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CHANNEL TRADING AND IMPERFECT COMPETITION: GOOD TRADES AND BAD TRADES

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Channel Trading and Imperfect Competition: Good Trades and Bad Trades

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Abstract

We investigate the potential economic effects of spectrum trading amongst firms who require spectrum licences as part of their activities. Trading takes place within the technical interference constraints enforced by a regulator. The model accommodates a variety of markets and firms, as well as both channel exchange and channel re-use (i.e. sharing across different markets). Our most detailed analytical results have focused on trade amongst oligopolists in a given (geographical) market. In this context, our results suggest that trade can enhance productive efficiency by placing licences in the hands of firms who value them most (i.e. low-cost firms). These are the 'good trades'. However, there is a danger that this process may cause higher consumer prices which, in turn, could offset the welfare effects of lower cost production, the 'bad trades'. An important outcome of our modelling is to make clear a role played by licences: they provide credible commitment mechanisms to restrict output.

JEL Classification: L10, L50, L96

Keywords: radio spectrum, spectrum trading, imperfect competition.

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1 Introduction

The objective of permitting trading in spectrum is to enable this scarce resource to be allocated to those who value it the most thereby improving efficiency. This section which draws on Leese *et al.* (2002) and Hurley *et al.* (2001) investigates the possible economic effects of such an arrangement in a market characterized by imperfect competition. We consider firms operating only in one local market. We examine the effect of a single trade over an interval for which no further spectrum is made available for this particular market. We first study a regime which imposes no restrictions on the amount of spectrum that can be exchanged. We show that a possible outcome is that trade will lead to more spare spectrum, lower output and a higher price facing consumers. One possible solution is to impose restrictions on the amount spectrum the can be traded. In some circumstances this may alleviate the previous problem, but under other circumstances a potentially more serious problem emerges: trade can result in an exchange that transfers spectrum from the more to the less efficient firm.

2 The Economic Model

We now turn to details of the economic model. We consider a single local market with N competing firms providing a homogeneous service at a market price P.[1] Firm k produces output q_k, $k = 1, 2, \cdots, N$ and output $Q = \sum_{k=1}^{N} q_k$. The demand curve is given by $Q = D(P)$; $D'(P) < 0$ and we assume that $\lim_{P\to\infty} PD(P) = 0$. In what follows we write the inverse demand curve as $P = D^{-1}(Q) = P(Q)$ for short. Units of output are customer-minutes of some service requiring radio channels as an input per unit of time (say, the financial year).

Dropping the firm subscript for now, on the supply side labour (L), capital (K) and radio spectrum (Z) combine as inputs to produce output given a production.

[1]Later we introduce sectors and in each sector we allow firms to provide the service across a number of local markets.

Let us first consider the following very general CES production function which we later specialize for reasons of tractability:

$$q = T\left[\gamma_1 L^\eta + (1-\gamma_1)[\gamma_2 Z^\xi + (1-\gamma_2)K^\xi]^{\eta/\xi}\right]^{\frac{1}{\eta}} \tag{1}$$

where T is a total factor productivity, a measure of technical efficiency. In (1) we have grouped capital and spectrum together with an elasticity of substitution equal to $\frac{1}{1-\xi}$. The elasticity of substitution between labour and the grouped inputs Z and K is $\frac{1}{1-\eta}$. Then if spectrum and capital are substitutes, but labour is a complement to the other inputs we would choose $\xi \in (0,1)$ and $\eta < 0$.

Alternatively we could model spectrum as a complement to the other two substitutable inputs by grouping inputs as follows:

$$q = T\left[\gamma_1 Z^\eta + (1-\gamma_1)[\gamma_2 L^\xi + (1-\gamma_2)K^\xi]^{\eta/\xi}\right]^{\frac{1}{\eta}} \tag{2}$$

In the limit as η and ξ tend to 0, both (1) and (2) tend to the Cobb-Douglas form

$$q = TL^{\theta_1}Z^{\theta_2}K^{\theta_3}\,;\ \ \sum_{i=1}^{3}\theta_i = 1$$

Given a production function in one of these forms[2] and given factor prices (w, r, a) per unit of labour, capital and spectrum respectively, we can formulate a *minimum cost function* per unit of output $c(w, r, a)$ in the standard way. Associated factor demands per unit of output are $L(w, r, a)$, $K(w, r, a)$ and $Z(w, r, a)$. Standard analysis gives $\frac{\partial L}{\partial w}, \frac{\partial K}{\partial r}, \frac{\partial Z}{\partial a} < 0$. We assume that each firm is a price taker in factor markets and in the market for licences which incorporates all local markets such as the one modelled in this section. We assume that the price elasticity of demand in the market, $\epsilon(Q) = -\frac{PdQ}{QdP}$, is constant with respect to total output Q. We assume that $\epsilon > 1$ for reasons which will become apparent.

