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OPTIMAL TAXATION AND DEBT MANAGEMENT WITHOUT COMMITMENT

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Optimal Taxation and Debt Management without Commitment*

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Abstract

This paper considers optimal fiscal policy in a deterministic Lucas and Stokey (1983) economy in the absence of government commitment. In every period, the government chooses a labor income tax and issues any unconstrained maturity structure of debt as a function of its outstanding debt portfolio. We find that the solution under commitment cannot always be sustained through the appropriate choice of debt maturities, a result which contrasts with previous conclusions in the literature. This is because a government today cannot commit future governments to a particular side of the Laffer curve, even if it can commit them to future revenues. We find that the unique stable debt maturity structure under no commitment is flat, with the government owing the same amount of resources to the private sector at all future dates. We present examples in which the maturity structure converges to such a flat distribution over time. In cases where the commitment and no-commitment solutions do not coincide, debt converges to the natural debt limit.

Keywords: Public debt, optimal taxation, fiscal policy

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1 Introduction

How should income taxes and government debt maturity be structured over time? In this paper, we study this question in a deterministic Lucas and Stokey (1983) economy in which a government without commitment dynamically chooses labor income taxes and issues any unconstrained maturity structure of debt as a function of its outstanding debt portfolio. We establish two main results. First, in contrast to many conclusions in the literature dating back to the work of Lucas and Stokey (1983), the solution under commitment cannot always be sustained through the appropriate choice of debt maturities.[1] Second, the unique stable debt maturity structure under no commitment is flat, with the government owing the same amount of resources to the private sector at all future dates. We present examples in which the maturity structure—which may or may not sustain the commitment solution—converges to a flat distribution over time.

We establish these results in the deterministic version of the model of Lucas and Stokey (1983). This is an economy with exogenous public spending and no capital in which the government chooses linear taxes on labor and issues public debt to finance government spending. In this environment, if the government could commit to policy at the beginning of time, then the choice of government debt maturity would be indeterminate. This is because many arbitrary debt maturity structures can satisfy the present value constraints of the government at a given point in time.

We do not assume that the government commits ex-ante to policy, and we instead consider the sequentially optimal policy. More specifically, we characterize the Markov Perfect Competitive Equilibrium (MPCE) in which, at every date, a government—which needs to honor the inherited debt repayments—chooses current taxes and an issued portfolio of maturities. In doing so, the government considers how current taxes and its financing strategy affect the price of bonds through expectations of future policy. We focus on characterizing the entire set of MPCE's, including those with potentially discontinuous policy functions both on and off the equilibrium path.[2] In addition, we allow for any unconstrained structure of maturity issuance. This means that the payoff relevant state—the government's portfolio of inherited maturities—is an infinite-dimensional and potentially complicated object.

Our first main result is that the solution under commitment cannot always be sustained

[1] Followup work which builds on these results includes, but is not limited to, Calvo and Obstfeld (1990), Alvarez et al. (2004), Persson et al. (2006), Diaz-Gimenez, Giovannetti, Marimon, and Teles (2008), and Debortoli et al. (2017), among others.

[2] In this regard, our approach is similar in spirit to that of Cao and Werning (2017) in their analysis of Markov equilibria in the hyperbolic consumption model.

through the appropriate choice of debt maturities. The reason is that a government today cannot commit future governments to a particular side of the Laffer curve, even if it can commit them to future revenues by an appropriate choice of debt. We establish this result in a simple example in which a government considers how to roll over its short-term debt. We show that if the level of the short-term debt is sufficiently large, the government today would like to commit all future governments to high tax rates on the downward sloping portion of the Laffer curve. Doing so reduces consumption tomorrow and reduces short-term interest rates today, allowing the government today to roll over its inherited debt at a lower cost. However, if given the option to reevaluate this policy, the government tomorrow strictly prefers to repay the inherited debt with a lower tax rate on the upward sloping portion of the Laffer curve since this increases consumption and welfare ex post. As such, the optimal policy under commitment cannot be sustained under lack of commitment.

This result contrasts with the arguments in the work of Lucas and Stokey (1983). They argue that the optimal policy under commitment can be made time-consistent with the appropriate choice of maturity. To construct this argument, they envision a government today selecting two objects to ensure that the optimal policy under commitment today satisfies the first order conditions of the government tomorrow. These two objects are a maturity structure of debt and a Lagrange multiplier on the future government's implementability constraint, which is the present value constraint of the government which incorporates this future maturity structure. Our simple example shows that this construction works if the implied future Lagrange multiplier on the implementability constraint is positive, which occurs whenever the initial short-term debt is low and future tax rates under commitment are on the upward sloping portion of the Laffer curve. However, when initial short-term debt is high, the implied future Lagrange multiplier on the implementability constraint is negative, as future tax rates under commitment are on the downward sloping portion of the Laffer curve. Since this Lagrange multiplier would never be negative ex post—because repaying public debt is costly—the construction in Lucas and Stokey (1983) fails, and the equilibrium under commitment does not coincide with that under lack of commitment. We note that our counterexample does not rely on the presence of non-concavities in the government's program and multiplicity of solutions at any date; we consider an example with isoelastic preferences in which the program is concave and the constraint set is convex at all dates.[3]

Motivated by this finding, we proceed to provide a general characterization of MPCE's.

[3] We conjecture that taking this multiplicity into account could make it even more challenging for today's government to induce commitment by future governments.

Our approach encompasses potential cases where the commitment and no-commitment solutions do not coincide and where policy functions are discontinuous, both on and off the equilibrium path. We characterize a stable maturity distribution, which is a time-invariant distribution of maturities which emerges when the inherited portfolio of maturities equals the issued portfolio. Given the Markov structure, such a stable distribution is associated with tax rates and interest rates that are both constant over time.

Our second main result is that the unique stable debt maturity structure in an MPCE is flat, with the government owing the same amount of resources to the private sector at all future dates. The fact that a flat maturity structure is stable follows from the arguments of Lucas and Stokey (1983): under a flat maturity structure, the government sequentially chooses a stable tax rate, and this tax rate coincides with the optimum under full commitment. In establishing this result, our contribution is to show that no other maturity structure admits a stable tax rate. The argument rests on showing that if the debt maturity were not flat, the government would pursue an unstable fiscal policy which decreases (increases) the market value of outstanding (newly-issued) government liabilities. A flat maturity structure is thus the unique stable maturity structure to emerge in any MPCE.

To provide some intuition for this result, suppose that the government enters the period with more long-term liabilities relative to short-term liabilities. Rather than maintain a stable tax rate, the government should pursue a policy which increases short-term interest rates. This relaxes the government budget constraint by reducing the market value of its outstanding long-term liabilities, making the government strictly better off. The opposite is true if the government enters the period with more short-term liabilities relative to long-term liabilities. In this case, rather than maintain a stable tax rate, the government should pursue a policy which reduces short-term interest rates. This policy relaxes the government budget constraint by increasing the market value of newly issued liabilities, making the government strictly better off.

