications of the adoption of the NBS as it was in the Pissarides model (except that in the latter's model, strategic bargaining analyses were not used): $w(t) = w^n(t)$, where

$$w^n(t) = arg \max_w \ [W(t) - U(t)]^\beta [J(t) - V(t)]^{1-\beta} \tag{16}$$

where U and V are the threatpoints of worker and firm respectively, and $\beta \epsilon [0, 1]$ is the bargaining power of the worker. In addition, I need to impose that this maximisation is subject to the constraints that guarantee trade is voluntary; that is, there are gains to trade:

$$\omega_w(w, t) = \mu(w) + W(t) > U(t) \tag{17}$$

$$\omega_f(w, t) = \ p - c(w) + J(t) > V(t) \tag{18}$$

where the instantaneous payoff functions following a successful match are on the LHS of the inequality sign in both these equations.

We will choose a utility function of the form $\mu(w) = \frac{w^{1-\rho}}{1-\rho}$ with $0 < \rho < 1$, thereby rendering our game to be more general than the Pissarides model. $c(w)$ can also be normalised. Redefine costs in terms of units of utility. Two points arise from this definition. First, in the vicinity of $w = 0$, (18) becomes $\ p - c(0)\, w + J(t) > V(t)$. Hence the reason for the additional assumptions initially of $c'(0) < 1$. Second, if $c(w) = cw$ in conjunction with $\mu(w) = \frac{w^{1-\rho}}{1-\rho}$, then with $c < 1$, there will always be some $p - c(w)$ and $V(t)$ such that $p - c(w) + J(t) > V(t)$. The same applies to (17). I generalise the Pissarides model which has *constant returns-to-scale* and *linear utility* with risk-neutral agents and nonlinear cost functions with decreasing returns-to-scale. Therefore, $ITE \neq NBS$, in contrast to the assumption of $ITE \equiv NBS$ in the Pissarides model. Thereby, introducing a *new* equilibrium concept.

Given bargaining power, an equilibrium can be defined as a list of nonnegative and bounded paths $[J(t), V(t), W(t), U(t), w^n(t)]_{t=0}^{\infty}$ satisfying for all t the dynamic programming equations in either discrete or continuous time and the maximisation problem in (16) subject to (17) and (18).

In a related model CW claim to have obtained limit cycles. But this gives rise to indeterminacy and instability. Pissarides (2000) also analyses a labour market model, but effectively imposes the steady-state wage-bargaining solution, that is, the Nash solution in an otherwise dynamic model. In addition, he also only analyses a two-dimensional non-linear model, when considering out-of-steady-state dynamics. Second, Pissarides (2000) does not conduct any simulation as noted. In the next section I apply some results of CW to a model of Pissarides. In the succeeding section after that I define a *saddle-path stable* and *unique* equilibrium with *no* limit cycles in a *fully* dynamic model, in contrast to past studies, including CW and Pissarides (2000), where all or either of these elements are absent/lacking.

6. A BARGAINING MODEL BETWEEN FIRMS AND WORKERS

If both firms and workers form a successful match, their instantaneous payoffs are $\omega_w(w,t)$ and $\omega_f\,(w,t)$ as given in (17) and (18) respectively, where ω_w is increasing and ω_f is decreasing in w and both explicitly vary with time. Agent i discounts the future at rate $r_i > 0$, so the payoff for i from trading at t discounted back to date 0 is $e_i^{-r_i^t}\omega\,(w,t)$. Assume $\omega_i \epsilon\ C^2$, ω_i concave in w for all t, $\omega_i(w,t)$ bounded in t, and $\frac{\partial\omega_i(w,t)}{\partial t}$ bounded for all (w,t). Since firms exploit all profit opportunities from new jobs, in equilibrium $V = 0$. Workers derive some utility from not trading, but this is normalised to zero for expositional purposes in this section. This should have no impact on the qualitative result of this chapter. Define $\varsigma\,(t) = \{w_i : \omega_i\,(w,t) \geq 0,\ i = w,f\}$, and assume that $\varsigma\,(t)$ is nonempty for all t and uniformly bounded in t.

6.1. The Bargaining Process

In making our treatment of wage-bargaining we employ a Rubinstein-type bargaining process of the type considered in CW with the following features:

(i) Random alternating offers, where with some probability π_w, nature chooses the worker to propose a value of w and the firm, with probability $\pi_f = 1 - \pi_w$.

(ii) There is no delay (immediate agreement)

When agreements are reached, agents trade and depart. But when agreements have not been reached immediately, agents prefer to pursue bargaining until they have reached one in accordance with constraints (17) and (18), which is consistent with individual optimising behaviour[6].

Our object is to characterize subgame perfect equilibria in strategies that are history independent, but typically nonstationary, since payoffs are time varying. In equilibrium firms and workers reach immediate agreement upon meeting. This category of equilibrium is referred to as the Immediate Trade Equilibrium (ITE).

Following CW, we define reservation values $w_w(t)$ and $w_f(t)$ such that at time t the worker will accept any $w \geq w_w(t)$, and the firm will pay any $w \leq w_f(t)$. In addition, the best proposal is always the reservation value of the other agent. This implies we identify a strategy profile with $[w_w(t), w_f(t)]_{t=0}^{\infty}$, where each agent proposes the other's reservation values and accepts each agent's own reservation values, when it is his turn to accept.

Theorem 1. *In an ITE, in the limit as* $\Delta t \to 0$, the expected terms of trade, $w(t)$ is a differential function of t, (Coles and Wright, 1998), which satisfies

$$\dot{w} = \pi_f \left[\frac{r_w \omega_w (w,t) - \partial \mu_w (w,t) / \partial t}{\partial \omega_w (w,t) / \partial w} \right] + \pi_w \left[\frac{r_f \omega_f (w,t) - \partial \omega_f (w,t) / \partial t}{\partial \omega_f (w,t) / \partial w} \right] \tag{22}$$

By Theorem 1, if we know $w(t) = \hat{w}$ at any given time $\hat{t}$, for example, then the entire path $[w(t)]_{t=0}^{\infty}$ can be found by iterating on (22). Next, we will establish precisely such a condition, and thereby, identify an ITE. The next result considers the case where μ_i settle down over time, that is, when $t \to \infty$.

However, (22) permits ambiguity[7], when we consider Binmore's continuum example. In addition, currently, the Theorem is only applicable when agents use Markov strategies. Hence, a uniqueness argument can be provided to render Theorem 1 more general[8].

Theorem 2 *In the* $lim_{t\to\infty}\ w(t) = lim_{t\to\infty} w^n(t)$, *that is, the steady-state of the ITE and the Nash Solution coincide* (Coles and Wright, 1998). *That is,* $\omega_i(w,t) \to \bar{\omega}_i(w)$ *as* $t \to \infty$, *and* $\bar{\omega}_i$ *satisfies all the assumptions on* ω_i, *then in the limit as* $\Delta t \to 0$, *if an ITE*

[6] Later in this section I also generalize the model to permit exogenous breakdown in bargaining.
[7] I will leave this for my future research.
[8] I will leave this for my future research.

exists it is unique and $w(t) \to \bar{w}$ as $t \to \infty$ and satisfies

$$\bar{w} = arg \ \max \ \bar{\omega}_w (w)_f^{\beta} \bar{\omega}(w)^{1-\beta}$$

where

$$\beta = \frac{\pi_w r_f}{\pi_w r_f + \pi_f r_w}$$

Proof as in CW.

Let us consider again the Nash solution, as shown in (16), with different β and threat points, T_i set to zero, to those in (16),

$$w^n(t) = arg \ \max_w \ [\omega_w(w,t) - T_w(t)]^{\beta} [\omega_f(w,t) - T_f(t)]^{1-\beta} \tag{22a}$$

where $T_i(t) = 0$ and $\beta = \frac{\pi_w r_f}{\pi_w r_f + \pi_f r_w}$

The previous result says that, when $\lim_{t\to\infty} w(t) = \lim_{t\to\infty} w^n(t)$; but the coincidence does not generally hold when $t < \infty$. The conditions in Theorem 2 apply to the Pissarides (2000) model, when $\lim_{t\to\infty} w(t) = \lim_{t\to\infty} w^n(t)$, where the steady-state wage is imposed.

