# Exercises on Background Material

University mathematics is *NOT* just about knowing facts, formulae and methods. However, if you do not know the basic techniques from A-level then you will quickly fall behind. Here are some simple exercises to check that you have the necessary background to start your degree. Expect to be taught some much more interesting and thought-provoking material when you get here!

1. Multiply out and simplify: $(x-2)^2(x+4)$.
2. Factorise completely: $2x^4 - 72$.
3. Solve the equation $3x^2 - 7x - 6 = 0$.
4. Find the complete set of values of $x$ for which $4x^2 \geq 1$.
5. Make $y$ the subject of the formula $x - ay = by + c$.
6. If $\mathrm{f}(x) = x^2 + 2$ and $\mathrm{g}(x) = 2x - 3$, find and simplify $\mathrm{f}(\mathrm{g}(x))$, $\mathrm{g}(\mathrm{f}(x))$ and $\mathrm{g}^{-1}(x)$.
7. (a) If $\log_a x + \log_a y = b$, express $y$ in terms of $a, b$ and $x$. (b) Simplify $\mathrm{e}^{2\ln x}$.
8. If $y = 4t^2 + 1$ and $x = 2t - 1$, express $y$ in terms of $x$.
9. Express $\dfrac{2x-1}{x^2-5x+6}$ in the form $\dfrac{A}{x-2} + \dfrac{B}{x-3}$.
10. Find the complete binomial expansion of $(3-2x)^4$.
11. Find the first three terms in the binomial expansion of $(1+2x)^{-2}$.
12. Divide $2x^3 + 3x - 4$ by $x - 2$.
13. Without doing a division, decide whether $x + 2$ is a factor of $x^3 + x^2 - 4x - 4$.
14. Sketch graphs of $y = 3 - x$ and $y = x^2 - 9$ and find where they intersect.
15. Find the sum of the first thirty terms of the following:
    (a) $3 + 7 + 11 + 15 + ...$, (b) $8 + 4 + 2 + 1 + ...$
16. Simplify $\sin(x+y) + \sin(x-y)$.
17. Find, in radians, all the solutions of the equation $\cos 2x = \cos x$ such that $0 \leq x \leq 2\pi$.
18. Differentiate $4x^3 + 2x\sin x + \mathrm{e}^{2x} - \dfrac{1}{x}$ with respect to $x$.
19. Find the gradient of the curve $y = \sin 3x$ at the point where $x = \frac{\pi}{6}$.
20. Find the coordinates of the turning points on the graph of $y = x^3 + 5x^2 - 8x + 1$.
21. Integrate $3x^5 - \cos 2x + \dfrac{2}{x}$ with respect to $x$.
22. Integrate $\dfrac{1}{x^2-4}$, using partial fractions or otherwise.
23. Find the area between the curve $y = x\mathrm{e}^{-x}$, the $x$-axis and the line $x = \ln 2$.
24. If $\dfrac{dy}{dx} = \dfrac{y}{x}$, express $y$ in terms of $x$ and an arbitrary constant.
25. $P$ is the point $(2, 3, 6)$ in 3-D space, and $O$ is the origin. Find the length $OP$.