In addition to these two main results, we examine the transition path of debt maturity away from a stable debt maturity, and we construct examples in which the optimal government debt maturity under no commitment converges to a flat distribution. In these examples, the initial debt maturity structure is declining in the horizon and maturities beyond a certain horizon are equal. In the cases where the commitment and no-commitment solutions coincide, this result follows from the arguments in Lucas and Stokey (1983): Optimal tax rates mirror initial maturities, and are therefore stable beyond a particular horizon. This eventual stability is guaranteed with a gradual convergence to a flat maturity under no commitment.

In the cases where the commitment and no-commitment solutions do not coincide, the argument is more subtle and follows from backward induction. Consider, for example, a government which inherits a nearly flat maturity structure where all debt payments due from tomorrow onward are the same, but debt due today is different. Such a government clearly desires a stable tax rate from tomorrow onward given these incoming maturities. However, if this desired tax rate exceeds the revenue-maximizing tax rate defining the peak of the Laffer curve, the government today realizes that it cannot commit future governments to its desired policy. Facing this binding upper bound on future tax rates, we show that the government chooses all future tax rates to equal the revenue-maximizing tax rate, and the government issues a flat maturity structure associated with the natural debt limit to achieve this outcome.

Related Literature

The main contribution of this paper is to characterize the set of MPCE's in the deterministic case of the Lucas and Stokey (1983) model. We depart from Lucas and Stokey (1983) by considering the entire set of MPCE's, not only the ones which coincide with the optimal ex-ante policy under full commitment. This allows us to establish that a flat maturity structure is the unique stable structure in the entire space of MPCE's, and to also provide examples under which convergence to a stable structure characterizes the MPCE. Our work also contributes to a literature on the optimal government debt maturity in the absence of government commitment. We depart from this literature in two ways. First, we consider the optimal maturity without imposing arbitrary constraints on maturities available to the government.[4] Second, our model is most applicable to economies in which the risk of default and surprise in inflation are not salient, but the government is still not committed to a path of taxes and debt maturity issuance.[5] In this regard, our paper complements the quantitative analysis of Debortoli et al. (2017). In contrast to this work, we consider a deterministic economy and ignore the presence of shocks.[6] This allows us to achieve theoretical characterization in an infinite horizon economy without confining the set of maturities available to the government. Our theoretical result that the optimal

[4]Krusell et al. (2006) and Debortoli and Nunes (2013) consider a similar environment to ours in the absence of commitment, but with only one-period bonds, for example.

[5]Other work considers optimal government debt maturity in the presence of default risk, for example, Aguiar et al. (2017), Arellano and Ramanarayanan (2012), Dovis (2017), and Fernandez and Martin (2015), among others. Bocola and Dovis (2016) additionally consider the presence of liquidity risk. Bigio et al. (2017) consider debt maturity in the presence of transactions costs. Arellano et al. (2013) consider lack of commitment when surprise inflation is possible. See also additional work cited in Footnote 1.

[6]Angeletos (2002), Bhandari et al. (2017), Buera and Nicolini (2004), Faraglia et al. (2010), Guibaud et al. (2013), and Lustig et al. (2008) also consider optimal government debt maturity in the presence of shocks, but they assume full commitment.

stable maturity structure is exactly flat is consistent with their quantitative result that the optimal maturity structure is nearly flat in the presence of shocks.

Our paper proceeds as follows. In Section 2, we describe the model. In Section 3, we define the equilibrium. In Section 4, we provide an example explaining why the solution under commitment cannot always be sustained through the appropriate choice of debt maturities. In Section 5, we provide the main result of the paper that the unique stable maturity distribution is flat. In Section 6, we provide examples in which the MPCE converges to a flat maturity distribution over time. Section 7 concludes. The Appendix provides all of the proofs and additional results not included in the text.

2 Model

We consider an economy identical to a deterministic version of Lucas and Stokey (1983) in which the government has no commitment to fiscal policy. There are discrete time periods $t = \{0, 1, ..., \infty\}$. The resource constraint of the economy is

$$c_t + g = n_t, \tag{1}$$

where c_t is consumption, n_t is labor, and $g > 0$ is government spending, which is exogenous and constant over time.

There is a continuum of mass 1 of identical households that derive the following utility:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right), \, \beta \in (0, 1) \, . \tag{2}$$

$u(\cdot)$ is strictly increasing in consumption, strictly decreasing in labor, globally concave, and continuously differentiable. We also assume that $u_{cc}(c, c+g) + u_{cn}(c, c+g) < 0$ so that the marginal utility of consumption is decreasing in consumption in general equilibrium. As a benchmark, we define the first best consumption and labor $\left\{c^{fb}, n^{fb}\right\}$ as the values of consumption and labor which maximize $u\left(c_t, n_t\right)$ subject to the resource constraint (1).

Household wages equal the marginal product of labor (which is 1 unit of consumption), and are taxed at a linear tax rate τ_t. $b_{t,k} \gtreqless 0$ represents government debt purchased by a representative household at t, which is a promise to repay 1 unit of consumption at $t + k > t$. $q_{t,k}$ is the bond price at t. At every t, the household's allocation and portfolio

$\{c_t, n_t, \{b_{t,k}\}_{k=1}^{\infty}\}$ must satisfy the household's dynamic budget constraint

$$c_t + \sum_{k=1}^{\infty} q_{t,k} \left(b_{t,k} - b_{t-1,k+1}\right) = (1 - \tau_t)\, n_t + b_{t-1,1}. \tag{3}$$

$B_{t,k} \gtreqless 0$ represents debt issued by the government at t with a promise to repay 1 unit of consumption at $t + k > t$. At every t, government policies $\{\tau_t, g_t, \{B_{t,k}\}_{k=1}^{\infty}\}$ must satisfy the government's dynamic budget constraint

$$g_t + B_{t-1,1} = \tau_t n_t + \sum_{k=1}^{\infty} q_{t,k} \left(B_{t,k} - B_{t-1,k+1}\right).^{7} \tag{4}$$

The economy is closed which means that the bonds issued by the government equal the bonds purchased by households:

$$b_{t,k} = B_{t,k} \ \forall t, k. \tag{5}$$

Initial debt $\{B_{-1,k}\}_{k=1}^{\infty} = \{b_{-1,k}\}_{k=1}^{\infty}$ is exogenous. We assume that there exist debt limits to prevent Ponzi schemes:

$$b_{t,k} \in \left[\underline{b}, \overline{b}\right] \ \ \forall t, k. \tag{6}$$

We will consider economies where these limits are not binding along the equilibrium path. The government is benevolent and shares the same preferences as the households in (2). We assume that the government cannot commit to policy and therefore chooses taxes and debt sequentially.