[2]Yet another form of the production function relevant for telecommunications radio services is to explicitly introduce base-stations as one of the forms of capital and to model the relationship between the number of stations, the available spectrum, the number and users and the bit speed of the service–see **?**).

We now specialize the economic model by considering a fixed technology for which spectrum is a pure complement and cannot be substituted by capital or labour. One unit of 'output' requires one radio channel and output capacity equals the total number of channels available. Thus for firm i to produce output q_i per period it requires $r_i \geq q_i$ radio channel licences. We assume that the licence fee a is independent of the firm and its location. Total costs include a set-up cost F_i so total costs are given by

$$C_i(q_i, a) = F_i + r_i a + c_i q_i \tag{3}$$

where $c_i = c_i(w, R)$ is the cost function associated with the CES production function of labour and capital in (2).

3 The Cournot-Nash Equilibria

3.1 The Unconstrained Cournot-Nash Equilibrium

In the unconstrained Cournot-Nash equilibrium (UCNE), each firm chooses output and purchases just sufficient licences to service this output; i.e. $r_i = q_{ij}$. In a Cournot-Nash equilibrium firm i then maximizes profits given by

$$\Pi_i = Pq_i - C_i(q_i, a) = (P - c_i - a)q_i - K_i \tag{4}$$

taking the output of all other firms, $\sum_{j \neq i}^{N} q_j = \tilde{q}_i$, say, as given.

Writing $P = P(Q) = P(q_i + \tilde{q}_i)$ and differentiating with respect to q_i, with $\tilde{q}_i$ fixed, the first-order condition for an internal maximum with $q_i \geq 0$ is

$$P' q_i + (P - c_i - a) = 0 \tag{5}$$

Then rearranging and using the assumption of a constant elasticity, firm i's *market share* (given the market-clearing price) is given by

$$\frac{q_i}{Q} = \frac{(P - c_i - a)\epsilon}{P} \tag{6}$$

Summing (6) over all firms leads to the mark-up pricing result:

$$P = \frac{\bar{c} + a}{1 - \frac{1}{\epsilon N}} \tag{7}$$

where $\bar{c} = \frac{\sum_{i=1}^{N} c_i}{N}$. Thus $\epsilon N > 1$ ensures that the price in a symmetric equilibrium is always positive[3] $\epsilon > 1$ is also necessary for a profit-maximizing level of output to exist when the market has only one firm. In what follows we therefore assume a price elasticity greater than unity.

To motivate channel trading we are interested in the case where asymmetries exist between firms. Then in a non-symmetric Nash equilibrium (6) indicates the intuitive result that, given the licence price a and the product market price P (which in equilibrium will depend on a, as (7) shows), the market share of firm i will increase as its productive efficiency rises (i.e. as c_i increases). As firm i becomes less efficient its optimal output will approach zero and closure of the firm occurs. Before this eventuality however, because there are fixed costs independent of output, profits will be driven down to zero. We therefore introduce a participation constraint $\Pi_i \geq 0$ for firm i in the Nash equilibrium.[4] Outputs are given by solving the following system (subsequently referred to as programme **UCNE**) of $N \leq N^{max}$ equations in q_i, $i = 1, 2, \ldots, N$, subject to zero profit participation constraints:

$$\begin{aligned} P(1 - \frac{q_i}{Q\epsilon}) - c_i - aq_i &= 0 \\ Q &= \sum_{i=1}^{N} q_i \\ P = D^{-1}(Q) &= \left[\frac{A}{Q}\right]^{\frac{1}{\epsilon}} \\ \Pi_i &\geq 0 \end{aligned}$$

We now make a convenient simplifying assumption to make the process of entry and exit tractable. We assume that

$$K_1 < K_2 < \ldots < K_{N^{max}}\,; \;\; c_1 < c_2 < \ldots < c_{N^{max}} \tag{8}$$

[3] It is also a sufficient condition for the second-order maximization condition to be satisfied.

[4] Note that in the presence of fixed costs, $\Pi_i \geq 0$ implies that $q_i > 0$.

so that the efficiency of firms 1 to N^{max} can be ranked unambiguously in terms of the cost parameters. Arranging firms in order of efficiency, if the profits of the least efficient are negative, this firm is eliminated and the procedure is repeated with the remaining $N^{max} - 1$. This iterative process is repeated until we arrive at the Cournot-Nash equilibrium with all firms having non-negative profits.

The number of firms who can participate will depend on the distribution of cost parameters K_i, c_i, the parameters describing demand conditions, A, ϵ and the licence price a. Starting with N^{max} potential firms defined by their cost parameters, firms will leave or enter the market as demand conditions and the licence price change. If the regulator releases a fixed number of radio channels for the market as whole, then the market-clearing licence price will depend on this number.

