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Fiscal Policy in a Monetary Union: Can Fiscal Cooperation be Counterproductive?*

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Abstract

We analyze the interaction of monetary and fiscal policies in a monetary union where the common central bank is more conservative than the fiscal authorities. When monetary and fiscal policies are discretionary, we find that the Nash equilibrium is sub-optimal with higher output and lower inflation than the cooperative Ramsey optimum. In a further example of counterproductive cooperative, we find that fiscal cooperation makes matters worse. We also examine cooperative and non-cooperative fiscal policy in the case where the central bank can commit and has the same preferences as the fiscal authorities.

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1 Introduction

The Economic and Monetary Union (EMU) in Europe has a common central bank that decides monetary policy, but each member country's government decides its own fiscal policy. The Maastricht Treaty stipulates that the European Central Bank (ECB) should be independent of day-to-day political control from the member countries. This raises some new issues for the conduct of monetary and fiscal policies in the EMU. First, the monetary policy of the ECB and the fiscal policies of the member countries are decided separately (as a non-cooperative game); this leads to a Nash or leadership equilibrium depending on the structure of the game. Second, the ECB is likely to be more conservative than the politicians who run the treasuries in the member countries, either by explicit mandate or by natural inclination. This conservatism may concern both the ideal levels of outputs and inflation and the tradeoffs among them. This conflict of objectives raises the possibility that the resulting equilibrium is suboptimal.

In this paper we examine the interaction of monetary and fiscal policies in a monetary union and find some new results and suggestions regarding the design of the policy institutions. We consider a model where monetary and fiscal policies affect output and inflation, and the policymakers have possibly conflicting objectives regarding outputs, inflation and the tradeoffs among them. Because some prices are set in advance, an unanticipated monetary expansion raises output and inflation. An unanticipated fiscal expansion of demand financed by lump-sum taxes puts an upward pressure on prices and expands supply. When monetary policy is discretionary, the conflict of objectives leads to a non-cooperative race between the monetary and the fiscal authorities. With fiscal policies trying to achieve output beyond the central bank's ideal, and the monetary policy trying to achieve inflation below the fiscal authorities' ideal, in the resulting Nash equilibrium both inflation and output can be more extreme than the ideal points of all policymakers. Most importantly, the Nash equilibrium is suboptimal.

The suboptimality of the Nash equilibrium arises irrespective of whether the fiscal authorities cooperate or not in choosing their policies. In fact, the Nash equilibrium without cooperation may be less extreme and welfare-superior to the Nash equilibrium with cooperation. This occurs because cooperation exacerbates the time-consistency problem of fiscal policies.

These results suggest that, when there is a conflict of objectives among the monetary and fiscal authorities, cooperation may fail to improve economic outcomes. Careful design of monetary and fiscal institutions so as to make the central bank and the governments agree on the ideal levels of output and inflation leads to better outcomes. In that case, the desired goals are achieved despite any disagreement about the relative importance of the two goals, despite lack of cooperation among the policy-makers and without the need for monetary commitment.

2 Literature Review

Several works have considered the interaction of monetary and fiscal policies in a monetary union. Sibert (1992), Levine and Brociner (1994) and Beetsma and Bovenberg (1998) consider monetary-fiscal interaction in a monetary union where the purpose of fiscal policy is the provision of public goods. This literature suggests that a monetary union with decentralized fiscal decisions and discretionary monetary policy produces an inflationary bias and excessive spending on public goods; fiscal coordination or fiscal leadership may discipline fiscal and monetary policy. In this paper, we focus on the countercyclical role of fiscal policy. We consider a central bank and a government with possibly conflicting goals over output and inflation, and study the equilibria with and without monetary commitment, including Nash and leadership equilibria.

Dixit and Lambertini (2003 a) study in detail the case where the monetary and fiscal authorities agree about the ideal levels of output and inflation; Dixit and Lambertini (2001) study the case where monetary and fiscal authorities disagree on their ideal outcomes. In these papers, however, fiscal policies do not have a time-consistency problem.

Cooper and Kempf (2000) analyze monetary and fiscal policy with and without a monetary union in a two-country setting where the monetary and fiscal authorities agree on the policy goals. Unlike the setting of our model, the two authorities share a budget constraint in Cooper and Kempf. Each person gets an idiosyncratic shock that determines their preference between home and foreign goods; moreover, there is a cash-in-advance requirement in the currency of the good to be purchased and the exchange rate market cannot be accessed after the idiosyncratic shock is realized. The benefits of joining the union are that individuals can hold the optimal quantity of money; the costs are that each fiscal authority is tempted to raise its own GDP via expansionary monetary policy, passing on some of the costs to the other country in the form of higher common prices. When the monetary authority has leadership, a monetary union is Pareto-improving; however, if the fiscal authorities have leadership or monetary transfers to the fiscal authorities are constrained, a monetary union is welfare improving only if the aggregate shocks are highly correlated.

3 The Model

We consider a world economy that consists of two countries, country 1 and country 2. These two countries are in a monetary union and therefore share a common currency. They have separate governments that run fiscal policies; monetary policy, on the other hand, is decided by a common and independent central bank. We now proceed to model country 1; country 2 is symmetric.

3.1 Consumers

The representative household in country 1 maximizes the discounted sum of utilities of the form

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U_{1,s}, \tag{1}$$

where the period utility function is

$$U_{1,t} = \left[\log C_{1,t} + \chi \log \frac{M_{1,t}}{P_t} - \frac{d}{1+\eta} \int_0^n N_{1,t}(i)^{1+\eta} di + \frac{\alpha}{1-1/\rho} G_{1,t}^{1-1/\rho} \right], \tag{2}$$

with $d > 0, \chi > 0, \eta \geq 0$ and $\rho > 1$. $0 < \beta < 1$ is a discount factor, $C_{1,t}$ is consumption, $M_{1,t}/P_t$ are real balances. $N_{1,t}(i)$ is the quantity of labor of type i supplied by the representative individual to domestic firms; it is assumed that each differentiated good uses a specialized labor input in its production. The assumption of differentiated labor inputs is not necessary but convenient, as households with identical initial assets supply the same quantities of labor and receive the same labor income. $\eta > 0$ is the elasticity of the marginal disutility of labor with respect to labor supply. $G_{1,t}$ is public spending. Hence, it is assumed here that period utility depends positively on public good provision by the own-country government, with the parameter α measuring the relative importance of public versus private consumption for welfare. Because we are interested in studying how monetary and fiscal polices can better stabilize output and inflation in response to shocks, our welfare analysis will be based on the limiting economy as α goes to zero and public spending ceases to raise social welfare *per se.*[1]

There is a continuum of differentiated goods distributed over the interval $[0, 1]$; a fraction n of these goods is produced in country 1 while the fraction $1 - n$ is produced in country 2. $C_{1,t}$ is the real consumption index

$$C_{1,t} = \left[\int_0^1 C_{1,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $C_{1,t}(i)$ is consumption of good i at time t and $\theta > 1$ is the constant elasticity of substitution among the individual goods. The representative household consumes all goods produced in the world economy. The price index P_t corresponding to the consumption index $C_{1,t}$ is

$$P_t = \left[\int_0^n P_{1,t}(i)^{1-\theta} di + \int_n^1 P_{2,t}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \tag{4}$$

which is the minimum cost of a unit of the aggregate consumption good defined by (3), given the individual goods prices $P_{1,t}(i), P_{2,t}(i)$.

[1] We do not need to assume that $\alpha \to 0$. In fact, our results hold true if $\alpha > 0$ as long as $\alpha < 1/(\theta - 1)$. Intuitively, if $\alpha < 1/(\theta - 1)$ government spending is a public good that raises social welfare but, in a sense, not enough; the natural rate of output is still suboptimally low so that monetary and fiscal policies are time inconsistent. See Appendix B for a detailed proof.