To, illustrate this, let us take the example, of risk-averse workers with foresight. Let $\omega(w) = \frac{w^{1-\rho}}{1-\rho}$, with $0 < \rho < 1$. and $\omega_f(w,t) = e^{-\delta t} - w$, so that the surplus to be divided is depreciating at rate δ (or, if $\delta < 0$, appreciating). In the Pissarides model, where $NS \equiv ITE$[9], when $\rho = 0$, which implies, workers are risk-neutral. Again assume $r_i = r$ and $\pi_i = \frac{1}{2}$, then (22) is

$$\dot{w} = \frac{r\omega_w [1 + (1-\rho)] - (1-\rho) e^{-\delta t} (r+\delta)}{2(1-\rho)}$$

Here, Theorem 2 implies $w(t) \to 0$ and the solution to the above differential equation, subject to this boundary condition, is

$$w^* = \frac{(r+\delta)(1-\rho) e^{-\delta t}}{r(2-\rho) + 2\delta(1-\rho)}$$

It is straightforward to establish that immediate trade is an equilibrium provided $(r+\delta) > 0$. In comparison, the Nash solution with the threatpoints and β that is applicable in steady states, implies that

[9] When $t \to \infty$ or $r_f = r_w$ (that is, the discount rates of agents are equal).

$$w^n = \frac{(1-\rho)\,e^{-\delta t}}{2-\rho}$$

for all t. While w^* and w^n converge to the same limit as $t \to \infty$, for finite t, $w^* > w^n$ if $\delta > 0$ and $w^* < w^n$ if $\delta < 0$.

Theorem 3. *Equivalence of ITE and Nash Solution along the entire path, not just in steady-state* (Coles and Wright, 1998).

Suppose $\omega_i(w,t) = \eta_i w + \varphi_i(t)$, where $\eta_w > 0 > \eta_f$, and $r_w = r_f = r > 0$. Then if an ITE exists with these functional forms, it is unique and $w(t) = w^n(t)$ with $T_i(t) = 0$ and $\beta = \pi_w$.

Proof as in CW.

In other words, $w^n(t) = w(t)$ for all t only when payoffs are linear in w, separable between w and t, and $r_w = r_f$[10].

Notice that the NBS solution (16) is similar to the one used in the Pissarides (2000) labour market model. The NBS used in the Pissarides model does coincide with an ITE, shown in Theorem 3. The conditions in Theorem 3 apply in the Pissarides model, except that the threat points are now $T_w = U$ and $T_f = V$ for the worker and firm respectively. This does not affect the logic of the Rubinstein-type solution though, since one simply measures the utility relative to these threat points. As we have shown that this changes, if wage, $w(t)$ is characterised as a differential function of time, in an ITE in the lim as $\Delta t \to 0$, the workers are risk-averse with single period utility given by $\frac{w^{1-\rho}}{1-\rho}$, $0 < \rho < 1$, as we have shown in an example earlier and if the cost function is for example, $c(w) = a_0 + a_1 w + a_2 w^2$.

Theorem 3 also corresponds to how wage-bargaining has been settled in the past in general, in the labour economics models, including all the trade union models and the main unemployment and the wage determination models. In a dynamic environment as in the Pissarides model, in general, where at least one of the functions, that is w or $c(w)$, is non-linear, we must use the forward-looking bargaining solution (22) to analyse dynamics.

[10] In the risk averse example, if we assume $\alpha = 1$, then the assumptions of Theorem 3 are satisfied, and we can confirm that $w^n(t) = w^*$ for all t, as shown below.

$w^n = \frac{e^{-\delta t}}{2}$

$w^* = \frac{(r+\delta)e^{-\delta t}}{2(r+\delta)}$

$w^* = \frac{e^{-\delta t}}{2} = w^n$

So far, we have assumed immediate trade. We need to check that this is consistent with equilibrium behaviour. Let $\pi_i(t) = e_i^{-r_i t}\omega_i[w(t), t]$ be the equilibrium payoff to i if agreement is made at time t, given w solves (22). Then an immediate trade for all t is an equilibrium, if

$$\pi_i(t) > 0 \qquad \textit{and} \qquad \pi_i'(t) < 0 \tag{23}$$

for all t and both firms and workers. The interpretation of this equation is that, agents would prefer to trade sooner than later.

The second inequality will hold for both agents, given w satisfies (22), if and only if the following condition holds as in CW:

$$\pi_i(t) = e_i^{-r_i t}\omega_i[w(t), t]$$

$$\left(r_w\omega_w - \frac{\partial\omega_w}{\partial t}\right)\frac{\partial\omega_f}{\partial w} - \left(r_f\omega_f - \frac{\partial\omega_f}{\partial t}\right)\frac{\partial\omega_w}{\partial w} < 0 \tag{24}$$

The agents will trade sooner as opposed to later if (23) is satisfied. The interpretation of (23) is that $e^{-r_i t}\omega_i(w(t), t)$ decreases in t $\forall\ w\epsilon\varsigma(t)$ and is strictly decreasing for one agent (Binmore, 1987).

We now consider the case where we permit exogenous breakdowns in the bargaining game. Let σ_i be the Poisson arrival rate with which i believes an exogenous breakdown will occur during bargaining, and b_i his utility in this event. Note, we now let r_i, π_i, σ_i and b_i be time varying. For brevity, this variance is not made explicit in the notations.

In this case, a simple generalisation of Theorem 1, yields, following CW

$$\dot{w} = \pi_f\left[\frac{(r_w+\sigma_w)\,\omega_w - \sigma_w b_w - \partial\omega_w/\partial t}{\partial\omega_w/\partial w}\right] + \pi_w\left[\frac{(r_f+\sigma_f)\,\omega_f - \sigma_f b_f - \partial\omega_f/\partial t}{\partial\omega_f/\partial w}\right] \tag{25}$$

Just as in the case with no breakdown, we can analogously show, that when $\omega_i(w, t)$ converges over time to $\bar{\omega}_i(w)$, then $\lim_{t\to\infty} w(t) = \bar{w}$ is the Nash solution with,

$$T = \frac{\sigma_i b_i}{r_i + \sigma_i}$$

$$\beta = \frac{\pi_w(r_f+\sigma_f)}{\pi_w(r_f+\sigma_f) + \pi_f(r_w+\sigma_w)}$$

Following Theorem 3, it can be shown that if payoff functions are linear and $r_w = r_f$ and

$\sigma_w = \sigma_f$, then $w(t) = w^n[t]$ along the entire path and not just in steady-state[11].

7. MARKET EQUILIBRIA

We are now in a position to characterize equilibrium in the labour market model, with the dynamic wage-bargaining model. Agents are of measure zero, that is, agents are one of the many in the economy, but of course, their actions are typical; the likelihood of any two traders meeting again after separation is zero. In addition, any delay to trade between two agents has no impact on the aggregate market outcome. Thus, while bargaining, each agent takes the expected value of returning to the market as given. We assume, now, that in the bargaining game $\pi_i = \frac{1}{2}$ and there are no exogenous breakdowns, that is, $(\sigma_i = 0)$. If the game is subject to delay, constraints (17) and (18) dictate that agents prefer to pursue bargaining than separate.

In discrete time, to describe an ITE, we need to determine $[W(t), U(t), J(t), (V)(t), w_w(t), w_f(t)]_{t=0}^{\infty}$. The instantaneous payoffs of a successful match are: $\omega_w(w,t) = \mu(w) + W(t)$ and $\omega_f(w,t) = (p - c(w)) + J(t)$, for the workers and firm respectively. In an equilibrium, beliefs must be consistent with market outcomes, that is, taking $[w_w(t), w_f(t)]_{t=0}^{\infty}$ as given, $[W(t), U(t), J(t), V(t)]_{t=0}^{\infty}$, must satisfy the dynamic programming equations, in discrete time and vice versa.

In the limiting case, as $\Delta t \to 0$; we need to determine $[W(t), U(t), J(t), V(t), w(t)]_{t=0}^{\infty}$, where the value functions solve the continuous time equations, and $w(t)$ is the value of $w_w(t)$ and $w_f(t)$ as $\Delta t \to 0$. If an ITE exists in the labour market, then w satisfies (22) ,which in this model becomes

$$\dot{w} = \frac{\left[r\mu(w) + rW(t) - \dot{W}\right]}{2\mu'(w)} - \frac{\left[r(p - c(w)) + rJ(t) - \dot{J}\right]}{2c'(w)} \tag{26}$$

It is crucial to note, in contrast to the Pissarides model, I generalise the cost function by using *non-linear cost function* and *risk-averse utility function*, which is also *non-linear*. The equilibrium is given by paths for $[W(t), U(t), J(t), (V)(t), w(t)]_{t=0}^{\infty}$ satisfying the continuous time equations and (26), subject to the constraints, which guarantee, that the trade is voluntary and the condition for immediate trade.

[11] All calculations of solutions are available upon request from the author.

We have reduced the dimensionality of the system, by defining, $x = U - W$. Subtraction of (14) and (15) is

$$\dot{x} = x\left(r + \theta q\left(\theta\right) + \lambda\right) + \mu\left(w\right) - z \tag{27}$$

inserting (9) and (14) into (26) yields

$$\dot{w} = \frac{r\mu\left(w\right) + \mu\left(w\right) + \lambda x}{2\mu'\left(w\right)} - \left(\frac{r\left(\left(p - c\left(w\right)\right) + \left(p - c\left(w\right)\right) - \lambda h\theta\right)}{2c'\left(w\right)}\right) \tag{28}$$

In addition, note $h/q\left(\theta\right) = J$. Thus, further simplifications to the system, as shown below, enabled us to depict the system in four dimensions, of $\left(x, \theta, w, u\right)$.

