Answers to Exercises on Background Material

1. $x^3 - 12x + 16$
2. $2(x^4 - 36) = 2(x^2 - 6)(x^2 + 6)$
3. $x = 3$ or $x = -\frac{2}{3}$
4. $x \geq \frac{1}{2}$ or $x \leq -\frac{1}{2}$. Don't assume you can solve as if it were an equation.
5. $x - c = y(b + a)$ so $y = \dfrac{x - c}{a + b}$
6. $\mathrm{f}(\mathrm{g}(x)) = 4x^2 - 12x + 11$, $\mathrm{g}(\mathrm{f}(x)) = 2x^2 + 1$ and $\mathrm{g}^{-1}(x) = \dfrac{x + 3}{2}$
7. (a) $y = \dfrac{a^b}{x}$
   (b) $\mathrm{e}^{2\ln x} = \mathrm{e}^{\ln x^2} = x^2$
8. $y = x^2 + 2x + 2$
9. $\dfrac{5}{x - 3} - \dfrac{3}{x - 2}$
10. $81 - 216x + 216x^2 - 96x^3 + 16x^4$
11. $1 - 4x + 12x^2$.
12. $2x^2 + 4x + 11 + \dfrac{18}{x - 2}$
13. $\mathrm{f}(-2) = 0$. Therefore $(x + 2)$ is a factor.
14. Intercepts at $(-4, 7)$ and $(3, 0)$
15. (a) 1830 (b) 16.0 (to 3 sf)
16. $2\sin(x)\cos(y)$
17. $x = 0,\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ 2\pi$
18. $12x^2 + 2\sin x + 2x\cos x + 2\mathrm{e}^{2x} + \dfrac{1}{x^2}$. (Using product rule for the second term)
19. Gradient is 0.
20. $(-4, 49), (\frac{2}{3}, -\frac{49}{27})$
21. $\dfrac{x^6}{2} - \dfrac{1}{2}\sin 2x + 2\ln x + c$.
22. $\dfrac{1}{4}\ln\left|\dfrac{x - 2}{x + 2}\right| + c$
23. $\dfrac{1}{2}(1 - \ln 2)$. (Integration by parts)
24. $\dfrac{dy}{y} = \dfrac{dx}{x}$, so integrating both sides, $\ln y = \ln x + c$
    $\mathrm{e}^{\ln y} = \mathrm{e}^{\ln x + c}$ $y = x.\mathrm{e}^c$ $y = kx$ where k is an arbitrary constant.
25. 7