The representative household in country 2 has symmetric preferences to those in (2) and (3). We allow for exogenous aggregate disturbances to the period utility function as the parameters d, θ, η are stochastic. Because of our assumption that these preference parameters are common to the two countries, these are monetary-union-wide shocks.

All households in country 1 begin with the same amount of financial assets. Hence, they will have the same intertemporal budget constraints and will therefore choose the same sequences of consumption, real balances and efforts. The budget constraint for the representative agent in country 1 is

$$\frac{B_{1,t+1}}{1+r_t} + \frac{M_{1,t}}{P_t} + C_{1,t} = B_{1,t} + \frac{M_{1,t-1}}{P_t} + \int_0^n \frac{W_{1,t}(i)}{P_t} N_{1,t}(i) di + \int_0^n \frac{\Pi_{1,t}(i)}{P_t} di + T_{1,t} - \tau_{1,t}. \quad (5)$$

Here $B_{1,t+1}$ is the purchase of a riskless bond that pays one unit of aggregate consumption at time $t+1$. This bond is the only asset available for borrowing or lending between the two countries and r_t is the net real interest rate. $W_{1,t}(i)$ is the nominal wage of labor of type i in period t and $\Pi_{1,t}(i)$ are nominal profits of the country 1 firm producing good i. We assume that each household in country 1 owns an equal share of all the firms in the country, but no shares in the firms in country 2. $T_{1,t}$ represents transfers received from the household in country 1 at time t and $\tau_{1,t}$ is a lump-sum tax levied by the government of country 1 at time t on the citizens of that country.

Households face four decisions. First, how to allocate consumption across the differentiated goods. Taking prices as given, the optimal consumption of good i produced in country j is given by

$$C_{1,t}(i) = \left(\frac{P_{j,t}(i)}{P_t}\right)^{-\theta} C_{1,t}, \quad (6)$$

where $j = 1$ if good i is produced in country 1 and $j = 2$ otherwise. Second, the household must decide the optimal amount of riskless bonds to purchase, $B_{1,t+1}$. The first-order condition delivers the Euler equation

$$\frac{1}{C_{1,t}} = \beta(1+r_t) E_t \frac{1}{C_{1,t+1}}. \quad (7)$$

Third, the household must decide the optimal level of money balances to carry into next period, $M_{1,t}$. After making use of (7), the first-order condition for optimal money balances is given by

$$\frac{M_{1,t}}{P_t} = \chi C_{1,t} \frac{1}{E_t \frac{i_{t+1}}{1+i_{t+1}}}, \quad (8)$$

where i_{t+1} is the nominal interest rate defined as

$$(1+i_{t+1}) = (1+r_t) E_t \frac{P_{t+1}}{P_t}. \quad (9)$$

Finally, the household must decide the optimal quantity of each type of labor to supply, taking wages and prices as given. The related first-order condition is given by

$$N_{1,t}(i) = \left[\frac{W_{1,t}(i)}{P_t C_{1,t} d}\right]^{\frac{1}{\eta}}. \quad (10)$$

3.2 Policymakers

There is a common central bank that runs monetary policy for the monetary union; in addition, there are two governments that run fiscal policies, one in each country. The central bank is instrument-independent in the sense that it chooses monetary policy freely and it does not share the government budget constraints. We also assume that the central bank is conservative in the sense that it maximizes a utility that is more conservative than society's – this will be explained in detail in section 4. Let total money supply in the monetary union be $M_t \equiv M_{1,t} + M_{2,t}$. The budget constraint for the central bank is

$$nT_{1,t} + (1-n)T_{2,t} = \frac{M_t - M_{t-1}}{P_t}. \tag{11}$$

Hence, the central bank rebates seignorage back to households in the two countries.

In each country, fiscal policy consists of public spending financed with lump-sum taxes. We realistically assume that each government spends only on the goods produced domestically. The budget constraint for the government in country 1 is

$$\tau_{1,t} = G_{1,t}, \tag{12}$$

and similarly for country 2. Government 1's demand for good i has the following form

$$G_{1,t}(i) = \left(\frac{P_{1,t}(i)}{P_t}\right)^{-\theta} G_{1,t},\ \forall i \in [0,n], \tag{13}$$

and zero for all the goods produced in country 2. Similarly, government 2's demand for good i is

$$G_{2,t}(i) = \left(\frac{P_{1,t}(i)}{P_t}\right)^{-\theta} G_{1,t},\ \forall i \in (n,1], \tag{14}$$

and zero otherwise. The governments are benevolent and choose public spending so as to maximize the utility function of the representative individual. Optimal public spending in country 1 is given by

$$G_{1,t} = (\alpha C_{1,t})^{\rho}, \tag{15}$$

and similarly for country 2. Optimal public spending becomes negligible as α goes to zero.

3.3 Firms

The production function for the goods produced in country 1 makes only use of labor and is given by

$$Y_{1,t}(i) = A_{1,t}N_{1,t}(i), \tag{16}$$

where $A_{1,t}$ is an exogenous stochastic technological factor common to all firms in country 1, i.e. a supply-side aggregate shock. In country 2, the production function is

$$Y_{2,t}(i) = A_{2,t}N_{2,t}(i), \tag{17}$$

where $A_{2,t}$ is an exogenous stochastic technological factor common to all firms in country 2. We will consider alternative assumptions about the correlation between the technological shocks in the two countries.

Nomimal profits at time t for firm i in country 1 are given by

$$\Pi_{1,t}(i) = P_t(i)Y_{1,t}(i) - W_{1,t}(i)N_{1,t}(i). \tag{18}$$

The first term on the right hand side of (18) represents revenues from selling the good; the second term on the right hand side is the cost of producing, which is the nominal wage bill for employed labor. In (18) we have assumed that the firm takes the nominal wage as given. Firms maximize the present value of current and future profits

$$\sum_{s=t}^{\infty} E_t Q_{1,t,s} \Pi_{1,s}(i), \tag{19}$$

where $Q_{1,t,s}$ is the stochastic discount factor for country 1

$$Q_{1,t,s} = \frac{\beta^{s-t} u'(C_{1,s}) P_t}{u'(C_{1,t}) P_s} = \Pi_{j=t}^{s} Q_{1,j,j+1}. \tag{20}$$

From equation (6) and (11) we can obtain the demand for good i produced in country 1 is

$$Y_{1,t}(i)^d = \left[\frac{P_{1,t}(i)}{P_t}\right]^{-\theta} (C_t^w + G_{1,t}) \tag{21}$$

where $C_t^w \equiv nC_{1,t} + (1-n)C_{2,t}$ is world private consumption. If prices are flexible, firms choose prices every period to maximize the present value of current and future profits; because they can choose prices every period, this implies that firms choose prices so as to maximize current profits. The first-order condition is

$$P_{1,t}(i) = \frac{\theta}{\theta - 1} \frac{W_{1,t}(i)}{A_{1,t}}. \tag{22}$$

With flexible prices, it is optimal for the firm to set its price as a markup over the marginal cost. The markup $\theta/(\theta-1)$ falls as θ grows: the markup falls as the monopolistic power of the firm becomes smaller, i.e. as goods become better substitutes. Notice that the nominal marginal cost is country-specific because technological shocks are country-specific and because different fiscal policies result in different nominal wages.

3.4 Equilibrium

Aggregate output in country 1 is defined as

$$Y_{1,t} \equiv \int_0^n \frac{P_{1,t}(i)}{P_t} Y_{1,t}(i)\, di. \tag{23}$$

Bonds are in zero net supply and clearing on the bond market implies that

$$nB_{1,t} + (1-n)B_{2,t} = 0, \qquad \forall t. \tag{24}$$

Making use of (11), (12), (18), (23) and (24), the consolidated budget constraint becomes

$$C_t^w = Y_t^w - nG_{1,t} - (1-n)G_{2,t}, \tag{25}$$

where

$$Y_t^w = nY_{1,t} + (1-n)Y_{2,t}.$$

Total private consumption in the world economy is equal to world production minus world government spending.