3 Markov Perfect Competitive Equilibrium

In this section, we formally define our equilibrium and then apply the primal approach to abstract away from bond prices and tax rates and characterize the equilibrium in terms of allocations. We conclude by providing a recursive representation of the equilibrium.

[7] We follow the same exposition as in Angeletos (2002) in which the government rebalances its debt in every period by buying back all outstanding debt and then issuing fresh debt at all maturities. This is without loss of generality. For example, if the government at $t - k$ issues debt due at date t of size $B_{t-k,k}$ which it then holds to maturity without issuing additional debt, then all future governments at date $t - k + l$ for $l = 1, ..., k - 1$ will choose $B_{t-k+l,k-l} = B_{t-k,k}$, implying that $B_{t-1,1} = B_{t-k,k}$.

3.1 Equilibrium Definition

We consider a Markov Perfect Competitive Equilibrium (MPCE) in which the government optimally chooses its preferred policy—which consists of taxes and an issued portfolio of debt—at every date as a function of current payoff-relevant variables, which consists of the inherited portfolio of debt. The government takes into account that its choice affects future debt and thus affects the policies of future governments. Households rationally anticipate these future policies, and their expectations are in turn reflected in current bond prices. Thus, in choosing policy today, a government anticipates that it may affect current bond prices by impacting expectations about future policy.

Formally, let $\mathbf{B}_t \equiv \{B_{t,k}\}_{k=1}^{\infty}$ and $\mathbf{q}_t \equiv \{q_{t,k}\}_{k=1}^{\infty}$. In every period t, the government chooses a policy $\{\tau_t, \mathbf{B}_t\}$ given $\mathbf{B}_{t-1}$. Households then choose an allocation and portfolio $\left\{c_t, n_t, \{b_{t,k}\}_{k=1}^{\infty}\right\}$. An MPCE consists of: a government strategy $\rho\left(\mathbf{B}_{t-1}\right)$ which is a function of $\mathbf{B}_{t-1}$; a household allocation and portfolio strategy $\omega\left(\mathbf{B}_{t-1}, \rho_t, \mathbf{q}_t\right)$ which is a function of $\mathbf{B}_{t-1}$, the government policy $\rho_t = \rho\left(\mathbf{B}_{t-1}\right)$, and bond prices $\mathbf{q}_t$; and a set of bond pricing functions $\left\{\varphi^k\left(\mathbf{B}_{t-1}, \rho_t\right)\right\}_{k=1}^{\infty}$ with $q_{t,k} = \varphi^k\left(\mathbf{B}_{t-1}, \rho_t\right)\ \forall k \geq 1$ which depend on $\mathbf{B}_{t-1}$ and the government policy $\rho_t = \rho\left(\mathbf{B}_{t-1}\right)$. In an MPCE, these objects must satisfy the following conditions $\forall t$:

1. The government strategy $\rho\left(\cdot\right)$ maximizes (2) given $\omega\left(\cdot\right)$, $\varphi^k\left(\cdot\right)\ \forall k \geq 1$, and the government budget constraint (4);

2. The household allocation and portfolio strategy $\omega\left(\cdot\right)$ maximizes (2) given $\rho\left(\cdot\right)$, $\varphi^k\left(\cdot\right)$ $\forall k \geq 1$, and the household budget constraint (3), and

3. The set of bond pricing functions $\varphi^k\left(\cdot\right)\ \forall k \geq 1$ satisfy (5) given $\rho\left(\cdot\right)$ and $\omega\left(\cdot\right)$.

3.2 Primal Approach

Any MPCE must be a competitive equilibrium. We follow Lucas and Stokey (1983) by taking the primal approach to the characterization of competitive equilibria since this allows us to abstract away from bond prices and taxes. Let

$$\{c_t, n_t\}_{t=0}^{\infty} \tag{7}$$

represent a sequence. We can establish necessary and sufficient conditions for (7) to constitute a competitive equilibrium. The household's optimization problem implies the following intratemporal and intertemporal conditions, respectively:

$$1-\tau_t = -\frac{u_n\left(c_t,n_t\right)}{u_c\left(c_t,n_t\right)} \text{ and } q_{t,k} = \frac{\beta^k u_c\left(c_{t+k},n_{t+k}\right)}{u_c\left(c_t,n_t\right)}. \tag{8}$$

Substitution of these conditions into the household's dynamic budget constraint implies the following condition:

$$u_c\left(c_t,n_t\right)c_t + u_n\left(c_t,n_t\right)n_t + \sum_{k=1}^{\infty}\beta^k u_c\left(c_{t+k},n_{t+k}\right)b_{t,k} = \sum_{k=0}^{\infty}\beta^k u_c\left(c_{t+k},n_{t+k}\right)b_{t-1,k+1}. \tag{9}$$

Forward substitution into the above equation and taking into account the absence of Ponzi schemes implies the following implementability condition:

$$\sum_{k=0}^{\infty}\beta^k\left(u_c\left(c_{t+k},n_{t+k}\right)c_{t+k} + u_n\left(c_{t+k},n_{t+k}\right)n_{t+k}\right) = \sum_{k=0}^{\infty}\beta^k u_c\left(c_{t+k},n_{t+k}\right)b_{t-1,k+1}. \tag{10}$$

By this reasoning, if a sequence in (7) is generated by a competitive equilibrium, then it necessarily satisfies (1) and (10). We prove in the Appendix that the converse is also true, which leads to the below proposition that is useful for the rest of our analysis.

Lemma 1 ***(competitive equilibrium)*** *A sequence* (7) *is a competitive equilibrium if and only if it satisfies* (1) $\forall t$ *and* (10) *at* $t=0$ *given* $\{b_{-1,k}\}_{k=1}^{\infty}$.

Note that this result rests on the fact that the satisfaction of (10) at $t=0$ guarantees the satisfaction of (10) for all future dates, since bonds can be freely chosen so as to satisfy (10) at all future dates for a given sequence (7).

3.3 Recursive Representation

We can use the primal approach to represent an MPCE recursively. Recall that $\rho\left(\mathbf{B}_{t-1}\right)$ is a policy which depends on $\mathbf{B}_{t-1}$, and that $\omega\left(\left(\mathbf{B}_{t-1}\right),\rho_t,\mathbf{q}_t\right)$ is a household allocation and portfolio strategy which depends on $\mathbf{B}_{t-1}$, government policy $\rho_t=\rho\left(\mathbf{B}_{t-1}\right)$, and bond prices $\mathbf{q}_t$, where these bond prices depend on $\mathbf{B}_{t-1}$ and government policy. As such, an MPCE in equilibrium is characterized by a consumption and labor sequence (7) and a debt sequence $\left\{\{b_{t,k}\}_{k=1}^{\infty}\right\}_{t=0}^{\infty}$, where each element at date t depends on history only through $\mathbf{B}_{t-1}$, the payoff relevant variables. Given this observation, in an MPCE, one can define a function $h^k\left(\cdot\right)$

$$h^k\left(\mathbf{B}_t\right) = \beta^k u_c\left(c_{t+k},n_{t+k}\right)|\mathbf{B}_t \tag{11}$$

for $k \geq 1$, which equals the discounted marginal utility of consumption at $t+k$ given $\mathbf{B}_t$ at t. This function is useful since, in choosing $\mathbf{B}_t$ at date t, the government must take into account how it affects future expectations of policy, which in turn affect current bond prices through expected future marginal utility of consumption.