Figures 1 to 3 examines the effect on the market of increasing the licence price increasing from $a = 0$ to $a = 0.35$. We choose a demand function $Q = AP^{-\epsilon}$ and parameters values: $N^{max} = 5$, $\epsilon = 1.1$, $c_1 = 0.80$, $c_2 = 0.81$, $c_3 = 0.82$, $c_4 = 0.83$, $c_5 = 0.85$, $K_i = 2.5$ and $A = 150$. Taking total output first, Figure 1 shows that a higher licence price forces up the price and lower output. Interestingly, from Figures 2 and 3, for our chosen elasticity $\epsilon = 1.1$, the positive effect of a price increase on the profits of the least efficient firm outweighs the negative effect of a higher licence price, allowing it to become viable and enter the market. Profits initially fall for the most efficient four firms but then start to rise. A high licence fee therefore has the effect of redistributing surplus from consumers to producers, as well as raising revenue for the regulator, and supports a more competitive (high N) market.

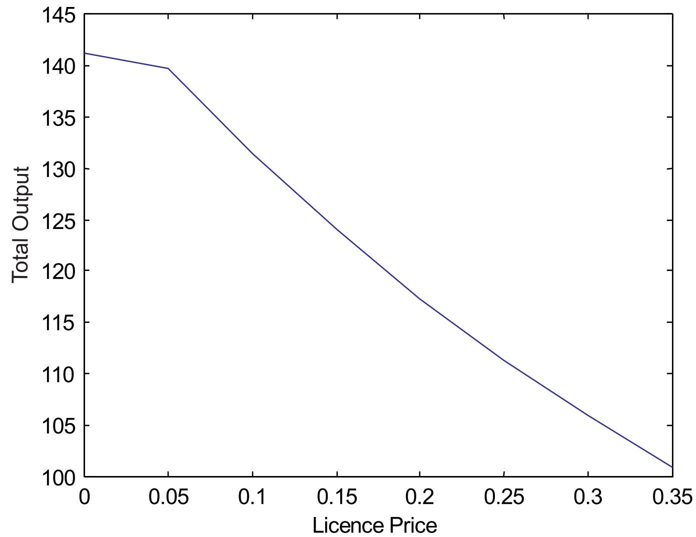

Figure 1: Output and Licence Price

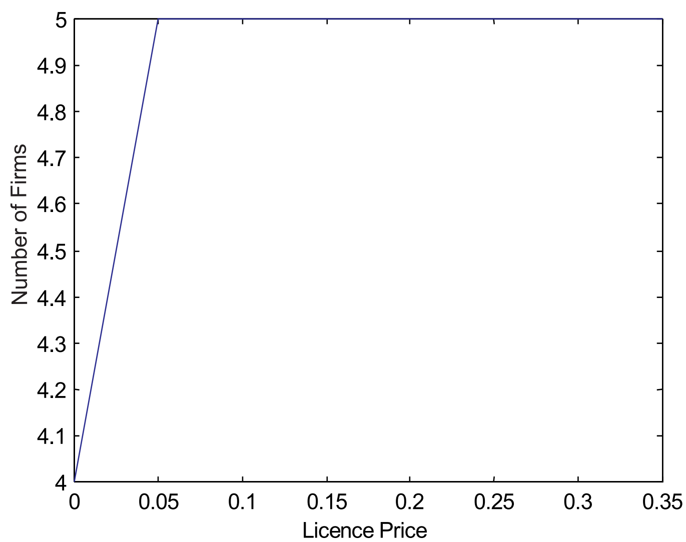

Figure 2: Participating Firms and Licence Price

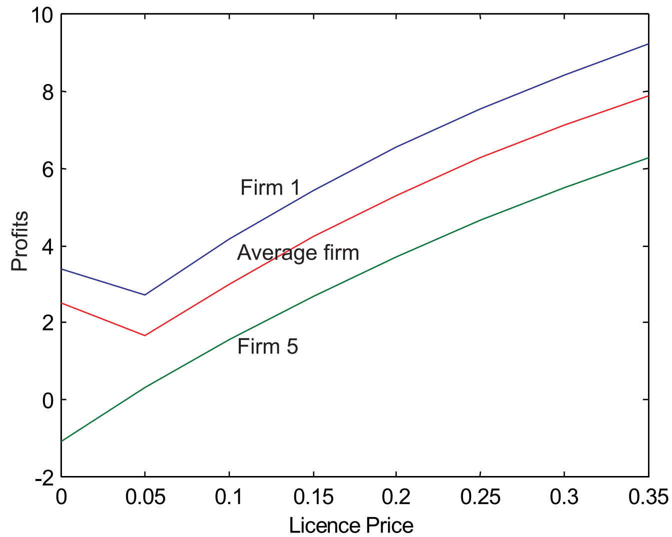

Figure 3: Profits and Licence Price

3.2 The Constrained Cournot-Nash Equilibrium

In a constrained Cournot-Nash equilibrium, firm i faces a capacity constraint $q_i \leq r_i$, where r_i is the number of radio channels for which it has licences. It then maximizes profits given by

$$\Pi_i = (P - c_i)q_i - K_i - ar_i \tag{9}$$

subject to this constraint, taking the output of all other firms, $\sum_{j \neq i}^{N} q_j = \tilde{q}_i$, as given, as before. Notice that licencing costs, ar_i, are now part of fixed costs and only affect the firm's participation constraint.