3.5 Steady State

At the randomless steady state, all differentiated goods have identical prices and wages across labor types are identical in the same country. Hence

$$\frac{W_1}{P} = \frac{\theta-1}{\theta}A_1, \tag{26}$$

where variables without a time subscript indicate steady-state values. Labor is also equalized across different types and is given by

$$N_1 = \left[\frac{(\theta-1)A_1}{\theta d C_1}\right]^{1/\eta}. \tag{27}$$

If money supply is constant in the steady state, the price level is

$$P = \frac{M(1-\beta)}{\chi C^w}. \tag{28}$$

At the steady state

$$C_1 = \frac{P_1}{P}Y_1 - G_1 + rB_1 \tag{29}$$

where $r = (1-\beta)/\beta$ and B_1 is steady-state bond holding by country 1. We focus on an initial steady-state where $B_1 = B_2 = 0$, which implies that $P_1/P = P_2/P = 1$. Hence

$$Y_1 = A_1\left[\frac{(\theta-1)A_1}{\theta d(Y_1 - G_1)}\right]^{1/\eta}, \tag{30}$$

where government spending is given by

$$G_1 = (\alpha C_1)^{\rho}. \tag{31}$$

An increase in government spending raises output in the steady state. Because higher government spending requires higher taxes, households reduce consumption and substitute out of leisure into work, thereby raising production. Hence, government spending does not crowd out private spending completely. As α approaches zero, steady-state output becomes

$$Y_1 = A_1 \left[\frac{\theta - 1}{\theta d}\right]^{\frac{1}{1+\eta}}, \tag{32}$$

Steady state output is suboptimally low due to the monopolistic power of producers. As the degree of substitutability among goods becomes large, i.e. $\theta \to \infty$, output approaches its efficient level

$$Y_1^* = A_1 \left[\frac{1}{d}\right]^{\frac{1}{1+\eta}}. \tag{33}$$

The efficient level of output can also be achieved by an appropriate production subsidy that offsets the distortion due to market power;[2] here we abstract from such subsidy. The steady state for country 2 is completely symmetric but a function of A_2, G_2.

3.6 The Dynamics of Prices

We assume that, in every period, a fraction $\phi \in [0, 1]$ of firms in each country cannot change their prices while the remaining fraction $1 - \phi$ of firms can adjust their prices; the following period, all firms are free to choose their prices. In other words, suppose an unanticipated shock occurs at time t; a fraction ϕ of firms cannot change their prices at t and they will adjust production to meet demand; these firms, however, can freely choose their desired prices in period $t + 1$. A fraction $1 - \phi$ of firms can freely choose their prices at time t and at time $t + 1$ (as well as in any other period).

This adjustment mechanism is relatively simple because it guarantees full adjustment in one period; in fact, we are able to solve analytically the model (which we would not be able to do with staggered-price setting á la Calvo (1983). Our assumption about the dynamics of prices implies

$$P_t^{1-\theta} = \left[\phi P_{t-1}^{1-\theta} + (1 - \phi)\tilde{P}_t^{1-\theta}\right], \tag{34}$$

where

$$P_{t-1}^{1-\theta} = nP_{1,t-1}^{1-\theta} + (1 - n)P_{2,t-1}^{1-\theta}, \qquad \tilde{P}_t^{1-\theta} = n\tilde{P}_{1,t}^{1-\theta} + (1 - n)\tilde{P}_{2,t}^{1-\theta}.$$

In each country, all suppliers that set new prices at t face exactly the same decision problem; hence, the newly set price $\tilde{P}_{j,t}$ is the same for all of them and is therefore not a function of i but it is a function of j, the country to which suppliers belong.

A supplier in country 1 that sets a new price at t chooses it so as to maximize current profits; the price chosen by such firm is as in (22). Let a small letter indicate the percentage deviation from the steady state value of the corresponding capitalized variable, for example $a_t \equiv dA_t/A$. We have that

$$\tilde{p}_{1,t} - p_t = w_{1,t} - a_{1,t}, \tag{35}$$

[2] The appropriate production subsidy is $1/(\theta - 1)$.

and a similar expression holds for country 2. Log-linearizing (34) and using (35) and its counterpart for country 2, we obtain that

$$\pi_t = \lambda(w_t^w - a^w), \qquad \lambda \equiv \frac{1-\phi}{\phi}, \tag{36}$$

where $w_t^w \equiv nw_{1,t} + (1-n)w_{2,t}, a_t^w \equiv na_{1,t} + (1-n)a_{2,t}$ and $\pi_t \equiv p_t - p_{t-1}$.

Appendix A log-linearizes the model around the steady state; aggregate inflation can be written as a function of monetary and fiscal policies and current shocks:

$$\pi_t = m_t + \sum_{i=1}^{2} c_i g_{i,t} + \omega_t + \gamma\beta E_t \pi_{t+1}, \tag{37}$$

where $\pi_t \equiv p_t - p_{t-1}$. All parameters are described in appendix A. Aggregate inflation is a sum of the component m_t, which is the controlled part of monetary policy and it is an increasing function of money supply, and a further contribution arising from fiscal policies $g_{i,t}$. $c_i > 0$: an increase in government spending financed by lump-sum taxes raises inflation. The term ω_t captures the effect of lagged prices, technological and government spending shocks.

Output in country 1 is given by

$$y_{1,t} = \bar{y}_{1,t} + \sum_{i=1}^{2} a_{1,i} g_{i,t} + b_i(\pi_t - \beta\pi_{t+1}^e). \tag{38}$$

A similar equation holds for country 2. The explanation of the parameters in the output equation (38) is as follows: [1] $\bar{y}_{1,t}$ is percentage deviation of the natural rate of output in country 1 at t from its steady-state value. The natural rate of output is the level of production that arises in the country with steady-state monetary and fiscal policies; this is suboptimally low. [2] The scalars $a_{1,i}$ is the direct effect of fiscal policy of country i on the GDP of country 1. When governments spend only on the goods produced at home, a fiscal expansion raises home GDP but its effect on the other country's GDP is uncertain: $a_{i,i} > 0, a_{i,j} \gtreqless 0$ for $i \neq j$. [3] π_{t+1}^e is firms' rational expectation of π_{t+1} as of time t. [4] The last term on the right-hand side of equation (38) is the usual supply effect of an unexpected increase in inflation; thus $b_i > 0$. All these parameters and the derivation of (38) are spelled out in detail in Appendix A.

4 Preferences of Policymakers

The central bank chooses a policy variable m_t, which stands for the base money supply, and determines a component of the price level; thus higher m_t means a more expansionary monetary policy. The fiscal authority in each country in the monetary union chooses a policy variable $g_{i,t}$; a larger $g_{i,t}$ means higher government spending and therefore a more expansionary fiscal policy. These policies affect the GDP levels and aggregate inflation according to equations (38) and (37) above.

The fiscal authorities are assumed to be benevolent. In the non-cooperative scenario, each fiscal authority choose fiscal policy to maximize social welfare of the country's citizens, namely the utility of the representative individual in the country. We approximate this utility by a second-order Taylor series expansion to the level of expected utility of the representative consumer in the country in the rational expectations equilibrium associated with given monetary and fiscal policies – see appendix B. We are interested in the welfare effects of output and inflation stabilization; for this reason, we consider the second-order approximation to the utility of the representative household as $\alpha \to 0$ and the direct welfare effect of public goods becomes very small.