Note that choosing $\{\tau_t, \mathbf{B}_t\}$ at date t from the perspective of the government is equivalent to choosing $\{c_t, n_t, \mathbf{B}_t\}$ where one can write, with some abuse of notation, $\mathbf{B}_t = \{b_{t,k}\}_{k=1}^{\infty}$, and this follows from the primal approach delineated in Section 3.2. Removing the time subscript and defining $\mathbf{B} \equiv \mathbf{B}_{t-1} = \{b_k\}_{k=1}^{\infty}$ as the inherited portfolio of bonds, we can write the government's problem recursively as

$$V(\mathbf{B}) = \max_{c,n,\mathbf{B}'} u(c,n) + \beta V(\mathbf{B}') \tag{12}$$

$$\text{s.t.}$$

$$c + g = n, \text{ and} \tag{13}$$

$$u_c(c,n)\,c + u_n(c,n)\,n - u_c(c,n)\,b_1 + \sum_{k=1}^{\infty} h^k(\mathbf{B}')\left(b'_k - b_{k+1}\right) = 0, \tag{14}$$

where (14) is a recursive representation of (9). Let $f(\mathbf{B})$ correspond to the solution to (12) – (14) given $V(\cdot)$ and $h^k(\cdot)$ $\forall k \geq 1$. It therefore follows that the function $f(\cdot)$ necessarily implies functions $h^k(\cdot)$ $\forall k \geq 1$ which satisfy (11). An MPCE is therefore composed of functions $V(\cdot)$, $f(\cdot)$, and $h^k(\cdot)$ $\forall k \geq 1$ which are consistent with one another and satisfy (11) – (14).

4 Commitment vs. Lack of Commitment

In this section, we provide an example to highlight why the solution under commitment cannot always be sustained through the appropriate choice of debt maturities. Our example implies that there does not always exist an MPCE which coincides with the solution under commitment.

4.1 Policy Under Commitment

Consider an economy in which preferences over consumption c and labor n satisfy

$$u(c,n) = \log c - \eta \frac{n^{\gamma}}{\gamma} \tag{15}$$

for $\eta > 0$ and $\gamma \geq 1$, which corresponds to a utility function analyzed in Werning (2007).

To facilitate the discussion, define c^{laffer} as

$$c^{laffer} = \arg\max_{c} c\left(1 - \eta\left(c+g\right)^{\gamma}\right). \tag{16}$$

The right hand side of (16) corresponds to the primary surplus of the government. Therefore, c^{laffer} is the level of consumption associated with the maximal tax revenue and the peak of the Laffer curve, which we label as τ^{laffer}. We assume that $g < \left(\frac{1}{\eta}\right)^{1/\gamma}$ to guarantee that $c^{laffer} > 0$. The function on the right hand side of (16) is strictly concave in c and admits a value of 0 if $c = 0$ (100 percent labor income tax) and a value of $-g$ if $c = c^{fb}$ (0 percent labor income tax). More broadly, if $c > c^{laffer}$, then the tax rate is below the revenue-maximizing tax rate and the economy is on the upward sloping portion (left hand side) of the Laffer curve. If $c < c^{laffer}$, then the tax rate is above the revenue-maximizing tax rate and the economy is on the downward sloping portion (right hand side) of the Laffer curve.

Suppose that $b_{-1,1} > 0$ and $b_{-1,k} = 0$ $\forall k \geq 2$. Using Lemma 1, we can consider the date 0 government's optimal policy under commitment, where we have substituted in for labor using the resource constraint:

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \eta \frac{\left(c_t+g\right)^{\gamma}}{\gamma}\right) \tag{17}$$

s.t.

$$1 - \frac{b_{-1,1}}{c_0} - \eta\left(c_0+g\right)^{\gamma} + \sum_{t=1}^{\infty} \beta^t \left(1 - \eta\left(c_t+g\right)^{\gamma}\right) = 0. \tag{18}$$

Equation (18) represents the date 0 implementability condition, which is the present value constraint of the government. Since $b_{-1,1} > 0$, the left hand side of (18)—which can be equivalently written in relaxed form as a weak inequality constraint—is concave, implying that the constraint set is convex. This leads to the below lemma which characterizes the unique optimum under commitment.

Lemma 2 *The unique solution to* (17) – (18) *satisfies the following conditions:*

1. $c_t = c_1$ $\forall t \geq 1$,

2. c_0 *and* $c_1 < c_0$ *are the unique solutions to the following system of equations for some*

$\mu_0 > 0$

$$\frac{1}{c_0} - \eta\left(c_0+g\right)^{\gamma-1} + \mu_0\left(\frac{b_{-1,1}}{c_0^2} - \eta\gamma\left(c_0+g\right)^{\gamma-1}\right) = 0, \tag{19}$$

$$\frac{1}{c_1} - \eta\left(c_1+g\right)^{\gamma-1} + \mu_0\left(-\eta\gamma\left(c_1+g\right)^{\gamma-1}\right) = 0, \text{ and} \tag{20}$$

$$1 - \frac{b_{-1,1}}{c_0} - \eta\left(c_0+g\right)^{\gamma} + \frac{\beta}{1-\beta}\left(1-\eta\left(c_1+g\right)^{\gamma}\right) = 0. \tag{21}$$

3. *There exists $b^*_{-1,1} > 0$ such that the solution admits $c_1 > c^{laffer}$ if $b_{-1,1} < b^*_{-1,1}$ and $c_1 < c^{laffer}$ if $b_{-1,1} > b^*_{-1,1}$.*

The first part of the lemma states that consumption—and therefore the tax rate—from date 1 onward is constant. Since initial debt due from date 1 onward is constant (and equal to zero), tax smoothing and interest rate smoothing from date 1 onward is optimal. The second part of the lemma characterizes the solution in terms of first order conditions for a positive Lagrange multiplier μ_0 on the implementability constraint (18). These conditions are necessary and sufficient for optimality given the concavity of the problem. Implicit differentiation of (19) and (20) taking into account second order conditions implies that initial consumption c_0 exceeds long-run consumption c_1, which means that the initial tax rate is below the future tax rate. Backloading tax rates is optimal since the reduction in future consumption relative to present consumption allows the government to roll over its initial short-term debt at a lower interest rate.