To carry out this constrained optimization programme, define the Lagrangian

$$L_i = \Pi_i + \lambda_i(r_i - q_i) + \mu_i \Pi_i \tag{10}$$

where $\lambda_i \geq 0$ and μ_i are Lagrangian multipliers associated with constraints $r_i \geq q_i$ and $\Pi_i \geq 0$ respectively. The Kuhn-Tucker first-order condition for a maximum is

$$(1 + \mu_i)\left[P(1 - \frac{q_i}{Q\epsilon}) - c_i\right] = \lambda_i \tag{11}$$

$$\mu_i \Pi_i = \lambda_i(r_i - q_i) = 0 \tag{12}$$

The left-hand-side of equation (11) now defines the function $(1+\mu_i)f(q_i)$. Suppose the participation constraint is satisfied; then $\mu_i = 0$. If $f(r_i) > 0$, then $q_i = r_i$ and firm i uses all its acquired channels producing at full-capacity. If $f(r_i) \leq 0$, then the capacity constraint no longer holds and $q_i < r_i$ implying spare radio channels and capacity. In this case, q_i is given by (11) with $\mu_i = \lambda_i = 0$. If in equilibrium $\mu_i > 0$, $\Pi_i \leq 0$, the firm exits and we put $q_i = 0$ for that firm.

The constrained Cournot-Nash equilibrium (subsequently referred to as programme **CCNE**) is then given by solving

$$\begin{aligned}
(1+\mu_i)\left[P(1-\frac{q_i}{Q\epsilon}) - c_i\right] &= \lambda_i \\
\mu_i\Pi_i = \lambda_i(r_i - q_i) &= 0 \\
Q &= \sum_{i=1}^{N} q_i \\
P &= D^{-1}(Q) = \left[\frac{A}{Q}\right]^{\frac{1}{\epsilon}}
\end{aligned}$$

which gives $3N$ equations in q_i, μ_i and λ_i, $i = 1, 2, \ldots, N$, given r_i, $i = 1, 2, \ldots, N$ and a^R. As for the unconstrained Cournot-Nash equilibrium, $N \leq N^{max}$ is the number of firms after exits for which the participation constraint $\Pi_i \geq 0$.

4 Channel Trading Games

4.1 Game 1: No Restrictions on Trade

We now allow firms to trade on a bilateral basis and we examine the effects of a single exchange. We assume that interference constraints are such that no channel reuse is possible. After agreeing to a transfer of channels at a particular price we assume that no collusion is allowed and in a new constrained equilibrium two firms re-optimize with respect to output independently. Before trade commences the regulator sells radio channels at price a. In a UCNE, firms acquire these channels to service their anticipated output.[5] An crucial assumption is that *no new licences are issued by the regulator after the initial sale of licences.* To summarise we have the following sequence of events:

Stage 1. All firms acquire spectrum just sufficient to service the UCNE at a given licence price a. Denote output equal to channel licences by $\bar{q}_i = \bar{r}_i$, $i = 1, 2, \cdots, N$. Let $\bar{Q}$ and $\bar{P}$ be the corresponding total output and market price respectively.

Stage 2. Firms $i = f, g$ trade and agree a trade of e channels without any restrictions on e, at an exchange price.

Stage 3. All firms (including $i = f, g$) independently choose output levels in a new Cournot-Nash equilibrium of this stage of the game found by solving the programme **CCNE** where the new constraints are:

$$q_f \le r_f = \bar{r}_f - e\,; \quad q_g \le r_g = \bar{r}_g + e \tag{13}$$

The appropriate equilibrium concept is a **sub-game perfect equilibrium** found by backward induction at stage 3. Each trade redistributes capacity which remains fixed in total. If firms continue to produce at full capacity as they do in the initial Cournot-Nash equilibrium then total output and price remain constant at their levels in the initial equilibrium. Consumers are unaffected by the trade, but firms benefit in this case. If firms produce below capacity after trade, then total output falls, the price rises and consumers lose out. The condition for price to remain unchanged at stage 3 $\lambda_i > 0 \;\; i = f, g$ i.e. from the first-order conditions in **CCNE**

$$\lambda_f = \bar{P}\left[1 - \frac{(\bar{q}_f - e)}{\bar{Q}\epsilon}\right] - c_f \quad > \quad 0 \tag{14}$$

$$\lambda_g = \bar{P}\left[1 - \frac{(\bar{q}_g + e)}{\bar{Q}\epsilon}\right] - c_g \quad > \quad 0 \tag{15}$$