The fiscal authority of country i maximizes

$$V_{F,i,t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} U_{F,i,s}, \qquad i = 1, 2, \tag{39}$$

where the period utility $U_{i,t}$ is approximated by

$$U_{i,t} = -L_{F,i,t},$$

where $L_{F,i,t}$ is the quadratic loss function

$$L_{F,i,t} = \frac{1}{2}\left[(\pi_t - \pi_{F,i})^2 + \theta_{F,i}(y_{i,t} - y_{F,i})^2 + 2\delta_i g_{i,t}\right], \qquad i = 1, 2. \tag{40}$$

$\pi_{F,i} = 0$ and it is socially optimal to minimize price level dispersion. The GDP that minimizes social losses in country i is $y_{F,i}$, which is the GDP that would arise in an economy with flexible prices and without monopolistic power by the firms; hence, $y_{F,i} \geq \overline{y}_{i,t}$ and extra output is desirable. Fiscal policy can raise output above its natural rate, but it creates social losses $\delta_i > 0$ because government spending is financed by lump-sum taxes that reduce private consumption. $\theta_{F,i} > 0$ parameterizes the social preference for the output versus the inflation goals. All parameters are spelled out in Appendix B.

If the fiscal authorities cooperate, they choose fiscal policies so as to maximize social welfare in the monetary union, which is given by

$$V^c_{F,t} = -E_t \sum_{s=t}^{\infty} \beta^{s-t} L_{F,s}, \tag{41}$$

where

$$L_{F,t} = \gamma L_{F,1,t} + (1-\gamma) L_{F,2,t}$$

and the parameter γ is the weight of country 2 in social welfare. For simplicity, we are going to assume that $\gamma = 0.5$ and both countries have equal weight in the cooperative social welfare function.

Monetary policy is chosen by a monetary authority that is conservative in a way that encompasses both Rogoff's and Svensson's definition, and minimizes a loss function

$$V_{M,t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} L_{M,s}, \tag{42}$$

where

$$L_{M,t} = \frac{1}{2}\left[\sum_{i=1}^{2}\theta_{M,i}(y_{i,t} - y_{M,i})^2 + (\pi_t - \pi_M)^2\right], \tag{43}$$

where $y_{M,i}$ is the central bank's output target for country i, $\theta_{M,i}$ the preference for the output-in-country-i versus the prive-level goals for the monetary authority and π_M the inflation target. The central bank is more conservative than society in the sense that $\theta_{M,i} \leq \theta_{F,i}, y_{M,i} \leq y_{F,i}$ for all i and/or $\pi_M \leq 0$.

The natural rate of output $\overline{y}_{i,t}$, the scalar parameters $a_{i,j}$ summarizing the fiscal policy effects on GDPs, the scalar parameters b_i for the supply effect of surprise inflation, the scalar parameters c_i of the effect of fiscal policy on inflation level, the scalar parameter δ_i for the deadweight losses of fiscal policies, the scalar parameters $\theta_{F,i}$ for the social preferences, the efficient levels of output $y_{F,i}$, the central bank's output targets $y_{M,i}$ and inflation targets $\pi_{M,i}$ and the scalar parameters $\theta_{M,i}$ for the central bank's preferences, are all stochastic shocks because they depend on the five stochastic preference and technology parameters of our structural model $(d, \theta, \eta, A_1, A_2)$. We denote the whole vector of these shocks by z_t. The policy variables m_t and $g_{1,t}, g_{2,t}$ are implemented after the shocks are observed, and therefore are written as functions $m(z_t)$ and $g_1(z_t), g_2(z_t)$ (although the functional form may be fixed before the shocks are observed in regimes where policies are precommitted). The resulting outcomes of GDPs and inflation are then also realization-specific or functions $y_1(z_t), y_2(z_t)$ and $\pi(z_t)$.

The condition of rational expectations is

$$\pi_t^e = E_{z_t}[\pi(z_t)] \equiv \int \pi(z_t), \tag{44}$$

where the integral is taken over the distribution of z, and is five-dimensional since all the components of z are functions of five underlying structural parameters $d, \eta, \theta, A_1, A_2$. In words, π_t^e is the firms' rational expectation of π_t as of time $t - s, s < t$.

4.1 Timing of events

In absence of commitment, we consider the case where monetary and fiscal policies are chosen simultaneously (Nash); at the same time, fiscal policies may be cooperative or not. If monetary policy is precommitted, then it has leadership with respect to setting the rule, and fiscal policies are followers in each state of the world (realization of the shocks); if fiscal policies are precommitted, they have leadership with respect to setting the rule, and monetary policy is the follower in each state of the world. Once again, fiscal policies can be choosen cooperatively or not whether they can be precommitted or discretionary.

Hence, the timing of events is as follows:

1. We consider two possible scenarios:

    (a) If there is commitment, the three policies are chosen in a coordinated manner maxizing social welfare in the union.

(b) If the policy regime is one of discretion, nothing happens at this step.

2. The private sector forms expectations π_t^e. When firms set their prices at time $s < t$, they rationally forecast future inflation.

3. The stochastic shock vector z_t is realized.

4. If the policy regime is one of discretion, the central bank chooses m_t and the fiscal authorities choose fiscal policies, cooperatively or not. All these policies are chosen simultaneously. If the policy regime is one of commitment, the central bank simply implements the monetary rule m_t that was chosen at step 1 and the fiscal authorities simply implement the fiscal rules chosen at step 1.

5 Joint Commitment

First we study the equilibrium with joint commitment of monetary and fiscal policies. This is done when all authorities can precommit so as to maximize social welfare in the union. This delivers the socially optimal and feasible allocation that we refer to as second best; hence, it is the natural benchmark against which to compare all other equilibria.

Let both the monetary and fiscal authorities minimize the social loss function (41) and recognize the rational expectations constraint. Since this is a separable problem, at step 1 the whole functions $m(\cdot), g_1(\cdot), g_2(\cdot)$ are chosento minimize the expected period loss function

$$\int L_{F,t}(z_t). \tag{45}$$

Substituting π_t^e into the objective complicates the algebra, because it then involves one integration inside another. We avoid this by regarding the authorities as if they had another choice variable, namely π_t^e, but their choice was subject to the constraint (44). After substituting the loss functions and using $\gamma = 1/2$, the common Lagrangean for this problem is:

$$\mathcal{L}_{\mathcal{F},\sqcup}^{\mathcal{JC}} = \int \left\{ \frac{1}{2} \left[L_{F,1,t} + L_{F,2,t} \right] + \lambda_t \pi(z_t) \right\} - \lambda_t \pi_t^e, \tag{46}$$

where λ_t is the Lagrangean multiplier. The first-order condition with respect to the function $g_1(z_t), g_2(z_t)$ and m_t are given by

$$2(\pi(z_t) + \lambda_t) + \frac{\delta_1}{c_1} + \sum_{i=1} 2\theta_{F,i}(y_i(z_t) - y_{F,i}) \left(\frac{a_{i,1}}{c_1} + b_i \right) = 0. \tag{47}$$

$$2(\pi(z_t) + \lambda_t) + \frac{\delta_2}{c_2} + \sum_{i=1} 2\theta_{F,i}(y_i(z_t) - y_{F,i}) \left(\frac{a_{i,2}}{c_2} + b_i \right) = 0. \tag{48}$$

$$2(\pi(z_t) + \lambda_t) + \sum_{i=1} 2\theta_{F,i}(y_i(z_t) - y_{F,i}) b_i = 0. \tag{49}$$

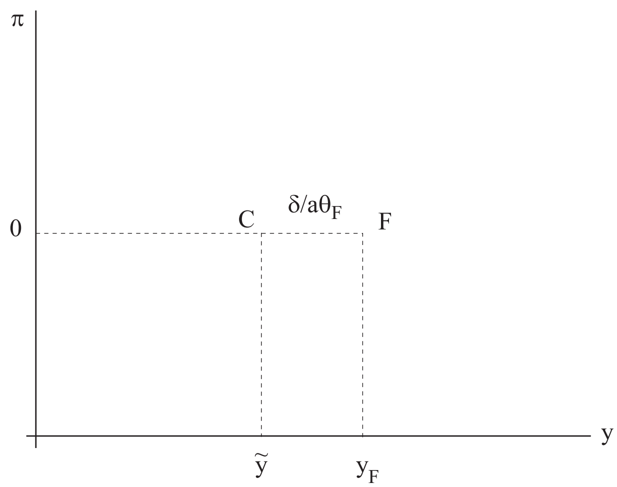

Figure 1: Full Commitment

The first-order condition with respect to π_t^e is

$$-\int(\lambda_t+\pi_t)\frac{\psi\beta}{(1+\psi)(1-\beta)}+\int(\lambda_{t-1}+\pi_{t-1})\frac{1-\beta+\psi}{(1+\psi)(1-\beta)}=0.$$