The last part of the lemma states that if initial short-term debt $b_{-1,1}$ is sufficiently high, then future consumption c_1 is below c^{laffer}, implying that the future tax rate τ_1 is above the revenue-maximizing tax rate at the peak of the Laffer curve τ^{laffer}. This result stems from the fact that the government under commitment accommodates increases in initial short-term debt $b_{-1,1}$ with a reduction in future consumption c_1 and an increase in the future tax rate τ_1. Mathematically, higher $b_{-1,1}$ tightens the implementability constraint (18) which increases the Lagrange multiplier μ_0 on this constraint. From (20), a higher value of μ_0 leads to a lower value of c_1, and beyond a certain level $b^*_{-1,1}$, c_1 declines below c^{laffer} and τ_1 rises above τ^{laffer}. Conceptually, for $c_1 > c^{laffer}$ and $\tau_1 < \tau^{laffer}$, the reduction in future consumption c_1 accommodates an increase in initial short-term debt $b_{-1,1}$ by increasing future revenues and decreasing short-term interest rates. Once c_1 declines beyond c^{laffer} and τ_1 rises above τ^{laffer}, the increase in initial short-term debt $b_{-1,1}$ is accommodated with lower short-term interest rates only. If $c_1 < c^{laffer}$ and $\tau_1 > \tau^{laffer}$, the government at date 0 could instead choose a value of $c_1 > c^{laffer}$ and $\tau_1 < \tau^{laffer}$ yielding the same future revenue to repay its issued debt. However, doing so is suboptimal and would lead to higher short-term interest rates, significantly reducing the resources raised at date 0 by issuing this debt.

4.2 Time-Consistency of Policy

We now show that the policy under commitment may not be time-consistent. To make this point as clearly as possible, we follow Lucas and Stokey (1983) and consider what happens if at date 1, policy is reevaluated and chosen by a government with full commitment from date 1 onward. Given an inherited portfolio of maturities $\{b_{0,k}\}_{k=1}^{\infty}$, the government at date 1 solves the following problem:

$$\max_{\{c_t,n_t\}_{t=0}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} \left(\log c_t - \eta \frac{(c_t+g)^{\gamma}}{\gamma} \right) \tag{22}$$

s.t.

$$\sum_{t=1}^{\infty} \beta^{t-1} \left(1 - \eta (c_t+g)^{\gamma} - \frac{b_{0,t}}{c_t} \right) = 0. \tag{23}$$

Letting μ_1 represent the Lagrange multiplier on (23), first order conditions with respect to c_t are:

$$\frac{1}{c_t} - \eta (c_t+g)^{\gamma-1} + \mu_1 \left(\frac{b_{0,t}}{c_t^2} - \eta\gamma (c_t+g)^{\gamma-1} \right) = 0 \ \forall t \geq 1. \tag{24}$$

We will say that optimal policy at date 0 is time-consistent if there exists $\{b_{0,k}\}_{k=1}^{\infty}$ such that the government at date 1 solving (22) $-$ (23) chooses $c_t = c_1$ for c_1 which satisfies (19) $-$ (21). In other words, the optimal date 1 policy coincides with the optimal date 0 policy.

Proposition 1 ***(time-consistency of optimal policy)*** *If $b_{-1,1} < b^*_{-1,1}$, then the optimal date 0 policy is time-consistent with $b_{0,k} = b_{0,1}$ $\forall k \geq 1$. If $b_{-1,1} > b^*_{-1,1}$, then the optimal date 0 policy is not time-consistent.*

If $b_{-1,1} < b^*_{-1,1}$, then the optimal date 0 policy can be sustained under lack of commitment with the government at date 0 issuing a flat maturity structure with $b_{0,k} = b_{0,1}$ $\forall k \geq 1$. Under such a flat structure, the government at date 1 optimally chooses to smooth tax rates into the future. Moreover, given that date 1 tax rates under commitment are on the upward sloping portion of the Laffer curve, the choice of such tax rates is time-consistent. The date 0 and date 1 government agree about the optimal tax rate to repay this debt.

If instead $b_{-1,1} > b^*_{-1,1}$, then the optimal date 0 policy cannot be sustained under lack of commitment. If the government at date 0 tried to induce the date 1 government into a smooth policy from date 1 onward by issuing a flat maturity structure with $b_{0,k} = b_{0,1}$ $\forall k \geq 1$, the date 1 government would never choose a value $c_1 < c^{laffer}$ and $\tau_1 > \tau^{laffer}$ and would instead repay the inherited debt with a value $c_1 > c^{laffer}$ and $\tau_1 < \tau^{laffer}$. Choosing a lower tax rate on the upward sloping portion of the Laffer curve increases consumption and increases welfare ex-post. Thus, while the date 0 government can commit the date 1 government to a smooth path of revenue and interest rates, it cannot commit the date 1 government to a particular tax rate. As such, the optimal date 0 policy is not time-consistent.

This result contrasts with the arguments in the work of Lucas and Stokey (1983). They argue that the optimal policy under commitment at date 0 can be made time-consistent at date 1 with the appropriate choice of maturities $\{b_{0,k}\}_{k=1}^{\infty}$ which satisfy the date 1 implementability condition (23) and the date 1 first order condition (24) for some Lagrange multiplier μ_1. In our example, this argument would imply that the issuance of a flat debt maturity at date 0 with $b_{0,k} = b_{0,1}$ $\forall k \geq 1$ would induce commitment at date 1.

To see why this argument cannot always work, consider the equations characterizing $b_{0,1}$ and μ_1 under this construction. Combining (20) and (24), it is clear that $b_{0,1}$ and μ_1 jointly satisfy

$$b_{0,1} = \left(1 - \frac{\mu_0}{\mu_1}\right) \eta\gamma c_1^2 \left(c_1 + g\right)^{\gamma-1}, \tag{25}$$

and (23) which reduces to

$$b_{0,1} = c_1 \left(1 - \eta \left(c_1 + g\right)^{\gamma}\right) \tag{26}$$

for μ_0 and c_1 which satisfy (19) – (21). Our simple example shows that this construction works if the implied future Lagrange multiplier μ_1 satisfying (25) – (26) is positive, which occurs whenever $b_{-1,1} < b^*_{-1,1}$. However, when $b_{-1,1} > b^*_{-1,1}$ the construction implies that (25) – (26) are satisfied by a negative multiplier μ_1. However, the solution to (22) – (23) under a positive debt portfolio $\{b_{0,k}\}_{k=1}^{\infty}$ would never admit a negative multiplier—since the shadow cost of inherited debt is positive—which is why the construction fails.

There are three important points to note regarding this counterexample. First, our counterexample does not rely on the presence of non-concavities in the government's program and multiplicity of solutions at any date. Our isoelastic preferences imply that the government's welfare is concave and constraint set convex, which guarantees that the solution to the government's problem at dates 0 and 1 is unique. We conjecture that taking this multiplicity into account (for example by considering examples with negative debt positions which make the implementability condition no longer a convex constraint) could make it even more challenging for today's government to induce commitment by future governments.