Incorporating the results from our analysis in the earlier sections, the full dynamical system is:

$$\begin{bmatrix} \dot{W} \\ \dot{U} \\ \dot{J} \\ \dot{V} \\ \dot{u} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} rW - \mu\left(w\right) + \lambda\left(W - U\right)] \\ rU + \theta q\left(\theta\right)\left(U - W\right) - z \\ rJ + \lambda\left(J - V\right) - \left(p - c\left(w\right)\right) \\ rV + q\left(\theta\right)\left(V - J\right) + h \\ \lambda\left(1 - u\right) - \theta q\left(\theta\right) u \\ \frac{r\mu(w)+\mu(w)+\lambda x}{2\mu'(w)} - \left(\frac{r(p-c(w))+(p-c(w))-\lambda h\theta}{2c'(w)}\right) \end{bmatrix}$$

where in order to maintain proper generality, $\mu\left(w\right) = \frac{w^{1-\rho}}{1-\rho}$. In contrast to the Pissarides model, my model is more general. The Pissarides model pertains to risk-neutral workers. CW also does not explicitly analyse risk-averse workers.

Following (27) and setting $V = 0$, implies, $J = \frac{h}{q(\theta)}$, we now have a system in four dimensions, which is

$$\begin{bmatrix} \dot{x} \\ \dot{J} \\ \dot{u} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} rx + \theta q\left(\theta\right) x + \lambda x + \mu\left(w\right) - z \\ rJ + \lambda\left(J\right) - \left(p - c\left(w\right)\right) \\ \lambda\left(1 - u\right) - \theta q\left(\theta\right) u \\ \left(\frac{r\mu(w)+\mu(w)+\lambda x}{2\mu'(w)}\right) - \left(\frac{r(p-c(w))+(p-c(w))-\lambda h\theta}{2c'(w)}\right) \end{bmatrix}$$

But

$$J = \frac{h}{q\left(\theta\right)} = h\left(q\left(\theta\right)\right)^{-1}$$

Then,

$$\dot{J} = \frac{-h\left(q\left(\theta\right)\right)^{-2} q'\left(\theta\right)}{\left(q\left(\theta\right)\right)^{2}} = \frac{h\dot{\theta}q'\left(\theta\right)}{\left(q\left(\theta\right)\right)^{2}}$$

and the above equation, becomes after rearrangement,

$$\frac{\dot{\theta}q'\left(\theta\right)}{\left(q\left(\theta\right)\right)^{2}} = -\frac{\left(r+\lambda\right)}{q\left(\theta\right)} + \frac{\left(p-c\left(w\right)\right)}{h}$$

$$\dot{\theta} = -\left(r+\lambda\right)\frac{q\left(\theta\right)}{q'\left(\theta\right)} + \frac{\left(p-c\left(w\right)\right)}{h}\frac{\left(q\left(\theta\right)\right)^{2}}{q'\left(\theta\right)}$$

Hence, the above dynamic system will now be in $\dot{\theta}$, instead of $\dot{J}$,

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \dot{u} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} x(r+\theta q\left(\theta\right)+\lambda)+\mu\left(w\right)-z \\ -\left(r+\lambda\right)\frac{q(\theta)}{q'(\theta)} + \frac{(p-c(w))}{h}\frac{(q(\theta))^{2}}{q'(\theta)} \\ \lambda\left(1-u\right)-\theta q\left(\theta\right)u \\ \left(\frac{r\mu(w)+\mu(w)+\lambda x}{2\mu'(w)}\right) - \left(\frac{r(p-c(w))+(p-c(w))-\lambda h\theta}{2c'(w)}\right) \end{bmatrix} \tag{29}$$

where the appropriate expression for $q\left(\theta\right)$ is $q\left(\theta\right) = 1 - e^{-\frac{1}{\theta}}$, $q'\left(\theta\right) = -\frac{e^{-\frac{1}{\theta}}}{\theta^{2}}$, which implies, $1 - e^{-\frac{1}{\theta}} / -\frac{e^{-\frac{1}{\theta}}}{\theta^{2}} = \theta^{2}\left(1 - e^{\frac{1}{\theta}}\right)$, $c\left(w\right) = a_0 + a_1 w + a_2 w^2$, as shown below. In a few experiments[12] I also exclude the fixed cost, where $c\left(w\right) = a_1 w + a_2 w^2$ and without loss in generality, normalise $\mu\left(w\right) = \frac{w^{1-\rho}}{1-\rho}$, $\mu'\left(w\right) = w^{-\rho}$. Then an immediate trade equilibrium is any solution to (29), that confines in both constraints (17) and (18), and also satisfies $\pi_w = e^{-rt}\left[\mu\left(w\right) + W\right]$ and $\pi_f = e^{-rt}\left[p - c\left(w\right) + J\right]$.

A special case is a steady-state, which is an equilibrium where w and x are constant. Then $(w, x) = (0, 0)$ is a steady-state. In the Pissarides model, the dynamical system, comprises of *purely* $\dot{\theta}$ and $\dot{u}$, where w and x are *constants.*

8. CALIBRATION OF THE MODEL

The choice of the U.S. data for my empirical analysis is due to all labour market simulation work being U.S. based. The past literature source/econometric studies is based in the U.S., provided all the data I required from one country, specifically, the U.S.. Needless, to say, it is not possible to obtain data for all the variables in my analysis from the UK, since no

[12] Details available from the author upon request.

simulation work is UK based. It is not possible to obtain data for all the variables in my analysis from one particular country, for example, the UK.

Prior to conducting the stability analysis it is useful to conduct a very brief tour of the available data pertinent to our study, in order to make an appropriate choice of the literature source/econometric studies, which would match/reflect the concepts of our model as far as possible. All the data sources pertaining to the literature source/econometric studies are of the U.S., except the econometric estimate of hiring costs, which is from Abowd and Kramarz (2000). The latter claims that all the microeconomic evidence for France has counterparts in the U.S., which are similar to those observed in France. Thus, the calibrations apply to the U.S. in most cases, except due to the scarcity of data on hiring costs (Hamermesh, 1993), the hiring costs is sourced from France's data[13].

8.1. Unemployment

We considered the official unemployment rate as the most appropriate data compatible with the concepts of our model. This is because other categories such as those responding affirmatively to the question if they 'wanted a job now' are inappropriate.

The reason for exclusion of the out-of-the-labour-force data in our study is obvious, notably, since the out-of-the-labour force flows exhibit distinct cyclical properties relative to flows between employment and unemployment. The unemployment to employment flows are countercyclical or acyclical, whilst the out-of-the-labour-force to employment flows are pro-cyclical.

It is worth noting that there is a high correlation between the official unemployment rate data and other configuration of unemployment rates data, namely, the official plus 'want a job now' unemployment rate data (Yashiv, 2005).

Given this choice, it is natural to consider only the official rate data when studying dynamic wage-bargaining in the labour market and the associated worker flows.

[13] Other authors also use Abowd and Kramarz (2000), when all their other data sources are obtained from the U.S. For example, Silva and Toledo (2005).

8.2. Vacancies

The pertinent concept of vacancies compatible to the concepts of the model is the one pertaining to those vacancies that are to be occupied by workers from outside the employment pool, but within the labour force. But the available and widely-used data series relates to another concept, which also includes vacancies that are subsequently occupied with workers in job-to-job transitions.

The available vacancy data series in the U.S. economy has two representations. One is the index of Help Wanted advertising in newspapers published by the Conference Board (see Abraham (1987) for an analysis and discussion of this series). A newer data series is the job openings data series available from the BLS since December 2000, utilising the Job Openings and Labour Turnover Survey (JOLTS)[14].

The third data series has the gross flows of workers from outside employment (unemployment and out of the labour force) to employment. The latter was recently compiled at the Boston Fed, based on the Current Population Survey (CPS) data, see Bleakley, Ferris and Fuhrer (1999). The hiring flows data series is negatively correlated with the first two series, that is, -0.27 with JOLTS series and -0.36 with the Help Wanted ads data series (Yashiv, 2005). The flows data series is considerably less persistent than the two vacancies series. The hiring flows data series is less volatile than the Help Wanted Index and the JOLTS data series (Yashiv, 2005).

In addition, there is a body of evidence on gross worker flows. For example, Fallick and Fleischman (2004), using CPS data in the period $1994 - 2003$, find that job-to-job transitions are massive, that is $2/5$ of new jobs represent employer changes. They also demonstrate that the cyclical properties of job-to-job transitions are distinct from the flows into and out of employment. Nagypal (2004), using microdatasets, also shows the prevalence of job-to-job transitions. Pissarides (1994) provides a possible explanation for the higher volatility of vacancies associated with both job-to-job transitions and to transitions from out of employment.