Using the first-order condition for the original Cournot-Nash equilibrium in **UCNE**

[5] Alternatively channels may be issued in a *ad hoc* fashion (which we actually assume in the results from the demonstrator described in the report to the Radiocommunications Agency.)

where firms produce at full capacity these conditions become:

$$\lambda_f = a + \left(\frac{\bar{P}}{\bar{Q}\epsilon}\right) e \quad > \quad 0 \tag{16}$$

$$\lambda_g = a - \left(\frac{\bar{P}}{\bar{Q}\epsilon}\right) e \quad > \quad 0 \tag{17}$$

Clearly (16) always holds. Condition (17) can be written as

$$a > \left(\frac{\bar{P}}{\bar{Q}\epsilon}\right) e \tag{18}$$

We have then shown that starting from a Cournot-Nash equilibrium with exogenous licence price a, price $\bar{P}$ and output $\bar{Q} = D(\bar{P})$ where $D(\cdot)$ has a constant elasticity ϵ, an exchange of e channels from firm f to firm g does not change the price in the post-trade equilibrium iff (17) holds.

Turning to stage 2, the number of channels exchanged, e, is agreed at a price arrived at by some bargaining process. The latter does not concern us here (but is modelled in Hurley *et al.* (2001) and Leese *et al.* (2002). Given our assumption of constant returns to trade the efficient bargain will see all channels transferred from the less efficient to the more efficient firm. Let firm g be the latter (i.e., $c_g < c_f$). It follows that $e = \bar{q}_f$ and firm f ceases to produce at stage 3. We can therefore write (18) as

$$\frac{a}{\bar{P}} > \frac{\bar{q}_f}{\bar{Q}\epsilon} \tag{19}$$

This condition for trading not to result in an increase in the price can be interpreted as follows. The left-hand side is the real price (i.e, the price relative to the price of output) of a unit of spectrum. The right-hand side is the market share of the inefficient firm multiplied by ϵ^{-1}. As ϵ increases, market falls and the price in a symmetric equilibrium approaches the total marginal cost $c+a$. Hence trading may result in a higher price if spectrum is cheap and/or the inefficient firm has a large market share and/or the elasticity of demand is low.

One possible policy response to this result is to limit the quantity of spectrum that can be exchanged at any time. The next section modifies the game to incorporate such a constraint.

4.2 Game 2: Quantity Restrictions on Trade

We now impose a constraint on the number of channels that they are permitted to exchange so that $e \leq \mu$, say, where μ is determined by the regulator. As μ is raised we then approach the case where a firm can, if it chooses, sell all its licences and cease to produce a service. With this restriction the details of the game are pretty much as for game 1, but now the condition for trading not to result in a decrease in price becomes

$$\frac{a}{\bar{\bar{P}}} > \frac{\mu}{\bar{Q}\epsilon} \tag{20}$$

where $\mu \leq \bar{q}_f$. Then if $\frac{\bar{q}_f}{\bar{Q}\epsilon} > \frac{a}{\bar{P}} > \frac{\mu}{\bar{Q}\epsilon}$, the restriction that $e \leq \mu$ is a sound policy that on this occasion prevents the harmful effects for consumers highlighted in game 1. However our final trading game points to any possible drawback from trading, namely that in some circumstances trading with quantity restrictions can lead to licences and capacity passing from an efficient to an inefficient firm.

4.3 Game 3: Arbitrary Initial Holdings of Licences with Quantity Restrictions on Trade

In our final game we stick with quantity restriction on trade but now at stage 1 assume that the initial holding of licences are arbitrary and can be in excess of the initial UCNE output levels; i.e., $\bar{r}_i \geq \bar{q}_i$, $i = f, g$. Suppose first that excess spectrum held by each firm exceeds the amount that can be traded; i.e.

$$\bar{r}_g - \bar{q}_g > \mu\,; \quad \bar{r}_f - \mu > \bar{q}_f \tag{21}$$

In this case if either firm acquired μ more channels they will still choose the same UCNE levels of output and nothing in the market will change.

Suppose now that

$$\bar{r}_g - \mu < \bar{q}_g\,; \quad \bar{r}_f - \mu > \bar{q}_f \tag{22}$$

The the efficient firm g is producing close to its capacity constraint, but the inefficient firm f is not. First suppose that up to μ channels pass from the inefficient firm f to the efficient firm g. Since firm f has excess capacity exceeding μ and firm g now has more capacity, neither firm would choose to revise its output and nothing changes in the market. However if channels were exchanged in the opposite direction, the efficient firm would face a constraint $r_g = \bar{r}_g - \mu < \bar{q}_g$ and would be forced to lower its output. Since an inefficient firm produces less (see (6)) it will not compensate completely for the reduction if firm g's output. Total output therefore falls, and the price prices. The two firms will agree to such an arrangement if the rise in price is sufficient to compensate for the higher costs of production (see the example in the next sub-section).