Second, our counterexample suggests that similar challenges to inducing commitment with maturities could emerge in other settings with different initial maturities and different preferences. Any government expected to run a primary surplus weakly below its short-term debt—as in date 1 in our setting—cannot commit to choosing a tax rate on a downward sloping portion of the Laffer curve. In those instances, a feasible and beneficial deviation to another tax rate associated with higher consumption always exists. Such a deviation reduces short-term interest rates which relaxes the government's budget constraint since the short-term debt weakly exceeds the primary surplus.

Finally, for illustration we made our arguments by examining whether a government with full commitment from date 1 onward would choose the same policy as the government with full commitment from date 0 onward. However, our arguments apply to an MPCE more generally in which a government reoptimizes at all future dates in an infinite horizon. As we show in the next section, the only way to ensure a stable tax rate from date 1 onward in a continuation MPCE is for the government at date 0 to issue a flat maturity structure with $b_{0,k} = b_{0,1}$ $\forall k \geq 1$, and the continuation equilibrium necessarily coincides with that under commitment starting from date 1. Our counterexample shows that once this flat maturity structure is issued, a future government would never choose a tax rate on the downward sloping portion of the Laffer curve.

In sum, it is generally not the case that the continuation equilibrium in an MPCE starting from some initial debt maturity $\{b_{-1,k}\}_{k=1}^{\infty}$ will necessarily coincide with the solution under commitment. For this reason, a complete analysis of MPCE's must consider the possibility that the commitment and no-commitment solutions do not coincide both on and off the equilibrium path.

5 Stable Government Debt Maturity

Motivated by our findings in the previous section, we proceed by providing a general characterization of MPCE's. In our approach, we do not impose conditions on the continuation strategies of future governments and allow for potentially discontinuous policy functions.

We focus on characterizing an economy in which the debt maturity structure is stable with $b_{t+1,k} = b_{t,k}$, $\forall t, k$, so that government debt maturity is time-invariant. Given the Markovian structure of the solution to the MPCE defined by $(12) - (14)$, such a stable maturity distribution is associated with tax rates and interest rates which are constant over time. We show that the unique stable maturity distribution is flat, with the government owing the same amount of resources to the private sector at all future dates.

5.1 Preliminaries

Before proceeding with our analysis, we establish a preliminary assumption which we utilize in deriving our results. Using our recursive notation introduced in Section 3, let us define $W\left(\{b_k\}_{k=1}^{\infty}\right)$ as the welfare of the government under full commitment given an initial starting debt position $\{b_k\}_{k=1}^{\infty}$:

$$W\left(\{b_k\}_{k=1}^{\infty}\right) = \max_{\{c_k, n_k\}_{k=0}^{\infty}} \sum_{k=0}^{\infty} \beta^k u\left(c_k, n_k\right) \tag{27}$$

$$\text{s.t.}$$

$$c_k + g = n_k, \text{ and} \tag{28}$$

$$\sum_{k=0}^{\infty} \beta^k \left(u_c\left(c_k, n_k\right) c_k + u_n\left(c_k, n_k\right) n_k\right) = \sum_{k=0}^{\infty} \beta^k u_c\left(c_k, n_k\right) b_{k+1}. \tag{29}$$

Given Lemma 1, the program in $(27) - (29)$ corresponds to that of a government under full commitment with $b_{-1,k} = b_k$. We now make an assumption regarding the solution to this program under a flat maturity structure, meaning a maturity structure in which b_k is the same for all k.

Assumption 1. Consider the solution to $(27) - (29)$ with $b_k = b \; \forall k \geq 1$. $\forall b \in \left[\underline{b}, \overline{b}\right]$, if the solution exists, then the solution is unique and admits $\{c_k, n_k\} = \{c^*(b), n^*(b)\}$ $\forall k \geq 1$, where

$$u_c\left(c^*(b), n^*(b)\right) c^*(b) + u_n\left(c^*(b), n^*(b)\right) n^*(b) = u_c\left(c^*(b), n^*(b)\right) b, \tag{30}$$

$$\text{and } c^*(b) + g = n^*(b). \tag{31}$$

This assumption states that if a government under full commitment is faced with a flat maturity structure, then there is a unique optimum in which the government chooses a constant allocation of consumption and labor in the future.[8] This assumption is intuitive.

[8] Assumption 1 requires that the solution exists. If the upper bound on individual maturities $\overline{b}$ exceeds the highest primary surplus which can be raised at the peak of the Laffer curve, then there is no solution

Under a flat maturity structure, every time period in the program in $(27)-(29)$ is identical in the objective function and in the constraint set, which suggests that the optimal solution is a time-invariant allocation. A sufficient condition for Assumption 1 is that the function $u_c(c, c+g)(c-b) + u_n(c, c+g)(c+g)$ is concave in c for all b, which is the case, for example, if the utility function satisfies (15) as in our example in Section 4 and if $\underline{b} = 0$ so that debt is non-negative.

5.2 Stability of Flat Maturity

We begin by establishing that there exists an MPCE with a flat debt maturity which is stable.

Lemma 3 *Suppose that* $\mathbf{B}$ *satisfies* $b_k = b \ \forall k$ *for some* $b \in \left[\underline{b}, \overline{b}\right]$. *Then,*

1. *In all solutions to* $(12)-(14)$, $c = c^*(b)$ *and* $n = n^*(b)$, *and*
2. *There exists a solution to* $(12)-(14)$ *which admits* $b'_k = b \ \forall k$.

The first part of the lemma states that in any MPCE, if the government inherits a flat maturity with $b_k = b \ \forall k$, then the unique optimal response of the government is to choose consumption and labor which coincide with the commitment optimum. The second part of the lemma implies that one optimal—but not necessarily uniquely optimal—strategy for the government is to choose $b'_k = b \ \forall k \geq 1$ so that the issued debt maturity structure is unchanged and continues to be flat. As such, there exists a stable MPCE with a flat government debt maturity. Importantly, this lemma implies that in any MPCE for which $\mathbf{B}$ is a flat government debt maturity, it is necessary that

$$V(\mathbf{B}) = W(\mathbf{B}) \tag{32}$$

so that there is no welfare loss for the present government due to lack of commitment by future governments.

The logic behind the proof of this argument follows from the arguments of Lucas and Stokey (1983) after applying Assumption 1: under a flat maturity structure, the government sequentially chooses a stable tax rate, and this tax rate coincides with the optimum under full commitment. Note, however, that in contrast to the arguments of Lucas and Stokey (1983), this lemma applies to any MPCE which is constructed. This lemma does not rely on making assumptions regarding the structure of future government

under a flat maturity for some high values of debt which satisfies the constraints of the problem.

strategies, which may not coincide with the commitment solution off the equilibrium path. Moreover, we emphasize that this lemma establishes the existence, but not the uniqueness of this stable MPCE.