[14] This survey defines a job as "open", conditional on it fulfilling the following criteria: (i) a specific position exists and there is work available for that position; which can be full or part time, permanent , temporary, short-term or seasonal; (ii) the job could start within 30 days, irrespective of whether the establishment has found a suitable candidate during that time and (iii) the employer is actively recruiting from external establishments to fill the position.

I conclude that in the light of our model, which is an aggregate[15], representative firm and worker type of model, job-to-job transitions are inappropriate, as its behaviour is distinct from the pertinent data series, where vacancies are filled by workers from outside the employment pool. Since the latter is unobserved, I use the vacancy rate from the literature that utilises the rate for the observed worker flow data series.

8.3. Wages

The existence of a variety of data series of wages with various cyclical properties was reported by several studies, for instance, Abraham and Haltiwanger (1995), Abraham, Spletzer and Stewart (1999), and Krueger (1999). But the analysis in these studies does not lead to any definitive conclusions as to which series is the most pertinent. The Bureau of Economic Analysis (BEA) series using total compensation[16] and wages, suggests that although the series are correlated 0.83 (Yashiv, 2005), there are a multitude of important differences. Specifically, the wage series declines more over time, it is lower by 10 percentage points on average and exhibits considerably more variation, that is a coefficient of variation of 0.037 as opposed to 0.016 with respect to the compensation series (Yashiv, 2005). Both series have an extremely weak correlation with the cycle. The compensation series has -0.05 correlation and the wage series has a 0.12 correlation with the employment rate (Yashiv, 2005).

Needless to say, econometric study using the U.S. data on wages settled on the basis of future financial performance, does not exist. The series of labour share using total compensation as opposed to wages, reconciles more with the concepts of the model.

Hence, I have used the compensation data series rate from the literature, as it incorporates all the firm's wage-related costs, which is the terms of trade, wage, w, in our model.

8.4. Other Data Series

With respect to the job destruction rate, λ, I use the rate which represents the flow from employment to unemployment. The discount rate r, in the model is the rate of time preference for both agents.

[15] Although we have bargaining between one worker and firm, they are representative of their respective type.

[16] Defined as the total compensation of employees relative to GDP, latter including wages/salaries, employer contributions for employee pension and insurance funds and government social insurance.

8.5. Calibration Values

To conduct the numerical simulation and hence, examine the model's performance, we need to calibrate the model. We parameterise the model to match the relevant U.S. data. That is, we assign values to the variables of the v/u ratio, θ, unemployment, u, wages, w, and the difference between the value functions of the unemployed and employed workers, at the steady-state values of these endogenous variables and the steady-state values of the exogenous variables. The latter are: the rate of interest, r; destruction rate, λ; the value of output, p; hiring cost rate, h; and unemployment benefit rate, z. I do this by experimenting with fundamental parameters such as a_0, a_1, a_2 and ρ. To attain this, I utilise, wherever possible, results from econometric studies and prior empirical estimates on U.S. quarterly data, which use the average values of the longest possible sample period available.

I proceed to explain the choice of the parameters. Since the firm's output can be sold at any price, with no loss in generality, I normalise the value of output, $p = 1$. Based on Yashiv (2005), I set the separation rate to $\lambda = 0.0404$. I normalise time period to be a quarter and thus, set the discount rate to $r = 0.01$, which reflects historical U.S. values. Surveys on hiring costs are scant (Hamermesh, 1993) as noted earlier. Based on Silva and Toledo (2005), I set the hiring costs to $h = 0.30$. They follow the econometric estimate of Abowd and Kramarz (2000). They estimate the cost of hiring as a fraction of the annual labour costs per worker to be 30 percent[17] for a representative sample of French establishments. Abowd and Kramarz (2000) also claim that all of the microeconomic evidence for France has counterparts in the U.S., which are similar to those observed in France as mentioned above. Based on Shimer (2004), unemployment benefit is set to $z = 0.40$. Based on Yashiv (2005), I set $u = 0.063$, $v = 0.047$ and hence, $\theta = v/u = 0.75$. Also based on Yashiv (2005), I set total compensation of employees as a proportion GDP, $w = 0.579$. Taking into account $z = 0.40$ and $w = 0.579$, I set x to be the difference between 0.40 and $w = 0.579$, implying the difference between the value functions of the unemployed and employed worker to be $x = -0.179$, on the grounds that it is not directly observable nor available/accessible. Table 1 describes these parameters, including their calibrated values and sources.

[17] This entails reported expenditure on job advertising, search firm fees and compensation of applicants.

TABLE 1
BASELINE CALIBRATION VALUES

Steady State Values of the Endogenous Variables and the Parameters of the Exogenous Variables			
Parameter/Variable	Notation	Value	Source
Difference between the value functions of the unemployed and employed workers	x	-0.179	Own Calculation
Vacancy-Unemployment ratio	θ	0.75	Yashiv (2005)
Unemployment	u	0.063	Yashiv (2005)
Wage	w	0.579	Yashiv (2005)
Discount rate	r	0.012	Shimer (2005)
Separation rate	k	0.0404	Yashiv (2005)
Value of Output	p	1	Quarterly Normalisation
Hiring Costs	h	0.30	Abowd and Kramarz (2000)
Unemployment Benefit	z	0.40	Yashiv (2005)
Coefficient of the quadratic term in the cost function	a_2	0.312	Own calibrated value
Coefficient of the first term in the cost function	a_1	0.623	Own calibrated value

9. STABILITY ANALYSIS

9.1. Constant w and x: Nash Bargaining Solution

I make it clear at the outset, that we analyse a *fully* dynamic labour market model in contrast to the Pissarides model, and we also generalise with *risk-averse* agents and *non-linear* cost functions. This, shows that $ITE \neq NBS$ and introduces a *new* equilibrium concept in labour economics. I have applied the CW general result to my model with *risk-averse* agents and *non-linear* cost functions in a *fully* dynamic labour market model. The issues that arise of immense interest are: is such a model *saddle-path stable*? In addition, are there limit cycles? CW in related literature show a limit cycle in their two-dimensional non-linear system. This implies there is indeterminacy and instability in their system. For a model to be theoretically consistent and dynamically stable, the equilibrium should be saddle-path stable and unique with *no* limit cycles. I now undertake to investigate if this is true in my model.

When my parameters, which will be my baseline value of parameters, and the parameters that I will be varying later in my full model with non-linear costs and utility function, will be ρ and a_2 in (ρ,a_2) parameter space (these are zero with linear costs and utility functions case) when my experiments are conducted, are given by,

$$r = 0.012, \rho = 0, a_0 = 0.02, a_1 = 0.623, a_2 = 0, p = 1, h = 0.30, \lambda = 0.0404, w = 0.579$$

the equilibrium is

$$\theta = 0.75, u = 0.07$$

The information about eigenvalues for the linearised equation and its nature and the fixed points are provided in Table 2. The eigenvalues are: 1.50 and -0.59 as shown in Table 2.

TABLE 2
Equilibria
SADDLEPOINT
Pissarides Model
Linear Costs and Utilty Function
$NBS \equiv ITE^{a}$

$c+=0$	$c-=0$	$im=0$
$r+=1$	$r-=1$	

Value of Fixed Points
$\theta = 0.75$
$u = 0.07$
Eigenvalues
1.50
−0.59

[a]When $t \to \infty$ and $r_f = r_w$.

where the information about the eigenvalues for the linearised equation are given in the middle; $c+$, the number of complex eigenvalues with positive real parts, is zero; $c-$, the number of complex eigenvalues with negative real parts, is zero; im, the number of purely imaginary eigenvalues with zero real part, is zero; $r+$, the number of positive real eigenvalues is 3; and $r-$, the number of negative real eigenvalues is, 1. The results of the simulations, with both linear cost and utility is unstable, namely, when $a_2 = 0$ and $\rho = 0$, as is the case when $NBS \equiv ITE$.

Numerical simulations of fixed points and eigenvalues using the software XPP, showed the typical equilibria to be saddlepoint, as we would expect, with one positive and one negative eigenvalue, and did not yield a stable limit cycle, that is, an isolated[18] periodic orbit[19] for the differential equations which is stable or attracting. That is, if the system is perturbed from its regular oscillatory state, the ensuing new path will be attracted back to the limit cycle. That is, all the neighbouring trajectories approach the limit cycle. A stable limit cycle model the system exhibiting self-sustained oscillations. That is, the system oscillates devoid of external periodic forcing[20]. If all the neighbouring trajectories approach the limit cycle, then the limit cycle is stable or attracting. The steady-state as remarked is a saddle point, that is there is one positive and negative eigenvalue.

9.2. The Full Model

For the remainder of the analysis, we assume that $\dot{w}$ and $\dot{x}$ are not zero and pursue and show dynamic equilibria. The equilibrium states are found by setting the RHSs of the remaining

[18] Isolated in the sense that there is not another closed path in its immediate neighbourhood. That is, the neighbouring trajectories are not closed, they either spiral toward or away from the limit cycle. In rare cases, half stable.