5 Simulation Results

This section reports simulations results for a simulation model constructed by Hurley *et al.* (2001). This model is based on Game 3 above and assumes, for programming convenience, that firms only trade one licence at a time. When firms trade they exchange the licence at a price determined by a Nash bargaining solution. The algorithm searches sequentially for trades and allows them if gains exist and interference constraints are not violated. Both intra-market trades (as analyzed above) and inter-market trades are allowed.

We choose initial values as follows: $A_j = 5,\ \epsilon_j = 1.2, c \in \{0.5, 1.0, 1.5\},\ n_j \in \{1, 2, 3, 6\}$. We assume six licences per market, which that may be the result of piecemeal policy over the years.[6] We then proceed to look at trades within a single market ('inter-market trades') and trades between markets 'intra-market trades'). Below we present some illustrative findings.

5.1 Intra-market trades

To begin, consider a market with two firms, with marginal costs of $\mathbf{c} = (1.0,\ 1.5)$, each holding three licences. The initial equilibrium has a market price of 2.14, profits

[6] Justification for these initial configurations can be found in Hurley *et al.* (2001).

of $\Pi = (1.46,\ 0.46)$ and consumer surplus of 21.47. Outputs are $\mathbf{x} = (1.28,\ 0.72)$. In this case, there is no incentive to trade: neither firm gains sufficiently from the price effects restricting its rival's output (by buying a licence) and then increasing its own.

A slight change generates trade, however (see Table 2). Suppose that $\mathbf{c} = (0.5,\ 1.0)$, each firm with three licences. Initial market price is 1.29, we have $\Pi = (2.13,\ 0.28)$ and consumer surplus of 23.77. Outputs are $\mathbf{x} = (2.71,\ 0.99)$—both high as a result of the firms' lower marginal costs in this example. There is now an incentive to trade from firm 1 to 2, the high cost firm. (Trade the other way would not alter constraints sufficiently to change outputs.) This restricts firm 1's output and allows the high cost firm gain sufficiently from the resulting high price to make trade worthwhile. This indicates the role of licences as credible commitments to constrain output. In fact, trade will continue here until firm 2 has all six licences and a high cost monopoly results. The final price is 6, firm 2's profit has doubled to 0.53, but consumer surplus has fallen to 17.47: total welfare (the sum of consumer surplus and profits) falls from 26.18 to 20.38.

	Initially	After all trading
Licences	(3, 3)	(0, 6)
Outputs	(2.71, 0.99)	(0, 0.58)
Profits	(2.13, 0.28)	(2.38, 0.53)
Mkt. P	1.29	6.00
C. surplus	23.77	17.47
Welfare	26.18	20.38

Table 2. Simulation Results for Intra-Market Trades

As we allow ϵ to rise, this result eventually disappears. The reason is straightforward: the more elastic is demand, the smaller is the price effect of restricting output, so the lower is the high cost firm's gain rom doing so. Indeed, as ϵ rises past 1.4, the incentives to trade reverse and a single trade from firm 2 to firm 1 takes place.

Suppose we now have more firms (say, $n = 6$), with marginal costs of $\mathbf{c} = (0.5,\ 0.5,\ ,0.5,\ 1.0,\ 1.0,\ 1.5)$, and each firm having one licence. A variety of possibilities now emerge, depending on the sequence of trades. However, the outcomes can be partitioned into two sets (see Table 3). In the first of these, one of the low cost firms monopolises the market (trade ends when it holds all six licences). The productive efficiency of this outcome is not always enough, however, to increase total welfare, because of the resulting increase in price. In the second set of outcomes, each of the three lowest cost firms finishes with two licences (the others have none). In this case, the benefits of low cost production are enough to offset the (weaker) effects of concentration on price, with the result that this set typically increases total welfare.[7]

5.2 Inter-market trades

Now suppose that there are four markets, with associated interference constraints as pictured in Figure 4. Whereas within a market channel re-use was ruled out, between markets re-use is possible subject to these interference constraints. According to figure 4 re-use (i.e., channel-sharing) is possible between markets located at A and C, and between B and D but not between A and B, B and C or C and D.

A key issue now emerges because it is possible for trading not to terminate. The reason is that the transfer of a licence across markets has effects on prices in both. Thus, firms that may not have been thinking of trading may suddenly become keen to do so. One solution to this is to introduce a threshold gain from trade below which trades do not take place.[8] In this case, convergence of the trading process

[7] 100 replications of the program produced the second outcome 25 times, a statistically significantly smaller number of times than the first outcome. To the extent that the regulator would prefer the second outcome, it would be interesting to examine institutional arrangements that encourage trading patterns which generate this.