The solution is

$$\lambda_t=\int\sum_{i=1}^{2}\frac{b_i\theta_{F,i}}{2}(y_{F,i}-y_{i,t}),\qquad \pi_t^e=0,\qquad \pi(z_t)=-2\lambda_t+\sum_{i=1}^{2}b_i\theta_{F,i}(y_{F,i}-y_{i,t}) \tag{50}$$

$$y_1(z_t)=\tilde{y}_{1,t}=y_{F,1}-\frac{\delta_1 a_{2,2}-\delta_2 a_{1,2}}{\|A\|\,\theta_{F,1}},\qquad y_2(z_t)=\tilde{y}_{2,t}=y_{F,2}-\frac{\delta_2 a_{1,1}-\delta_1 a_{2,1}}{\|A\|\,\theta_{F,1}}$$

where

$$\|A\|\equiv a_{1,1}a_{2,2}-a_{1,2}a_{2,1}$$

If $\delta_1=\delta_2=0$:

$$y_{1,t}=y_{F,1}\quad y_{2,t}=y_{F,2},\quad \pi_t=0$$

The fully optimal, nonlinear rules for monetary and fiscal policies deliver zero inflation on average. The equilibrium with joint commitment is shown in Figure 1.

The lagrangean multiplier of the rational expectations constraint is positive if output is below its efficient level and negative otherwise. The output gap in country i is higher the larger the welfare cost of public spending δ_i, the less important is output in social preferences $\theta_{F,i}$, and the smaller the direct impact of fiscal policy on own output $a_{i,i}$; however, the output gap is smaller increases as the effect of the other's country fiscal policy on own GDP rises.

The rational expectations constraint is binding when all the m, g are chosen ex-ante optimally. More precisely, λ_t is the average inflation reduction achieved by joint commitment.[3]

[3] In fact, λ_t measures the inflation bias that arises with discretionary monetary and fiscal policies. This result is shown in section 6.

6 Discretionary Policies: Nash equilibrium

6.1 Fiscal Non-cooperation

Now we consider the case where the fiscal authorities do not cooperate when they choose fiscal policies. After the realization of the stochastic shock vector z_t, the fiscal authority of country 1 chooses $g_{1,t}$ so as to minimize $L_{F,1,t}$, taking $g_{2,t}, m_t$ as given; similarly, the fiscal authority of country 2 chooses $g_{2,t}$ so as to minimize $L_{F,2,t}$, taking $g_{1,t}, m_t$ as given; the central bank chooses m_t so as to minimize $L_{M,t}$ taking $g_{1,t}, g_{2,t}$ as given. The authorities act simultaneously; when they choose their policies, private sector's expectations π_t^e are fixed.

The first-order condition for fiscal policy in country 1 can be found by differentiating (39) with respect to $g_{1,t}$, recognizing the dependence of π_t on it; this gives

$$\theta_{F,1}(y_1 - y_{F,1})\left(\frac{a_{1,1}}{c_1} + b_1\right) + \pi + \frac{\delta_1}{c_1} = 0. \tag{51}$$

The government of country 1 does not take into account the effect of its own policy on the output of country 2. The first-order condition for fiscal policy in country 2 is given by

$$\theta_{F,i}(y_2 - y_{F,2})\left(\frac{a_{2,2}}{c_2} + b_2\right) + \pi + \frac{\delta_2}{c_2} = 0. \tag{52}$$

Equation (51) defines the reaction function for the government of country 1 in the $(y_{1,t}, \pi_t)$ space and equation(52) defines the reaction function for the government of country 2 in the $(y_{2,t}, \pi_t)$ space.

The first-order condition for monetary policy is obtained by differentiating (43) with respect to m_t, which gives

$$\pi_t = \pi_M - \sum_{i=1}^{2} \theta_{M,i} b_i \, (y_{i,t} - y_{M,i}). \tag{53}$$

This defines the reaction function for the monetary authority (MRF) in the $(y_{1,t}, y_{2,t}, \pi_t)$ space.

Under non-cooperation the fiscal authority only considers the effects of its own fiscal policy on its own welfare. Hence, the fiscal reaction function of country 1 depends only on country 1's output and on inflation; similarly, the fiscal reaction function of country 2 depends only on country 2's output and inflation. The monetary reaction function, on the other hand, generally depends on both countries' GDP and on inflation.

Figure 2 plots the reaction function of country 1 (FRF_1) and the MRF in the $y_{1,t}, \pi$ space; in drawing the MRF, we take $y_{2,t} - y_{F,2}$ as given. The MRF is the solid line through point M, the bliss point for the conservative monetary authority; because $b_1 > 0$, MRF is negatively sloped. If y_2 increases, the MRF shifts vertically downward thereby lowering inflation and raising output in country 1, ceteris paribus.

FRF_1 is the solid line below point F_1, the bliss point for the fiscal authority in country 1. With $\delta_1 > 0$, FRF_1 does not pass through point F_1 because it is suboptimal to raise

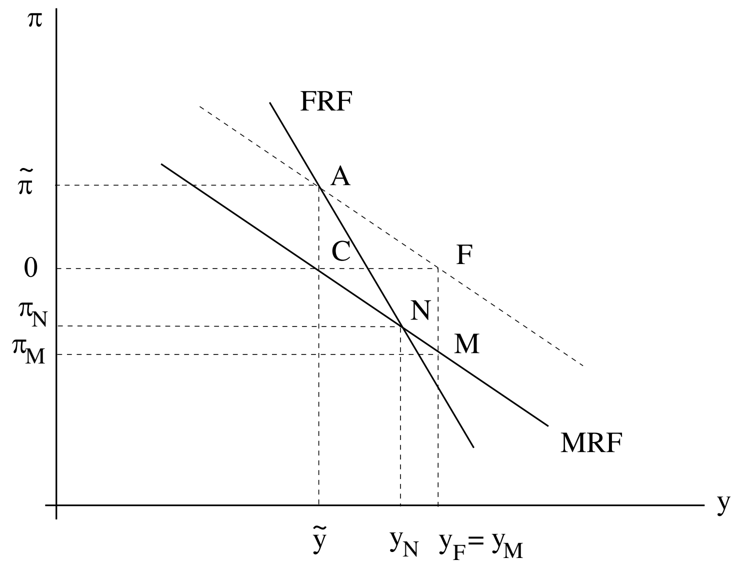

Figure 2: Nash Non-cooperative Equilibrium

public spending so as to raise output to $y_{F,i}$. The second best allocation is point C, where $y_{1,t} = \tilde{y}_1 < y_{F,1}$ and $\pi_t = 0$. It is easy to check that FRF_1 is steeper than MRF. Graphically, the Nash equilibrium occurs at the intersection of the two reaction functions MRF and FRF_1, and it is labeled N.[4]

The Nash equilibrium with non-cooperation has the following characteristics: output is higher and inflation is lower than socially optimal. These were also features of the Nash equilibrium with cooperation. As with cooperation, the Nash equilibrium may be extreme. In fact, figure 2 depicts a Nash equilibrium where inflation is in between the preferences of monetary and fiscal authorities. But the equilibrium may also occur on the right of point M, thereby delivering output and inflation that are beyond what both the central bank and the fiscal authority desire. This is shown in Figure 3.