[19] A periodic orbit is the orbit of any point through which a periodic solution passes. A periodic solution is a solution which is periodic in time $\phi(t) = \phi(t+T)$, for a fixed positive constant T. T is a period of $\phi(t)$.

[20] Another example, would be the beating of the heart.

four differential equations, that is, the full model, to zero, and solving for x, θ, w and u. This can only be performed computationally. Further numerical simulations with a wide range of parameter values of the combination of the pair (ρ, a_2), were conducted, where ρ is the risk aversion parameter of the Constant Relative Risk Aversion utility function of the workers, with $0 < \rho < 1$ and a_2 is the coefficient of the non-linear wage parameter of the firm's non-linear cost function, as mentioned earlier, where $a_2 \geq 0$. Of course, when $a_2 \geq 1$, we have decreasing returns to scale in the firm's cost function and $a_2 < 0$, due to increasing returns to scale. To preserve generality I also constructed examples with $0 < \rho \leq 1$ and $-3 \leq a_2 \leq 3$, in (ρ, a_2) parameter space. Depending on the accurate/precise combination of the pair (ρ, a_2), the system (29) can lead the labour market into a unique equilibrium with saddle-path stability. We briefly demonstrate this, with the dynamics associated with our dynamical system in (29). We discuss the combination of pairs (a_2, ρ) that lead the system to a unique equilibrium with saddle-path stability.

We combine the dynamics of unemployment with those of labour market tightness, the difference between the value functions of the unemployed and employed workers, and wages, as in (29). We first discuss the results of the combination of the pair (ρ, a_2) which leads the labour market to a unique and stable equilibrium with saddle-path stability. The conditions for a stable and unique equilibrium depend on the magnitude of the eigenvalues of the RHS of (29). If the number of eigenvalues outside the unit circle is equal to the number of non-predetermined variables, the system (29) has a unique equilibrium which is stable. That is, if we have one stable root and three unstable roots[21]. All examples constructed to examine if the model yields a limit cycle or a unique saddle-path stability, include fixed costs in the firm's cost function.

When the baseline value of the parameters are the same as above; and the parameters varied are given by,

$$\rho = 0.01 \text{ and } a_2 = 0.312$$

the equilibrium is

$$x = -0.19, \theta = 0.74, u = 0.07 \text{ and } w = 0.52$$

[21] Instability occurs when the number of eigenvalues of the RHS of (32), outside the unit circle is greater than the number of non predetermined variables. That is, if we have four unstable roots, when the economy is pushed off its steady state following a shock, it will not converge back to it, and results, in explosive dynamics, that is, with the orbits tending to infinity.

The matrix has eigenvalues[22], $1.54, 0.47, -0.59$ and 0.80, see Table 3. The equilibrium is a saddlepoint (that is, one negative eigenvalue associated with one pre-determined variable (u) and three positive eigenvalues associated with three non pre-determined variables). They are consistent with the equilibrium in Maple (where the orbit of u was converging towards the equilibrium), while attempting to see if there is a limit cycle. The three positive eigenvalues are likely to pertain to the forward-looking variables and the negative eigenvalue (that is, a stable root/root in the unit circle of the complex plane) relates to the unemployment differential equation, u as expected is consistent with the $\dot{\theta}, w$ and $\dot{u}$ in the Pissarides model, except that, as will be noted, the wage there, in their otherwise dynamical model is the steady-state wage, in a two-dimensional dynamical system. Our results are also consistent with the values I obtained for comparable parameters, in (ρ, a_2) parameter space, when I performed numerical simulations of eigenvalues, using the software XPP.

TABLE 3

Equilibria

SADDLE-PATH STABLE

Non-Linear Costs and Utility Function

$c+ = 0$	$c- = 0$	$im = 0$
$r+ = 3$	$r- = 1$	
	Fixed Point Values	
	$x = -0.21$	
	$\theta = 0.74$	
	$w = 0.52$	
	$u = 0.07$	
	Eigenvalues	
	1.54	
	0.47	
	-0.59	
	0.80	

I have varied a wide range of parameters in (ρ, a_2) parameter space, see Table A.1 in Appendix B, which reports the results from simulations of the model with dynamic bargaining in labour markets. In the values of the (ρ, a_2) combinations[23], that is, from $a_2 = 0.01$ to $a_2 = 3$ and $\rho = 0$ to $\rho = 0.09$, the solution is uniquely saddle-path stable. This also implies that the cost function, $c(w)$ exhibits decreasing returns- to-scale. A production function which does not imply an advantage to large firms exhibits what is called constant returns-to-duplication (Phillips, 1997) but also decreasing returns-to-scale.

The conclusion is that for calibrated parameter values the system has a unique equilibrium and is saddle-path stable. Of course, the three positive eigenvalues and the other negative eigenvalue do not necessarily mean that the system is unstable or stable (since any periodic orbit always has an eigenvalue of 1 along the orbit). The other two eigenvalues can be unstable

[22] Eigenvalues informs us of the local stability of the fixed point. XPP computes the Jacobi matrix numerically and then utilises standard eigenvalue routines to compute the eigenvalues of the resulting matrix.

[23] Except when $a_2 = 0$ to $a_2 = -3$ and $\rho = 0$ to $\rho = 0.09$, in the (ρ, a_2) parameter combinations, the solution/equilibrium of the system is unstable.

(i.e., > 1 in magnitude with the one eigenvalue stable). The steady-state is a saddle point. The result of the unstable solution, with linear cost functions, at $a_2 = 0$ and $\rho = 0.09$ in the (ρ, a_2) combination is consistent with expectations, since it further confirms the presence of non-linearities in the model and is not in support of the NBS or the Pissarides model. Similarly, the results of the solution, with both linear cost and utility is unstable, that is, where $a_2 = 0$ and $\rho = 0$, as is the case when $NBS \equiv ITE$[24] and the Pissarides model. This is also consistent with expectations and confirms support for our model as opposed to the NBS or Pissarides model[25].

The numerical simulations and numerical calculations of eigenvalues in XPP, suggest that limit cycles do not exist for realistic values of the parameters. XPP calculates the equilibria of our system. Calculations are performed using the values provided for the initial conditions as a first guess, applying the Newton's method. When a value is found, XPP then finds the eigenvalues, which were given above. The program continues to integrate to calculate the equilibria of our system beyond the current numerical parameters I specified. When the equilibrium is computed, the information on the value of the fixed point and its stability are provided, see Appendix A. Since the eigenvalues on the whole of the non-predetermined variables are unstable, it would be hard to find a limit cycle. Similar results were obtained for several experiments I conducted with a wide range of parameter values in (ρ, a_2) parameter space, to examine if a limit cycle can be detected and the stability if the equilibria. Further numerical simulations, on a wide range of parameters in XPP, also did not yield a limit cycle, as shown in Table A.1 in Appendix B. In addition, numerical simulations in the software DsTool, also suggest that limit cycles are non-existent for sensible values of the parameters. The ranges I conducted my search are similar to the ranges I constructed in XPP in (ρ, a_2) parameter space.

There are no limit cycles, in the various experiments I conducted, but the equilibrium is unique and saddle-path stable. Overall, the model, where naturally all four variables are dynamic, in contrast to models in the past, performs remarkably well, in reflecting the observed behaviour of wage-bargaining in labour markets found in the U.S. data. As will be seen below, my equilibrium values for most variables match exactly the real life data of the U.S..

[24] Where $r_f = r_w$ and $t \to \infty$.

[25] Or any other wage bargaining model in labour economics.

Naturally, the (ρ, a_2) parameter combination is the most important for our analysis, as these introduce non-linearities that cause the ITE to depart from the NBS. I then varied a wide range of ρ and a_1 parameters from the (ρ, a_1) combination. Table $A.2$ in the Appendix reports the results. Again, from $a_1 = 0.1$ and $\rho = 0$ to $\rho = 0.09$, in (ρ, a_1) parameter space[26], the system has a unique equilibrium and the solution is saddle-path stable. This reaffirms the consistent remarkably good performance of the model. For example, the equilibrium values from my simulation match precisely the real life time-series data of the U.S. with respect to most variables, except θ and w, where it is 0.01 and 0.05 out, as alluded to above.

We use the combined dynamics of the four variables in (29). The equation for the evolution of unemployment is stable with driving force θ. Substitution of wages and job values J from (11) into (9) yield an unstable equation in θ, with no other unknowns in it. The critical point of the four-dimensional dynamical system is that it yields a unique equilibrium which is saddle-path stable. The simulation were also conducted in the same software XPP, where Newton's method is used to find fixed points and then numerically linearises about them to determine stability as mentioned earlier.