[8] One possible interpretation of this would be a lump-sum tax imposed on the 'capital gains' resulting from a trade.

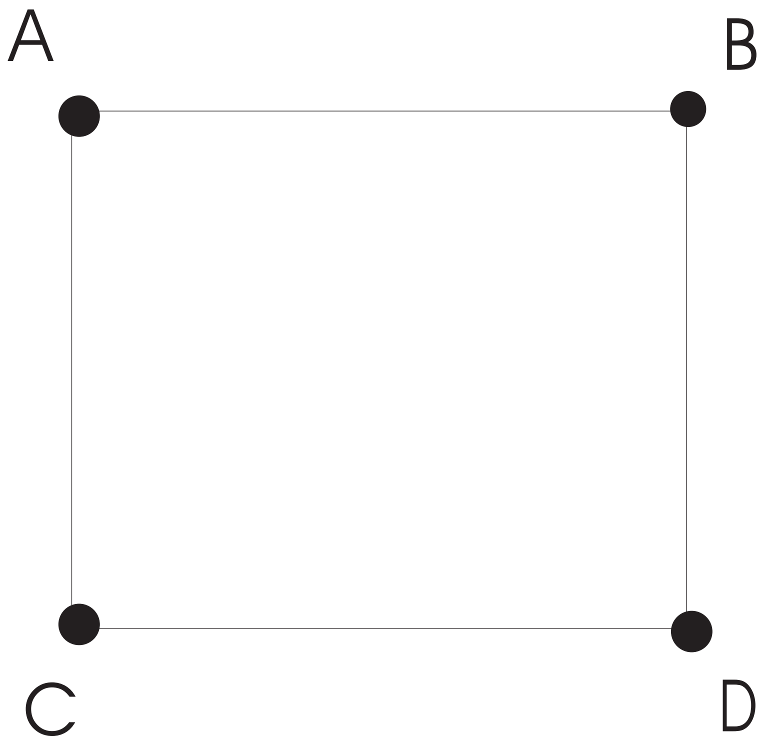

Figure 4: Interference Constraints

becomes more common (in the sense that trading becomes increasingly infrequent as searches for trades take place).

	Initially	Outcome A	Outcome B
Licences	(1, 1, 1, 1, 1, 1)	(6, 0, 0, 0, 0, 0)	(2, 2, 2, 0, 0, 0)
Outputs	(1, 1, 1, 0.63, 0.63, 0)	(6, 0, 0, 0, 0, 0)	(2, 2, 2, 0, 0, 0)
Profits	(0.6, 0.6, 0.6, 0.1, 0.1, 0)	(1.3, 0, 0, 0, 0, 0)	(0.6, 0.5, 0.5, 0.2, 0.1, 0.2)
Mkt. P	1.14	3.00	0.86
C. surplus	24.35	20.07	25.77
Welfare	26.45	23.41	27.92

Table 3. Simulation Results for Intra-Market Trades

To give an example, assume there are two firms in each market in Figure 3 (i.e. 8 firms in total) and recall that, given the figure, channels cannot be re-used at adjacent nodes, but can be at diagonally opposite ones. One firm in each market has marginal cost of 0.5, the other has marginal cost of 1.0. There are 9 channels: Market A has channels 1 and 2; B has channel 3; C has channels 1, 2 and 9; and D has channels 3, 4, 5, 6, 7 and 8. This allocation satisfies the interference matrix constraints in Figure 3.) The details of how these are split amongst the firms can be found in Hurley *et al.* (2001). Now suppose a trading threshold of 1; i.e. only trades yielding gains of at least this much can take place. In this case, one trade takes place (from the high cost firm in D to that in B). This raises output and lowers price in B sufficiently to increase total welfare (aggregated across the four markets). A lower trading threshold (0.75) encourages a second trade and welfare decreases in our examples. Moving to a threshold of 0.5 encourages enough trade that local monopolies can emerge. In particular, we find that the low cost firm in each case acquires three licences each but that total welfare drops. Once the threshold is at 0.2, trade ceases to terminate—an interesting result given that the number of markets, and firms in each, is relatively small.

6 Summary and Policy Issues

We have presented a model to investigate the potential economic effects of spectrum trading amongst firms who require spectrum licences as part of their activities. Trading takes place within the technical interference constraints enforced by a regulator. The model can, in principle, accommodate a variety of markets and firms, as well as both channel exchange and channel re-use (i.e. sharing across different markets). Our most detailed analytical results have focused on trade amongst oligopolists in a given (geographical) market. In this context, our results suggest that trade can enhance productive efficiency by placing licences in the hands of firms who value them most (i.e. low-cost firms). However, there is a danger that this process may cause higher consumer prices which, in turn, could offset the welfare effects of lower cost production. Subsequent discussion suggests that similar forces are likely to prevail in the other market settings we cover. An important outcome of our modelling is to make clear a role played by licences: they provide credible commitment mechanisms to restrict output. Unlike in other Cournot settings, the sale of a licence forces a firm's output down (assuming it was fully utilising its licences): the licences act as capacity constraints.