5.3 Uniqueness of Stable Flat Maturity

We now turn to the possibility that another stable MPCE which does not admit a flat maturity structure exists. Note that under such a maturity structure $\{b_k\}_{k=1}^{\infty}$, consumption is constant over time, which implies that the current price of a bond maturing in k periods is β^k.

Lemma 4 *Suppose that given* $\mathbf{B}$*, there exists a solution to* $(12)-(14)$ *with a stable debt maturity structure* $b'_k = b_k\ \forall k$ *and* $b'_l \neq b'_m$ *for some* l, m*. Then there exists another solution to* $(12)-(14)$ *with* $b'_k = \widehat{b}\ \forall k$ *where*

$$\widehat{b} = \sum_{k=1}^{\infty} \beta^{k-1} (1-\beta)\, b_k. \tag{33}$$

This lemma states that under any MPCE with a stable distribution of debt which is not flat, the government can choose the same current tax rate and deviate to a flat issuance of debt maturity and achieve the same welfare. More precisely, the government can issue a flat maturity with the same market value, as determined by (33). Moreover, Lemma 3 characterizes future welfare and future allocations following the issuance of a flat maturity today, which means that bond prices are not affected by the deviation.

This lemma implies that if there is a stable distribution of debt which is not flat, then the corresponding welfare is equal to that achieved under a flat maturity distribution with the same market value. Moreover, from (32), welfare under this MPCE equals that under commitment associated with a flat maturity distribution with the same market value:

$$V(\mathbf{B}) = W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k = \widehat{b}\ \forall k} = \frac{u(c(\widehat{b}), n(\widehat{b}))}{1-\beta}. \tag{34}$$

With these results in mind, we now develop an induction argument to show that the unique stable distribution of debt is flat. The argument rests on showing that if a distribution of debt is not flat, the government can deviate from a stable fiscal policy in order to frontload or backload consumption so as to change the value of its inherited or newly-issued debt portfolio.

Lemma 5 *Suppose that given* $\mathbf{B}$*, there exists a solution to* (12) − (14) *with a stable debt maturity structure* $b'_k = b_k \; \forall k$ *and for which* $\{c, n\} \neq \left\{c^{fb}, n^{fb}\right\}$*. Then,* $\mathbf{B}$ *must satisfy* $b_1 = \widehat{b}$ *for* $\widehat{b}$ *defined in* (33)*.*

This lemma states that in any stable distribution of debt maturity in which the tax rate is not zero (so that consumption and labor do not equal the first best), short-term debt b_1 equals the annuitized value of total debt $\widehat{b}$. This means that the primary surplus equals the short-term debt b_1 and net debt issuance is zero. If the primary surplus is in excess of, or below, this short-term debt then the government can pursue a deviation from its smooth consumption strategy to boost welfare.

For example, if the primary surplus is in excess of what the government immediately owes, then in equilibrium, the government buys back some of its long-term debt. In this circumstance, the government can deviate to tilt the path of consumption so as to increase short-term interest rates and reduce the value of the long-term debt which it buys back. If instead the primary surplus is below what the government owes, then in equilibrium the government issues fresh debt in order to repay current short-term debt. In this circumstance, the government can deviate to tilt the path of consumption so as to decrease short-term interest rates and increase the value of newly issued debt. Thus, if the primary surplus equals the amount of short-term debt that is due, the government will not engage in such deviations.

Note that in constructing these deviations, we utilize the result in Lemma 3 which allows us to characterize the continuation equilibrium if the government issues a flat government debt maturity today as part of its deviation. As such, we can explicitly show that these deviations increase welfare by relaxing the government's budget constraint. The reason why our argument does not hold under a stable distribution of debt maturities with zero taxes is that in this case, it is not possible to relax the government budget constraint further.

We now expand this lemma to consider longer maturities.

Lemma 6 *Suppose that given* $\mathbf{B}$*, there exists a solution to* (12) − (14) *with a stable debt maturity structure* $b'_k = b_k \; \forall k$ *and for which* $\{c, n\} \neq \left\{c^{fb}, n^{fb}\right\}$*. If* $b_l = \widehat{b} \; \forall l \leq m$*, then* $\mathbf{B}$ *must satisfy* $b_{m+1} = \widehat{b}$ *for* $\widehat{b}$ *defined in* (33)*.*

This lemma considers the stable distribution of government debt maturity when all maturities below m have the property that the amount owed equals the primary surplus of the government. The lemma states that if this is true, then the bond of maturity $m+1$ must also equal the primary surplus of the government.

The argument, which relies on a proof by contradiction, starts from the fact that under a stable maturity, government welfare satisfies (34). Now if the amount owed at date $m+1$ does not also equal the primary surplus, then a feasible deviation exists for the government which can increase welfare above (34), leading to a contradiction. More specifically, if $b_l = \widehat{b}$ $\forall l \leq m$ but $b_{m+1} \neq \widehat{b}$, a feasible strategy for the government today is to continue to choose the same consumption and labor allocation today $\{c(\widehat{b}), n(\widehat{b})\}$ but to deviate by not retrading the inherited maturity structure (i.e., letting the bonds mature to next period). Such a deviation is feasible whatever the expectations of future policy and their impact on current bond prices since the government is not rebalancing its portfolio.

Without specifying the exact form of the continuation equilibrium, we can show that this deviation must necessarily increase welfare. The argument rests on putting a lower bound on the welfare of future governments based on the feasible policies at their disposal. More specifically, note that after this initial deviation, future governments also have the opportunity to pursue the same strategy of choosing consumption and labor equal to $\{c(\widehat{b}), n(\widehat{b})\}$ and not rebalancing the portfolio of maturities. This is true up until some future date m periods in the future. Therefore, the welfare of the government today from pursuing the deviation must weakly exceed

$$\sum_{l=0}^{m-1} \beta^l u(c(\widehat{b}), n(\widehat{b})) + \beta^m V(\widehat{\mathbf{B}}\,(m)) \tag{35}$$

where $\widehat{\mathbf{B}}\,(m)$ satisfies $\widehat{b}\,(m)_k = b_{k+m}$ $\forall k \geq 1$.

At that point m periods in the future, if the government pursued a stable policy from thereafter, the market value of debt would equal $\widehat{b}/\,(1-\beta)$ and welfare $V(\widehat{\mathbf{B}}\,(m))$ would be given by (34). Were the government to choose $\{c(\widehat{b}), n(\widehat{b})\}$ at that date so as to satisfy (34), the fact that $b_{m+1} \neq \widehat{b}$ means that the primary surplus would either be above or below the short-term debt. However, by the arguments of Lemma 5, the government could choose at this point a non-stable policy which either decreases the market value of inherited debt or increases the market value of newly-issued debt. Such a policy would provide a continuation value $V(\widehat{\mathbf{B}}\,(m))$ which strictly exceeds (34). Based on this logic, the initial deviation which provides the government at least (35) makes the government strictly better off since (35) strictly exceeds (34). This completes the argument, since it contradicts the fact that government welfare equals (34) under the MPCE.