If the Nash equilibrium without cooperation is such that $y_{2,t} < y_{F,2}$, the FRF_1 moves outward with respect to the position it would have with cooperation; in this case, the Nash equilibrium with non-cooperation has lower inflation and higher output than the Nash equilibrium with cooperation. On the other hand, if $y_{2,t} > y_{F,2}$, the Nash equilibrium without cooperation has lower output and higher inflation than the one with cooperation. This implies that the Nash equilibria with and without cooperation cannot be ranked from a social welfare point of view, as non-cooperation may result in less extreme economic outcomes. Intuitively, time inconsistency makes fiscal policies too expansionary, thereby raising output above optimality; the common, conservative central bank runs a contractionary monetary policy that brings inflation below optimality. Fiscal cooperation worsens time inconsistency as each fiscal authority is now tempted to expand its fiscal policy even further.

[4]In drawing Figures 2 we have assumed that the central bank is appropriately conservative and $\theta_{M,i} = \theta_{F,i}, y_{M,i} = y_{F,i}$ and $\pi_M = -\lambda$; our results, however, do not depend on these assumptions.

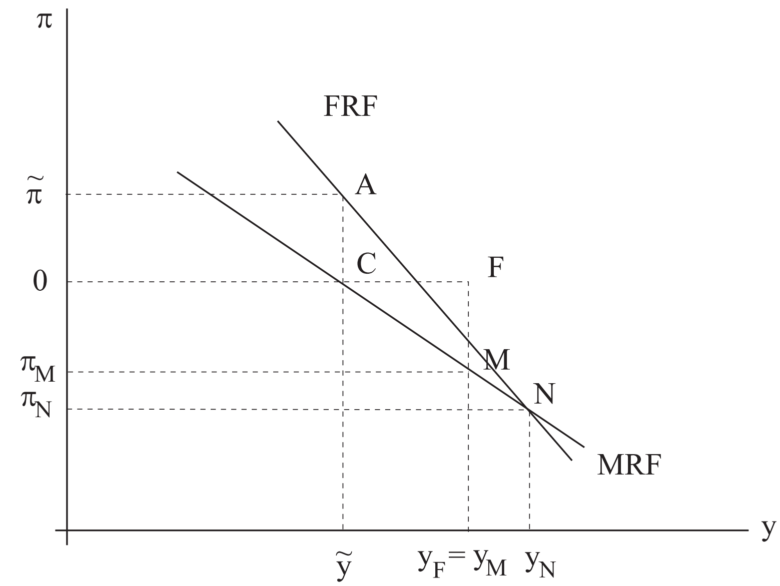

Figure 3: Extreme Nash Equilibrium with Fiscal Non-cooperation

The intuition behind our result is a reminiscent of the second-best theory: when economic outcomes are not first best, adding another distortion, i.e. non-cooperation, may actually improve social welfare. Kehoe (1989) and Rogoff (1985b) have similar results.

6.2 Fiscal Cooperation

In this policy regime, after each realization of the stochastic shock vector z_t, the fiscal authorities choose $g_{i,t}$, taking m_t as given, so as to minimize the union-wide loss function $L_{F,t}$; the monetary authority chooses m_t, taking $g_{i,t}$ as given, so as to minimize its loss function $L_{M,t}$. The fiscal authorites act cooperatively while the monetary and fiscal authorities authorities act non-cooperatively and simultaneously; however, when their choices are made, the private sector's expectations π_t^e are fixed. After completing the analysis of the policy equilibrium and economic outcome for an arbitrarily given state z_t, we can find π_t^e from the rational expectations condition (44).

The first-order conditions for fiscal policies are obtained by differentiating (41) with respect to $g_{1,t}$ and $g_{2,t}$, recognizing the dependence of π_t on them; this gives

$$\pi_t = \frac{1}{2}\left[-\sum_{i=1}^{2}\theta_{F,i}(y_i - y_{F,i})\left(\frac{a_{i,1}}{c_1} + b_i\right) - \frac{\delta_1}{c_1}\right], \tag{54}$$

$$\pi_t = \frac{1}{2}\left[-\sum_{i=1}^{2}\theta_{F,i}(y_i - y_{F,i})\left(\frac{a_{i,2}}{c_2} + b_i\right) - \frac{\delta_2}{c_2}\right]. \tag{55}$$

These define the reaction functions of the fiscal authorities ($FRFs$) in the $(y_{1,t}, y_{2,t}, \pi_t)$ space. One can obtain the reaction functions in terms of the policy variables $(m_t, g_{1,t}, g_{2,t})$ by substituting $y_{1,t}, y_{2,t}$ and π_t into (54) using (38) and (37).

Parameter	Value	Mean
β	0.98	
ϕ	0.5	
n	0.5	
d		1
θ		11
η		1
A_1		1
A_2		1

Table 1: Parameter Values

Because the fiscal authorities cooperate, they recognize the impact of their own policies on common inflation, on own output and on the other country's output; at the same time, each fiscal authority internalizes the welfare cost of a fiscal expansion (δ_i/c_i).

The first-order condition for monetary policy is obtained by differentiating (43) with respect to m_t, which gives (53).

The Nash equilibrium outcomes $y_{1,t}^C, y_{2,t}^C$ and π_t^C are found by solving (54), (55), (53) and (44) together and the solution is given in Appendix **??**. Making use of (38) and (37) and (44), we can find the policy variables m_t and $g_{1,t}, g_{2,t}$ that emerge in the Nash equilibrium. This is also done in Appendix **??**.

Because $a_{2,1}/c_1 + b_2 < 0$, government spending in country 1 raises if $y_{2,t} > y_{F,2}$ with respect to the case without cooperation; moreover, the social cost of public spending is now half what it was in the case without cooperation, which shifts the FRF toward the right. Whether the Nash equilibrium with fiscal cooperation is more extreme than that without cooperation depends on these effects.

6.3 Simulation and Welfare Comparison

This section compares social welfare under the discretionary regimes of Nash with and without fiscal cooperation from an ex ante point of view. We wish to shed some light on whether fiscal cooperation is ex-ante preferable to fiscal non-cooperation independently of the realization of the shocks.

We run a Monte Carlo simulation using the parameter values derived within the structural model. Changes in the parameters of the structural model necessarily imply changes in the elements of z_t, which are jointly distributed. For the steady state, we calibrate our model using the parameter values typically used in the literature; these are summarized below; also, see Gali (2001). We then assume preference shocks that deliver output fluctuations within the range of +/- 6% of steady-state output, which are roughly consistent with the fluctuations of U.S. output around a quadratic trend.

The parameter values are summarized in Table 1.

Table 2 shows the average difference in the social loss function between joint commitment

Variable	Mean	Variance	$\% \leq 0$
$L_1^{jc} - L_1^n$	-0.0044	- 5.1249	100
$L_1^{jc} - L_1^c$	-408.0622	-4.28	100
$L_1^n - L_1^c$	-408.0666	-4.26	100

Table 2: Welfare Comparison

and Nash without cooperation, joint commitment and Nash with cooperation and Nash without and with cooperation. Joint commitment is always best; then Nash without fiscal coordination follows. Nash with fiscal coordination is a distant third in all cases (we run 1000 draws). In fact, the Nash equilibrium with fiscal coordination is always extreme while the Nash equilibrium without cooperation is not.

6.4 Interpretation

Our simulations showed that the Nash equilibrium with cooperation has the following characteristics: output is higher than optimal in both countries and inflation is lower than optimal. Formally, $y_{1,t}^C > y_{F,1}, y_{2,t}^C > y_{F,2}, \pi_t^C < 0$. Hence, in the Nash cooperative equilibrium, inflation is lower than optimal.

A Nash equilibrium with high output and low inflation may sound good; however, the Nash equilibrium is *suboptimal*. In fact, household work too much in this equilibrium and they would happily substitute labor for leisure; at the same time, lower-than-optimal inflation implies price dispersion that distorts consumption choices.