We first discuss the results of the parameter combination of the pair (ρ, a_2) which led the labour market to a unique equilibrium and saddle-path stability. The sign pattern of a first-order linear approximation to the four differential equations are three positive eigenvalues and one negative eigenvalue as noted. Since the number of eigenvalues outside the unit circle is equal to the number of non-predetermined variables of the system, the equilibrium is unique and saddle-path stable. The combination of the pair (ρ, a_2) that lead to the solution of saddle-path stability are from $a_2 = 0.1$ to $a_2 = 3$ and $\rho = 0$ to $\rho = 0.09$. All areas from $a_2 = 0.1$ to $a_2 = 3$ and $\rho = 0$ to $\rho = 0.09$ in (ρ, a_2) parameter space, are associated with a unique and well behaved rational expectation equilibrium. This could be the result of a one-off moderately favourable tax-reduction, which could induce the dynamics associated with an increase in productivity, for example. Specifically, a fall in tax-reduction shifts the wage curve up and job creation curve to the right, causing an immediate rise in both θ and w. Both θ and w jump to their new equilibrium, while there is an anticlockwise rotation of the job creation line in u, v space.

The saddle point arises since one of the variables, unemployment is sticky and stable,

[26] From $a_1 = 2$ to $a_1 = 3$ and $\rho = 0$ to $\rho = 0.09$ in (ρ, a_1) parameter space, the solution is unstable. This confirms the non-linearities inherent in the system.

whilst the others, vacancies, difference between the value functions of an unemployed and employed worker and wages are forward-looking and unstable. Firms in this model treat vacancies as an asset, since it is the price that has to be paid now in order to attract employees in the future. The expected arrival of employees is the rate of return on this asset. In common with other assets, there is an instability inherent in the supply of vacancies. If the arrival rate of employees is expected to fall, then firms will want to be left with a lower supply of vacancies, in anticipation of less demand for it. But if the firm wants to hire more workers sooner, the firm needs to create more vacancies. Thus, an expected fall in the arrival rate of employees leads to the creation of more vacancies and to an immediate fall in the arrival rate of employees to each vacancy.

The expected changes in the arrival rate of employees reflects the expected capital gains or losses on the firm's outstanding vacancies. The unique feature of vacancies is that the firms allocate the current vacancies in accordance to their future needs. For example, if the arrival rate of employees is expected to fall, the firm creates more vacancies now, and hires more employees now. The upshot is that, as a consequence, vacancies overshoot their equilibrium value when an adjustment is anticipated to occur.

The perfect foresight path in the neighbourhood of the equilibrium is stable and unique. The number of stable roots in (29) is equal to the one predetermined variable. The initial condition on the predetermined variable, and the stipulation that the perfect foresight path should converge, uniquely defines an initial point in (θ, u, w, x) space, from which adjustment to equilibrium occurs. In the absence of anticipated changes in the exogenous variables, the initial point will always be on the saddle-path, since this is the unique convergent path.

In system (29), the saddle-path is easily found, due to the independence of the other three equations from unemployment. This can be easily shown, that is, the exogenity of unemployment from the other three variables, in XPP and Maple. Since θ, w and x are the unstable variables, if θ, w and x are not in equilibrium, it will diverge.

Let us consider the impact of a change in productivity on wages, tightness, the difference in the value functions of the unemployed and employed workers and unemployment. If the initial equilibrium is at $[w(0), \theta(0), u(0), x(0)]$, in u, v space, vacancies will increase as firms create more vacancies to take advantage of higher productivity. This leads to a decrease in unemployment simultaneously, lowering vacancies. Obversely, in the case of a fall in productivity, the adjustment dynamics will move the economy in the reverse direction. That is, there will be a fall in the job creation in the u, v space, that is, there will be fewer vacancies, due to firm closures. Unemployment increases, then more vacancies are created as firms anticipate the demand for them will be high. Then this leads to fall in unemployment and vacancies as they are matched with employees[27].

Since the bottom right entry of the Jacobian matrix of (29) is very complicated, it is infeasible to investigate the characteristic equation analytically. The situation is made worse by the fact that the steady-state (equilibrium) cannot be found analytically (the steady-state has to be substituted into the Jacobian prior to finding the eigenvalues).

As is well known, due to the complex nature of, and the fact, that we have a four-dimensional system, it is infeasible to check all of these, in particular whether the solution stays within both constraints (17) and (18), and to verify by numerical integration the existence of a limit cycle. With regard to the latter, in the extensive experiments I conducted by me, using both a linear cost function of constant returns to scale for the firm of the form, $c(w) = w$ and the non-linear cost function of the form $c(w) = a_0 + a_1 w + a_2 w^2$, and also that workers are risk-averse and have single period utility function of the form, $\mu(w) = \frac{w^{1-\rho}}{1-\rho}$, with $0 < \rho < 1$, where $\rho = 0$, which is the risk-neutral case, the $ITE \equiv NBS$, for $\rho > 0$,

[27] For some combination of the pair (ρ, a_2) of the four equation system, leads the economy into instability. The combination of parameter values of a_2 and ρ for which this is applicable are only for $a_2 = 0$ (this is expected and supports our non-linear cost function and not the NBS or Pissarides model with linear cost functions) to $a_2 = -3$ (where there are increasing returns to scale, in the firm's cost function), for $\rho = 0$ and to $\rho = 0.09$ in (ρ, a_2) combination as mentioned. Instability was triggered naturally by the choice of the pairs (ρ, a_2). These choices were attempted, since I wanted to provide a comprehensive account of what happens in the labour market, when one uses a wide range of parameters. That is, it is purely conducted for expositional purposes. Typically, these combination pairs does not imply a strong enough response to induce the necessary changes in wages, vacancies, unemployment and the difference between the value functions of the unemployed and employed worker.

The sign pattern of a first-order linear approximation to (29) of these choices of parameter values in (ρ, a_2) parameter space, is that, there are four positive fixed points, when x should be negative in the model. This implies that when the economy is pushed off its steady state following a shock, it cannot converge back to it, but ends up with explosive dynamics as mentioned, that is the orbits go away from the equilibrium to infinity.

The first thing to note is that the variable unemployment which should be sticky and stable, is not. This implies that the unemployed workers are increasing at a fast rate. This has detrimental effects on the economy: First, firms will not be able to supply sufficient vacancies to provide sufficient jobs for the increase in the arrival rate of employees, that is they are unsustainable when unemployment increases considerably. Second, it engenders an unfavourable signal to both the incumbent and unemployed workers, thereby reducing their incentive to remain in employment or exert the necessary effort as expounded in efficiency wage theory. In the case of the former and in the case of the latter, it discourages the volatility of the unemployed workers, and discourages the unemployed workers from applying to posts. Third, if there is a sudden increase in unemployment, labour market tightness fluctuates, with tightness oscillating from low to high. Fourth, wages similarly will fluctuate from low to high, with it being low when unemployment is high and high when unemployment is low. There is a body if evidence to suggest that unemployment has a negative impact on wages (for example, the empirical Chapter 3 of this thesis). Fifth, the difference in the value functions of the unemployed and employed workers would also naturally fluctuate with the other three variables fluctuating.

the workers are risk-averse; to numerically prove that there is a limit cycle, we did not find a limit cycle. In principle, we should detect a limit cycle, but as is well known, see for example, McCord, Mishaikow and Mrozak (1995) and Jordan and Smith (1999), we may not find a periodic solution, although there is one in principle. We have no grounds to completely rule out stable limit cycles.

CW, using a two-dimensional dynamical system, claim to have detected a limit cycle; which is reputably considerably easier, than in higher order systems. Equally, proving it analytically is also considerably easier, as the straightforward application of Poincare-Bendixson Theorem, will theoretically prove its existence, which was indeed claimed to have been applied by CW to show the limit-cycle's existence. No such standard proof can be applied to dynamical systems of three dimensions or higher. Of course, in our system, the Hopf-Bifucation theorem can be applied to show the existence of a limit cycle, if one is found by numerical integration. Furthermore, with respect to the CW study, although they provide a distinct analysis of monetary theory, the theory there developed makes no attempt to calibrate the model adequately. In addition, with regard to the CW paper, they did find a limit cycle for their two-dimensional model, but they used values of the parameters, which I feel are unrealistic, given their initial condition. The numerical value used for their cost function, was a -27.04902, which is inconsistent with their condition, $0 < c'(0) < 1$.