We have also presented numerical results to illustrate the outcomes trade might create. In the context of intra-market trades, we showed that trade need not occur and that, if initial allocations are not optimal, they may induce high-cost monopolies. Other examples illustrated the trade-off from out theoretical model: productive efficiency versus price rises through increased concentration: we found cases where trade increased and decreased welfare. We have also seen that the outcomes of trading my be pathological, with some outcomes being more desirable than others. This suggests that the design of trading institutions matters. Turning to inter-market trades, we have seen that trade generally will not terminate, because of external price effects as licences are transferred across markets. The imposition of suitable trading thresholds can overcome this problem. Further simulation work is required to gain a full understanding of the complex forces underlying these results, but our illustrative results suggest that trade may have a variety of outcomes.

Finally, let us think about potential developments of our trading model. To begin with, our model assumes complete information between potential traders. This may, perhaps, be feasible amongst local taxi firms (at least as a first approximation), where market conditions and competitors may be well known. It is less likely, however, to prevail for inter-market trades or for trades between network operators, where the potential for commercially sensitive information and strategic behaviour may be significant. In such circumstances, the costs of reaching agreement can be significant, as experience negotiating network access terms in New Zealand has demonstrated (see Spiller and Cardilli (1997)).

This suggests that the efficiency of the trading process we have modelled may be compromised be two types of transactions cost associated with information asymmetries: bargaining costs and search costs. In the first case, our use of the Nash bargaining solution may need modification to allow for other potential disagreements between negotiators. Such 'non-cooperative bargaining' involves considerable technical complexity (see e.g. Osborne and Rubinstein (1990); Kennan and Wilson (1993)) and may, for practical purposes, constrain the situations that can be modelled. One possible means of overcoming the potential hold-out problems that can arise here is provided by the US Clean Air mini-auctions: the evidence suggests that these helped identify an appropriate range of prices for pollution permits and, therefore, kick-started the pattern of bilateral trades.

Turning to search costs, our model assumes that parties can identify potential traders costlessly. While, again, this may not be unreasonable in a small local market, it will be harder to achieve in a densely populated local market or in many inter-market trades. Theoretical work on 'matching' in markets provides some useful algorithms for resolving these problems (see Osborne and Rubinstein (1990), ch. 9) and it would be sensible to investigate how easily these could be incorporated into the current demonstrator.

Another aspect of trading that we have not considered is the temporal aspect where firms trade and bargain taking into account the implications of each trade for future production and trades. Simultaneous trades also raise complications. We have circumvented these problems by assuming that trades are sequential and the time between each trade is sufficiently great to warrant a myopic calculation of the consequences of the next trade. These assumptions are clearly restrictive. By relaxing our assumption that firms are myopic, spectrum would become an asset, and we would need to consider the potential for intermediate 'spectrum agents' and expectations-based trading.

In terms of extensions, it would also be possible to consider other forms of market competition (such as Bertrand price competition) and product differentiation within a geographical market. Both of these would fit the current framework, and would allow the model to cover a particularly wide variety of market situations.

Finally, it is important to make clear how our current work (and potential extensions) can link into recent policy consultations in the UK. The Radiocommunications Agency (2001) consultative document (pp. 33–38) raises several questions in relation to spectrum trading. To illustrate how our work may be adapted to consider such questions, we suggest three links here. To begin, there are questions of whether trading may damage allocative efficiency by encouraging anti-competitive practices (para. 106); this is a danger illustrated by our current demonstrator. Next, the document asks how such trade might affect investment by existing and potential operators (para. 111); this is a question that could be examined by adding an initial investment period to our existing set-up (and note our comments about dynamic efficiency above).

As our third illustration, Question xxxvii asks what "market infrastructure" may be needed to facilitate trade. This echoes mini-auctions used under the US Clean Air Act and suggests that we should consider the role for market intermediaries to lubricate trade. Our work also suggests another intriguing institutional factor that may lubricate trade: the initial allocation of spectrum amongst firms. Simple reflection on graphs like that in Figure 3 indicate that there may be circumstances where interference constraints reject otherwise productively efficient trades. A solution to this would be for the regulator to make available extra measures of spectrum (perhaps more than that required to meet current demands). This could allow firms to 'trade round' interference problems and place licences with low-cost firms.

It is clear that much interesting work remains to be done before the net effects of spectrum trading can be fully understood. Hopefully, however, the present paper demonstrates the potential benefits of integrating economic and channel assignment tools for analysing the issues involved. The model is flexible enough to be extended in a variety of ways and may, therefore, provide a useful framework for future research in this important policy area.

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