Why does the Nash equilibrium fail to achieve the second best? It is the time-inconsistency of policies *and* the conflict of objectives between the policymakers. With some prices preset, fiscal policies are more expansionary than under joint commitment because the fiscal authorities believe this will boost demand and therefore output. In a rational expectations' equilibrium, the governments' incentive to raise spending are perfectly anticipated by rational firms and they result in higher output *and* higher inflation.

Notice that suboptimality of the Nash equilibrium arises even if the monetary authority is appropriately conservative in the sense that its monetary policy is consistent with the second best;[5] higher-than-optimal public spending raises output and inflation, which makes monetary policy more contractionary than under joint commitment.

Dixit and Lambertini (2003b) study monetary-fiscal interactions in a closed economy when fiscal policy consist of a production subsidy that generates deadweight losses. They find that the Nash equilibrium has output lower and prices higher than optimal and than

[5]Given the output goal $y_{M,1}$ and the weight on it $\theta_{M,1}$, the central bank is appropriately conservative (so that its policy is consistent with the second best) when

$$\pi_M = \pi_M^{AC} = \int \sum_{i=1}^{2} \theta_{M,i} b_i (\tilde{y}_{1,t} - y_{M,i}). \tag{56}$$

what either authority wants. Hence, their Nash equilibrium is suboptimal and extreme, but in a different way than here. Their Nash equilibrium lies above and to the left of point C in Figure 2, while the Nash equilibrium here lies below and to the right of point C. In Dixit and Lambertini, time inconsistency makes fiscal policy tighter than optimal so that production subsidies are too low in the Nash equilibrium; since output is lower than optimal, monetary policy is more expansionary than optimal, thereby raising prices above their optimal level. Fiscal policy works differently here: time inconsistency makes fiscal policy more expansionary than optimal, thereby raising output and making monetary policy tighter than optimal. As a result, the Nash equilibrium has higher output and lower inflation than at the second best.

7 Concluding Comments

We would like to conclude with some implications of our results for the design of institutions in a monetary union such as the EMU and suggestions for future research.

Central bank independence, given to the ECB by the Maastricht Treaty, implies that different authorities will choose monetary and fiscal policies in a non-cooperative manner. In this setting, making the central bank extra conservative (in the sense of low ideal output and inflation) is likely to make things worse. The non-cooperative interaction between the central bank and the fiscal authorities leads to a race between expansionary fiscal policy that aims to raise output and contractionary monetary policy that aims to reduce inflation. The resulting Nash equilibrium is characterized by both inflation and output that are more extreme than the ideal levels of all authorities in the monetary union. More importantly, the Nash equilibrium is suboptimal. This result occurs independently of fiscal cooperation; in fact, fiscal cooperation may make things worse.

How to avoid such extreme outcomes? If the authorities' preferences can be chosen in advance and can be made to coincide, the ideal goals for inflation and output can be attained. But if the policy preferences are fixed and in disagreement, then the outcomes can only be influenced by how institutions are designed.

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Appendix

A Log Linearization around the Steady State

This section log linearizes around the steady state and solves for inflation and output in each country. Log-linearization of (25) and (8) gives

$$y_t^w = c_t^w + g_t^w \tag{A.1}$$

and

$$c_t^w = \mu_t - p_t + \beta E_t \hat{i}_{t+1} \tag{A.2}$$

where $\hat{i}_{t+1} \equiv di_{t+1}/i$, where $i = r$ because we focus on a steady state with constant money supply, and μ_t is the deviation in money supply.

Log-linearizing the Euler equation (7) we find

$$E_t c_{1,t+1} - c_{1,t} = (1-\beta)\hat{r}_t, \tag{A.3}$$

and log-linearizing the Fisher relationship we obtain

$$E_t\hat{\hat{\imath}}_{t+1} = E_t\hat{r}_t + \frac{1}{1-\beta}E_t\pi_{t+1}, \tag{A.4}$$

where $\hat{r}_t \equiv dr_{t+1}/r$. Combining (A.1), (A.2), (A.4) and (A.3) we obtain

$$y_t^w = g_t^w + (1-\beta)(\mu_t - p_t) + \beta E_t\pi_{t+1}. \tag{A.5}$$

From (16) and its country 2's counterpart and log-linearizing we get

$$y_t^w = a_t^w + n_t^w. \tag{A.6}$$

Log-linearizing the world average of the first-order conditions with respect to labor one obtains

$$y_t^w = \frac{1}{1+\eta}[w_t^w - p_t + g_t^w + \eta a_t^w]. \tag{A.7}$$

We have two expressions for y_t^w, (A.5) and (A.7); we can equalize them to obtain an expression for $w_t^w - p_t$ to substitute into (36) to find inflation as a function of monetary and fiscal policies, the exogenous shocks and future inflation. More precisely

$$\pi_t = m_t + \sum_{i=1}^{2} c_i g_{i,t} + \omega_t + \gamma\beta\pi_{t+1}^e \tag{A.8}$$

where

$$\psi \equiv \lambda(1+\eta)(1-\beta), \quad m_t = \frac{\psi}{1+\psi}\mu_t, \quad c_1 = n\frac{\lambda\eta}{1+\psi}, \quad c_1 = (1-n)\frac{\lambda\eta}{1+\psi},$$

$$\gamma \equiv \frac{1-\beta+\psi}{1+\psi}, \quad \omega_t = -\frac{\psi}{1+\psi}p_{t-1} - \frac{\lambda}{1+\eta}1+\psi a_t^w.$$

Output in country 1 can be found by $y_{1,t} = y_t^w + (1-n)(y_{1,t} - y_{2,t})$, where

$$y_{1,t} - y_{2,t} = \frac{\theta}{\theta-1}\left[g_{1,t} - g_{2,t} + \frac{b_{1,t+1}}{1-n} + c_1 - c_2\right], \tag{A.9}$$

where $c_1 - c_2$ is the difference in steady-state consumption changes

$$c_1 - c_2 = \frac{\theta-1}{\theta(1+\eta)-1}\left[(1+\eta)(a_1-a_2) + \frac{rb_1\theta\eta}{(1-n)(\theta-1)}\right], \tag{A.10}$$

where b_1 is the change in country1's net asset position in the new steady state.

B Social Welfare Function

We follow Woodford (2003) and consider a second-order Taylor series approximation to the objective

$$U_t = u(C_t;\epsilon_t) - \int_0^1 v(N_t(i);\epsilon_t)di + \alpha x(G_t;\epsilon_t) \tag{B.11}$$

with

$$u(C_t;\epsilon_t) = \log C_t, \quad v(N_t(i);\epsilon_t) = \left(\frac{d}{1+\eta}\right) N_t(i)^{1+\eta}, \quad x(G_t;\epsilon_t) = \frac{1}{1-1/\rho} G_t^{1-1/\rho}.$$

The approximation is made around the steady-state level of output Y for each good and the mean values for the exogenous shocks. Here we derive the welfare criterion that applies to a limiting cashless economy and therefore we abstract from the welfare consequences of monetary frictions.