In my model, values of the parameters are consistent with first, the experiments with, $0 < c'(0) < 1$, and second the experiments with, $c'(0) < 1$[28], and the important result is that both sets of experiments show that the system has a unique equilibrium and is saddle-path stable, but do not seem to produce a limit cycle in either set of experiments. Moreover, CW used two special functions, b_0 and b_1; a cost function, which is inconsistent with their cost function in their model, namely, $c(q)$ in their model, with b_0 and b_1 functions, and two special parameters, which are both inconsistent to their model. Their functions are $b_0(q, y, z) = q + (y - 2ez)(e - y/2z) + z(e - y/2 * z)^2$ and $b_1(q) = 2ez$ and their parameters were $d = 0$ and $e = 0.001$. Devoid of these functions, but including their a value, which as I noted above is inconsistent to their conditions, the equilibrium in their model is also a saddle point. In my model, the functions employed are consistent with the functions in the model, namely, $c(w) = a_0 + a_1 w + a_2 w^2$ and $q(\theta) = 1 - e^{-\frac{1}{\theta}}$, and I produce a unique equilibrium

[28] Details available from the author upon request.

with saddle-path stability when experimenting with a wide range of parameters, but do not appear to produce a limit cycle.

In the past, related literature has shown there is a possibility of limit cycles, as shown in CW, which would give rise to *indeterminacy* and *instability*. To claim to represent reality, the equilibrium should be *saddle-path stable* and *unique* with *no* limit cycles. In my model, we have a well-behaved equilibrium.

Policy prescriptions can be made on the basis of whether policy critically influences wages in equilibrium. The supply of jobs is a variable and subject to profit maximisation. The wage, w determined in the model, absorbs/incorporates all variations in parameters, including policy parameters relevant at the time of bargaining. Hence, there will be no qualitative difference to the results. The policy implication of varying interest rate, r and unemployment benefit, z can be shown over time in XPP[29].

10. CONCLUSIONS

The chapter has analysed a labour market model with random matching and strategic bargaining. The solution to the bargaining problem was characterised in terms of a dynamical equation. It was also shown that the system in (ρ, a_2) parameter space has a unique equilibrium and is saddle-path stable for all four variables, including the dynamic variable for workers with both risk-neutral and risk-averse single period utility and both linear and non-linear cost functions of the firm, which has not been attempted in the past. But overall, this chapter had shown that there is a well behaved unique and stable equilibrium for plausible ranges of parameter values. Such a characterisation for the wage is important in a dynamic labour market model, besides informing us as to what was originally available for sharing between the bargainers, but also what, why and how it was shared by the same. It is found in this chapter that the solution did coincide with the wage-bargaining analyses in steady-state, but not when $t < \infty$, that is, out of steady-state, except when both the bargainers have equal rates of time preference and the agents are risk-neutral.

We analysed constructing a wide range of examples to show uniqueness and saddle-path stability of equilibrium in our full labour market model and no limit cycles, as in the past

[29] This is in my future research agenda.

related literature of CW. Limit cycles would give rise to indeterminacy and instability. But my model has a well behaved equilibrium. This shows that forward-looking behaviour is consistent with stable and unique outcomes in wage-bargaining. Our analysis complements remarkably well the forward-looking behaviour, empirically established in Chapter 3 of this thesis. Our analysis with one worker and firm bargaining, also shows there is an efficient outcome in

terms of both wages and employment.

APPENDICES

APPENDIX A

1. *Solving of the fixed points, eigenvalues and its stability*:

XPP finds fixed points as follows:

First by solving

$$G(X) = 0$$

where $G(X) = F(X)$ for differential equations $X' = F(X)$ and $G(X) = \lambda - F(X)$. Newton's method is iterative and satisfies the scheme

$$X_{k+1=} X_k - J^{-1} G(X_k)$$

where J is the matrix of partial derivates of G evaluated at X_k. XPP uses three parameters to implement Newton's method, namely the maximum number of iterates, the tolerance and a parameter for the numerical computation of the matrix J, referred to as epsilon in XPP. If successive iterates falls within the tolerance, then the convergence is assumed and the root is found. The matrix J is found by perturbing each of the variables by an amount proportional to epsilon and utilising this perturbation to approximate a derivative.

Following the finding of a fixed point, the matrix J is evaluated once again and the eigenvalues of J are computed using standard linear algebra routines. This information is utilised to compute the stability of solutions.

APPENDIX B

TABLE A.1

STABILITY PROPERTIES[a]

ρ	Equilibria[b,c]	a_2 0	1	2	3
		Risk-Neutral Workers With Linear utility Function $NBS \equiv ITE^d$ and Pissarides Model With linear Costs Function			
	Fixed Points				
	θ	0.75	0.44	0.36	0.34
	u	0.07	0.09	0.11	0.11
	w		0.57	0.50	0.44
	x		-0.39	-0.26	-0.12
0	Eigenvalues				
		1.50	3.38	4.94	5.89
		-0.59	0.42	0.38	0.37
			1.59	3.33	-0.36
			-0.43	-0.38	5.05
		Risk Averse Workers			
	Fixed Points				
	θ	1.19	0.44	0.36	0.34
	u	0.06	0.09	0.11	0.11
	w	0.39	0.57	0.50	0.44
	x	0.01	-0.39	-0.26	-0.12
0.01	Eigenvalues				
		-0.72	3.37	4.92	5.87
		0.92	0.42	0.38	0.37
		0.80	1.58	3.32	-0.36
		0.53	-0.43	-0.38	5.03
	Fixed Points				
	θ	1.20	0.44	0.36	0.34
	u	0.06	0.09	0.11	0.11
	w	0.39	0.57	0.50	0.44
	x	0.02	-0.38	-0.26	-0.12
0.02	Eigenvalues				
		-0.72	3.35	4.90	5.86
		0.92	0.42	0.38	0.37
		0.80	1.58	3.31	-0.36
		0.53	-0.43	-0.38	5.02
	Fixed Points				
	θ	1.21	0.44	0.36	0.34
	u	0.06	0.09	0.11	0.11
	w	0.38	0.57	0.50	0.44
	x	0.02	-0.38	-0.26	-0.12
0.03	Eigenvalues				
		-0.72	3.34	4.89	5.84
		0.91	0.42	0.38	-0.36
		0.80	1.57	3.30	0.37
		0.53	-0.44	-0.38	5.01
	Fixed Points				
	θ	1.22	0.44	0.36	0.34
	u	0.06	0.09	0.11	0.11
	w	0.38	0.57	0.50	0.44
	x	0.03	-0.38	-0.26	-0.12
0.04	Eigenvalues				
		-0.72	3.32	4.87	5.83
		0.91	0.42	0.38	-0.36
		0.80	1.56	3.29	0.37
		0.53	-0.44	-0.38	4.99
	Fixed Points				
	θ	1.23	0.44	0.36	0.34
	u	0.06	0.09	0.11	0.11
	w	0.38	0.57	0.50	0.44
	x	0.03	-0.37	-0.25	-0.11
0.05	Eigenvalues				
		-0.72	3.30	4.85	5.81
		0.91	0.42	0.38	-0.36
		0.79	1.55	3.28	0.37
		0.53	-0.44	-0.38	4.98

TABLE A.1

(CONTINUED)

ρ	Equilibria	a_2 0	1	2	3
		Risk Averse Workers			
	Fixed Points				
	θ	1.24	0.44	0.36	0.34
	u	0.06	0.09	0.11	0.11
	w	0.37	0.57	0.50	0.44
	x	0.04	-0.37	-0.25	-0.11
0.06	Eigenvalues				
		-0.73	3.29	4.85	5.81
		0.91	0.42	0.38	-0.36
		0.79	1.55	3.28	0.37
		0.53	-0.44	-0.38	4.98
	Fixed Points				
	θ	1.25	0.45	0.37	0.34
	u	0.06	0.09	0.11	0.11
	w	0.37	0.57	0.50	0.44
	x	0.04	-0.37	-0.25	-0.11
0.07	Eigenvalues				
		-0.73	3.27	4.81	5.78
		0.91	0.43	0.38	-0.36
		0.79	1.54	3.26	0.37
		0.53	-0.44	-0.38	4.95
	Fixed Points				
	θ	1.25	0.45	0.37	0.34
	u	0.06	0.09	0.11	0.11
	w	0.37	0.56	0.50	0.44
	x	0.04	-0.36	-0.25	-0.11
0.08	Eigenvalues				
		-0.73	3.25	4.80	5.76
		0.91	0.43	0.39	-0.36
		0.78	1.53	3.25	0.37
		0.53	-0.44	-0.38	4.94
	Fixed Points				
	θ	1.26	0.45	0.37	0.34
	u	0.06	0.09	0.11	0.11
	w	0.36	0.56	0.50	0.44
	x	0.05	-0.36	-0.25	-0.11
0.09	Eigenvalues				
		-0.73	3.23	4.78	5.74
		0.90	0.43	0.39	-0.36
		0.79	1.52	3.24	0.37
		0.53	-0.44	-0.38	4.93

[a] I have also conducted experiments of parameter values in between $a_2 = 0$ to $a_2 = 1$ and $a_2 = 1$ to $a_2 = 2$ and $a_2 = 3$ for values of $= 0$ to ρ $= 0.09$ in (ρ, a_2) parameter space and found the equilibrium of the system to be unique and saddle-path stable. Details available from the author upon request.
[b] The nature of eigenvalues for all variations in (ρ, a_2) parameter space, are 3 positive and 1 negative eigenvalue; except where NBS $\equiv$ ITE, Pissarides model with linear costs and utility function.
[c] The a_2 values from -1 to -3, that is decreasing cost functions and ρ values from 0 to 0.09 in (ρ, a_2) parameter space, the solution is unstable.
[d] When $r_f = r_w$ and $t \to \infty$, with linear costs and utility function.