We will proceed briefly; for details, see Woodford (2003). Let $\epsilon_t = (d, \eta, \theta, A_t)$ denote the complete vector of preference and technological shocks that we normalize so that $E(\epsilon_t) = 0$ and let a $^-$ denote steady-state value and, for simplicity, we drop time subscripts; a second-order expansion of the first and last term on the right-hand side of (B.11) is given by

$$\bar{u}+u_C\tilde{C}_t+u_\epsilon\epsilon_t+\frac{1}{2}u_{CC}\tilde{C}_t^2+\frac{1}{2}\epsilon_t^{'}u_{\epsilon\epsilon}\epsilon_t+u_{C\epsilon}\tilde{C}_t\epsilon_t+\alpha\left[\bar{x}+x_G\tilde{G}_t+x_\epsilon\epsilon_t+\frac{1}{2}u_{GG}\tilde{G}_t^2+\frac{1}{2}\epsilon_t^{'}u_{\epsilon\epsilon}\epsilon_t+u_{G\epsilon}\tilde{G}_t\epsilon_t\right],$$

where $\tilde{C}_t \equiv C_t - \bar{C}_t$ and $\tilde{G}_t \equiv G_t - \bar{G}_t$. At the steady state, $\bar{C} = \bar{Y} - \bar{G}$. We assume that $\tilde{G}$ is small enough, specifically of order $\mathcal{O}(||\epsilon||^2)$. After using Taylor expansion

$$\frac{Y_t}{\bar{Y}} = 1 + \hat{Y}_t + \frac{\hat{Y}_t^2}{2},$$

where $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$ (and similarly for other variables) and neglecting terms that are of order $\mathcal{O}(||\epsilon||^3)$ or higher) order, we obtain

$$u(C_t;\epsilon_t)+\alpha x(G_t;\epsilon_t) = \bar{Y}u_C\left(\hat{Y}_t+\frac{1}{2}\hat{Y}_t^2\right)-u_C\tilde{G}_t+\frac{1}{2}u_{CC}\bar{Y}^2\hat{Y}_t^2+u_{C\epsilon}\epsilon_t\bar{Y}\hat{Y}_t+\alpha x_G\tilde{G}_t+\alpha u_{G\epsilon}\epsilon_t\tilde{G}_t. \tag{B.12}$$

Let

$$s_t \equiv -\frac{u_{C\epsilon}\epsilon_t}{u_{CC}\bar{Y}}, \qquad \frac{1}{\sigma} \equiv -\frac{\bar{Y}u_{CC}}{u_C},$$

where s_t is of order $\mathcal{O}(||\epsilon||)$. Taking the limit as $\alpha \to 0$, (B.12) simplifies to

$$u(C_t;\epsilon_t) + \alpha x(G_t;\epsilon_t) = \bar{Y}u_C\left[\hat{Y}_t\left(1+\frac{s_t}{\sigma}\right)+\frac{1}{2}\hat{Y}_t^2\left(1-\frac{1}{\sigma}\right)-\frac{G_t}{\bar{Y}}\right]. \tag{B.13}$$

A second-order Taylor expansion of each $v(N_t(i);\epsilon)$, using the fact that $N_t(i) = Y_t(i)/A_t$, gives

$$\bar{Y}(i)v_Y\left\{\hat{Y}_t(i)\left(1+\frac{v_{Y\epsilon}\epsilon}{v_Y}\right)+\frac{\hat{Y}_t(i)^2}{2}\left(1+\frac{v_{YY}\bar{Y}}{v_Y}\right)\right\}. \tag{B.14}$$

Let

$$q_t \equiv -\frac{v_{Y\epsilon}\epsilon_t}{v_{YY}\bar{Y}},$$

where q_t is of order $\mathcal{O}(||\epsilon||)$. Since $v_{YY}\bar{Y}/v_Y = \eta$, we have that

$$\int_0^1 v(N_t(i);\epsilon_t)di = \bar{Y}v_Y\left[E\hat{Y}_t(i) + \frac{1+\eta}{2}(E\hat{Y}_t(i)^2 + var\hat{Y}_t(i)) - \eta q_t E\hat{Y}_t(i)\right]. \tag{B.15}$$

Using the Taylor series approximation

$$\hat{Y} = E\hat{Y}_t(i) + \frac{1}{2}\frac{\theta-1}{\theta}var\hat{Y}_t(i),$$

with

$$var\hat{Y}_t(i) = \theta^2 var\log P_t(i) = \theta^2\frac{\phi}{1-\phi}\pi_t^2,$$

where $\pi_t \equiv p_t - p_{t-1}$ and substituting these expressions in (B.14) also using

$$\frac{v_Y}{u_C} = \frac{\theta-1}{\theta},$$

which is the monopolistic distortions that we assume to be of order $\mathcal{O}(||\epsilon||)$, we obtain

$$\int_0^1 v(N_t(i);\epsilon_t)di = \bar{Y}u_C\left\{\left(1-\eta q_t - \frac{\theta-1}{\theta}\right)\hat{Y}_t + \frac{1+\eta}{2}\hat{Y}_t^2 + \frac{1}{2}\theta^2\frac{\phi}{1-\phi}\left(\eta+\frac{\theta-1}{\theta}\right)\pi_t^2\right\}. \tag{B.16}$$

Next, we subtract (B.16) from (B.12) we obtain that U_t is approximated by

$$-\frac{\bar{Y}u_C}{2}\left\{\hat{Y}_t^2\left(\eta+\frac{1}{\sigma}\right) - 2\hat{Y}_t\left(\eta q_t + \frac{s_t}{\sigma} + \frac{\theta-1}{\theta}\right) + 2\frac{G_t}{\bar{Y}} + \theta^2\frac{\phi}{1-\phi}\left(\eta+\frac{\theta-1}{\theta}\right)\pi_t^2\right\}.$$

Notice that $\sigma = 1$ with $u(C) = \log C$. The output terms above (together with a constant) come from the term $[\hat{Y}_t - \hat{Y}_t^n - \log(Y_t^*/\bar{Y})]^2$, where $\hat{Y}_t^n = \log(Y_t^n - \bar{Y})$, where Y_t^n is the equilibrium level of output at t under complete price flexibility. Let y_t be gap between current output and output under complete flexibility and let y_F be the gap between steady-state and efficient output. We can write

$$U_t = -\Omega L_{F,t},$$

where

$$L_{F,t} = \frac{1}{2}\left[\pi_t^2 + \theta_F(y_t - y_{F,t})^2 + 2\delta g_t\right], \tag{B.17}$$

where

$$\Omega = \frac{1-\phi}{\phi\theta[\theta(1+\eta)-1]} > 0,\ \ \theta_F = \frac{(1+\eta)(1-\phi)}{\theta\phi[\theta(1+\eta)-1]} > 0,$$

$$\delta = +\frac{(1-\phi)}{\phi\theta[\theta(1+\eta)-1]} > 0,\ \ y_F = \log\frac{Y^*}{\bar{Y}} = \log A_{1,t} + \frac{1}{1+\eta}\log\frac{\theta}{\theta-1}.$$

Social welfare is lower: a) the larger the gap between actual and the efficient level of output; b) the higher price dispersion that materializes with changes in the price level; c) the larger public spending.

Finally, we briefly discuss the case where $\alpha > 0$. In this case, government spending is a public good that raises social welfare directly. A benevolent fiscal authority chooses government spending according to the first-order condition (15); steady-state output is

$$\bar{Y} = \left[\frac{(1+\alpha)(\theta-1)}{d\theta}\right]^{\frac{1}{1+\eta}}.$$

As long as $\alpha < \theta/(\theta-1)$, steady-state output is below efficiency and there is an output gap that fiscal policy can close but at the cost of over-providing the public good. U_t is approximated by

$$-\frac{\bar{Y}u_C}{2}\left\{\hat{Y}_t^2\left(\eta+\frac{1}{\sigma}\right)-2\hat{Y}_t\left(\eta q_t+\frac{s_t}{\sigma}+\frac{\theta-1}{\theta}\right)+\theta^2\frac{\phi}{1-\phi}\left(\eta+\frac{\theta-1}{\theta}\right)\pi_t^2\right\},$$

but $[\hat{Y}_t - \log(Y_t^*/\bar{Y})]^2 = \hat{y}^2 - 2\frac{\hat{Y}}{1+\eta}\log\frac{(1+\alpha)(\theta-1)}{\theta} + \text{constant term} = -2\frac{\alpha}{1+\eta}\hat{G}$ if α is small. The period social loss function is as (B.17) with

$$\delta = +\frac{\alpha(1-\phi)}{\phi\theta[\theta(1+\eta)-1]} > 0, \qquad g_t \equiv \hat{G}_t.$$