TABLE A.2

STABILITY PROPERTIES[a]

		a_1		
ρ	Equilibria[b]	1	2	3
	Risk-Neutral Workers NBS $\equiv$ ITE[c] and Pissarides Model With linear Costs and Linear utility Function			
	Fixed Points			
	θ	0.47	0.35	0.32
	u	0.09	0.11	0.12
	w	0.51	0.38	0.28
	x	-0.23	0.06	0.33
0	Eigenvalues			
		3.00	5.25	6.54
		0.44	-0.37	0.36
		-0.45	2.96	5.43
		1.28	0.37	-0.35
	Fixed Points			
	θ	0.47	0.35	0.32
	u	0.09	0.11	0.12
	w	0.50	0.37	0.28
	x	-0.22	0.07	0.33
0.01	Eigenvalues			
		2.98	5.23	6.53
		0.44	-0.37	0.36
		-0.45	2.95	5.43
		1.28	0.38	-0.35
	Fixed Points			
	θ	0.47	0.35	0.32
	u	0.09	0.11	0.12
	w	0.50	0.37	0.28
	x	-0.22	0.07	0.33
0.02	Eigenvalues			
		2.97	5.21	6.51
		0.44	-0.37	0.36
		-0.46	2.95	5.43
		1.27	0.38	-0.35
	Fixed Points			
	θ	0.47	0.35	0.32
	u	0.09	0.11	0.12
	w	0.50	0.37	0.28
	x	-0.22	0.07	0.33
0.03	Eigenvalues			
		2.95	5.19	6.49
		0.44	-0.37	0.36
		-0.46	2.95	5.42
		1.27	0.38	-0.35
	Fixed Points			
	θ	0.47	0.35	0.32
	u	0.09	0.11	0.12
	w	0.50	0.37	0.28
	x	-0.21	0.07	0.33
0.04	Eigenvalues			
		2.94	5.17	6.48
		0.44	-0.37	0.36
		-0.46	2.95	5.42
		1.27	0.38	-0.35
	Fixed Points			
	θ	0.48	0.35	0.32
	u	0.09	0.11	0.12
	w	0.50	0.37	0.28
	x	-0.21	0.07	0.33
0.05	Eigenvalues			
		2.92	5.15	6.46
		0.44	-0.37	0.36
		-0.46	0.38	5.42
		1.26	2.95	-0.35
	Fixed Points			
	θ	0.48	0.35	0.32
	u	0.09	0.11	0.12
	w	0.50	0.37	0.28
	x	-0.21	0.07	0.33
0.06	Eigenvalues			
		2.90	5.13	6.44
		0.44	-0.37	0.36
		-0.46	0.38	5.42
		1.26	2.95	-0.35
	Fixed Points			
	θ	0.48	0.35	0.32
	u	0.09	0.11	0.12
	w	0.50	0.37	0.28
	x	-0.20	0.08	0.33
0.07	Eigenvalues			
		2.89	5.11	6.43
		0.44	-0.37	0.36
		-0.46	0.38	5.41
		1.26	2.94	-0.35

TABLE A.2
(CONTINUED)

ρ	Equilibria	a_1 1	2	3
		Risk Averse Workers		
	Fixed Points			
	θ	0.48	0.35	0.32
	u	0.09	0.11	0.12
	w	0.49	0.37	0.28
	x	-0.20	0.08	0.33
0.08	Eigenvalues			
		2.87	5.08	6.41
		0.44	-0.37	0.36
		-0.46	0.38	5.41
		1.25	2.94	-0.35
	Fixed Points			
	θ	0.48	0.36	0.32
	u	0.09	0.11	0.12
	w	0.49	0.37	0.28
	x	-0.19	0.08	0.33
0.09	Eigenvalues			
		2.86	5.06	6.39
		0.44	-0.37	0.36
		-0.46	0.38	5.40
		1.25	2.94	-0.35

I have also conducted experiments of values of a_1 in between $a_1 > 0$ to $a_1 = 1$ and $a_1 = 2$ to $a_1 = 2$ and $a_1 = 3$ for values of $\rho = 0$ to $\rho = 0.09$ in (ρ, a_1) parameter space and found the equilibrium of the system to be unique and saddle-path stable. Details available from the author upon request.

[b]The nature of the eigenvalues for all variations of the parameters in (ρ, a_1) parameter space, are 3 positive and 1 negative eigenvalues, except in the models with linear cost and utility function.

[c]where $r_f = r_w$ and $t \to \infty$, with linear costs and utility functions.

REFERENCES

Abowd, J., and F. Kramarz (2000): "The Costs of Hiring and Separations," Mimeographed, Cornell University and CREST.

Abraham, K. (1987): "Help-Wanted Advertising, Job Vacancies and Unemployment," *Brookings Papers on Economic Activity*, 1, 207 – 248.

Abraham, K. G., and J. C. Haltiwanger (1995): "Real Wages and the Business Cycle," *Journal of Economic Literature*, 33 (3), 1215 – 1264.

Abraham, K. G., J. C. Spletzer and J. C. Stewart (1999): "Why Do Different Wage Series Tell Different Stories?," *American Economic Review, AEA Papers and Proceedings*, 89 (2), 34 – 39.

Bleakley , H, A. E. Ferris and J. C. Fuhrer 1999): "New Data on Worker Flows During Business Cycles," *New England Economic Review*, July/August, 49 – 76.

Brown, D., P. Ingram, and J. Wadsworth (2004): "The Price is Right? Pay Settlements and Nominal Wage Rigidity in Britain," *British Journal of Industrial Relations*, 42(3), 507 – 525.

Coles, M., and R. Wright (1998): "A Dynamic Equilibrium Model of Search, Bargaining and Money," *Journal of Economic Theory*, 78, 32 – 54.

Fallick, B., and C. A. Fleischman (2004): "Employer to Employer Flows in the U.S. Labour Market: The Complete Picture of Gross Worker Flows," Mimeographed, The Federal Reserve Board.

Hamermesh, D. S. (1993): *Labor Demand.* Princeton: Princeton University Press.

Jordan, D. W., and P. Smith (1999): *Nonlinear Ordinary Differential Equations.* Oxford: Oxford University Press.

Krueger, A. B. (1999): "Measuring Labor's Share," *American Economic Review, American Economic Review Papers and Proceedings*, 89 (2), 45 – 51.

Lindbeck, A., and D. Snower (1988c): *The Insider-Outsider Theory of Unemployment.* Cambridge: The Massachusetts Institute of Technology Press.

McCord, C., K. Mischaikow, and M. Mrozek (1995): "Zeta functions, Periodic Trajectories, and the Conley Index," *Journal of Differential Equations,* 121, 258 – 292.

Nagypal, E. (2004): "Worker Allocation Over the Business Cycle: The Importance of Job to Job Transitions. Part 1: Evidence," unpublished manuscript, Department of Economics, Northwestern University.

Nickell, S., and G. Quintini (2003): "Nominal Wage Rigidity and the Rate of Inflation," *Economic Journal*, 490, 762 – 781.

Pissarides, C. A. (1994): "Search Unemployment with On-the-Job Search," *Review of Economic Studies*, 61, 457 – 475.

Pissarides, C. A. (2000): *Equilibrium Unemployment Theory.* The Massachusetts Institute of

Technology Press.

Shapiro, C., and J. E. Stiglitz (1984): "Equilibrium Unemployment as a Worker Discipline Device," *American Economic Review*, 74 (3), 433 – 444.

Shimer, R. (2004): "The Cyclical Behaviour of Equilibrium Unemployment and Vacancies," *American Economic Review*, forthcoming.

Silva, J. I., and M. Toledo (2005): "Labor Turnover Costs and the Behaviour of Vacancies and Unemployment," Mimeographed, Universidad Autonoma de Barcelona.

Slichter, S. (1950): " Notes on the Structure of Wages," *Review of Economics and Statistics*, 32, 80 – 91.

Smith, J. C. (1994): "Pay Settlements and Company Performance," Ph.D. Thesis, Cambridge University.

Smith, J. (2000): "Nominal Wage Rigidity in the United Kingdom," *Economic Journal*, 110, 176 – 195.

Stigler, G. (1961): "The Economics of Information," *Journal of Political Economy*, 3, 213 – 225.

Yashiv, E. (2005): "Forward-Looking Hiring Behaviour and the Dynamics of the Aggregate Labor Market," Scheduled to be presented at the Society for Economic Dynamics 2005 Meeting, Budapest.