Proposition 2 ***(flat maturity)*** *Suppose that conditional on* $\mathbf{B}$*, there exists a solution to* $(12)-(14)$ *with a stable debt maturity structure* $b'_k = b_k$ $\forall k$ *and for which* $\{c, n\} \neq$

$\{c^{fb}, n^{fb}\}$. *Then it is necessary that* $b_k = \widehat{b}\ \forall k$ *so that the government debt maturity is flat.*

This proposition represents the main result of the paper. It states that if the distribution of government debt is stable and if the equilibrium does not entail first best consumption and labor, then government debt must be flat. The reasoning for the proposition follows from induction arguments which appeal to Lemmas 5 and 6. Intuitively, if government debt is not flat, then there are opportunities for the government take advantage of this fact to decrease market value of its inherited portfolio or increase the market value of its newly-issued portfolio. Note that this result holds in any MPCE and does not appeal to any assumptions regarding the behavior of future governments.

Our result relies on the stable distribution of debt not being associated with first best consumption and labor. Under such a stable distribution, taxes are zero, the market value of debt is sufficiently negative to finance the stream of government spending forever, and the marginal benefit of resources for the government is zero. For this reason, the stable distribution of government debt maturity is undetermined in this circumstance. While such stable distribution potentially exists, we can rule such a stable distribution out if there are exogenous bounds on government debt which prevent such asset accumulation for the government.

Corollary 1 *Suppose that* $\underline{b} > -g$. *Then if conditional on* $\mathbf{B}$, *there exists a solution to* $(12)-(14)$ *with a stable debt maturity structure* $b'_k = b_k\ \forall k$, *it is necessary that* $b_k = \widehat{b}\ \forall k$ *so that the government debt maturity is flat.*

Finally, returning to Lemma 3, note that Proposition 2 also implies that starting from a flat government debt maturity, the unique continuation equilibrium involves a flat government debt maturity. Therefore, in any MPCE, a flat government debt maturity is an absorbing state.

Corollary 2 *Suppose that* $\mathbf{B}$ *satisfies* $b_k = b\ \forall k$ *for some* b *and that* $\{c, n\} \neq \{c^{fb}, n^{fb}\}$. *Then, in all solutions to* $(12)-(14)$ $b'_k = b\ \forall k$.

Starting from a flat government debt maturity, the current government would like to guarantee a constant level of consumption and labor going forward. Choosing a tilted maturity structure cannot guarantee such a continuation equilibrium going forward, since future governments will deviate from a smooth policy in order to relax the government budget constraint. For this reason, it chooses a flat maturity structure, and a flat maturity structure is an absorbing state.

6 Transition Path

We have established that the unique stable distribution of government debt maturity in any MPCE must be flat. In this section, we explore the transition path of debt maturity starting from a position which is not flat. A natural question concerns whether an MPCE can converge to a stable distribution over time. A complete analysis of MPCE's in an infinite horizon economy with an infinite choice of debt maturities is infeasible in the cases where the commitment and no-commitment solutions do not coincide; this is because techniques of Lucas and Stokey (1983) do not apply. Given this limitation, we analyze transitions using two examples: A three-period quasilinear economy which we theoretically characterize using backward induction and a T-period economy with more general preferences which we solve numerically. In both exercises, the initial debt maturity structure is declining in the horizon and maturities beyond a certain horizon are equal.

6.1 Three-Period Example

Suppose that preferences satisfy (15) and are quasilinear with $\gamma = 1$. The horizon is finite with $t = 0, 1, 2$. We impose bounds on government debt where $b_{t-1,k} \in \left[-g, c^{fb}\right]$ $\forall t, k$ for $c^{fb} = 1/\eta$ given by the preference structure.[9] In this setup, c^{laffer} defined in (16) satisfies

$$c^{laffer} = \frac{1 - \eta g}{2\eta}. \tag{36}$$

We consider an economy in which $b_{-1,1} \geq b_{-1,2} = b_{-1,3} \geq 0$. Thus, the initial debt maturity structure is declining in the horizon and maturities from date 1 onward are equal. We observe that in this environment, the solution under commitment admits $c_1 = c_2$, and this follows by analogous logic as in the example of Section 4. Furthermore, using the same arguments as in that section, we can construct examples in which initial debt $\{b_{-1,1}, b_{-1,2}, b_{-1,3}\}$ with elements within $\left[-g, c^{fb}\right]$ implies a solution under commitment with $c_1 = c_2 < c^{laffer}$.[10]

We characterize the path of consumption and debt using backward induction. At date 2, the government inherits debt $b_{1,1}$ and chooses a value of consumption which satisfies

[9] These debt limits are non-binding along the equilibrium path, but they allow us to characterize continuation equilibria off the equilibrium path.

[10] For example, if $b_{-1,1} = c^{fb} > b_{-1,2} = b_{-1,3} = 0$, we can show that $c_1 = c_2 < c^{laffer}$ if the discount factor β is sufficiently low.

the implementability condition:

$$c_2\left(1-\eta\left(c_2+g\right)\right)=b_{1,1}. \tag{37}$$

Conditional on $b_{1,1}$, the value of consumption c_2 satisfying (37) is unique since $c_2 \geq c^{laffer}$, where this follows by analogous logic as in Section 4. A government lacking commitment at date 2 would never choose a tax rate on the downward sloping portion of the Laffer curve; a tax rate on the upward sloping portion associated with $c_2 \in \left(c^{laffer}, c^{fb}\right)$ raises the same revenue and makes the government strictly better off.

Taking this into account, the government at date 1 maximizes welfare by solving the following problem, where we have substituted in for labor n_t using the resource constraint (1):

$$\max_{c_1,c_2,b_{1,1}} \sum_{t=1,2} \beta^{t-1}\left(\log c_t-\eta c_t\right) \tag{38}$$

s.t.

$$\sum_{t=1,2} \beta^{t-1}\left(1-\eta\left(c_t+g\right)-\frac{b_{0,t}}{c_t}\right)=0, \tag{39}$$

$$c_2\left(1-\eta\left(c_2+g\right)\right)=b_{1,1}, \tag{40}$$

$$b_{1,1} \in \left[-g, c^{fb}\right], \text{ and} \tag{41}$$

$$c_2 \geq c^{laffer}. \tag{42}$$

Note that analogous arguments to those of Section 4 imply that (39) can be written in relaxed form as a weak inequality constraint. The next lemma characterizes the solution to this problem by providing conditions on c_1 and c_2 which must hold given any inherited debt $\{b_{0,1}, b_{0,2}\}$ at date 1.

Lemma 7 *For any $\{b_{0,1}, b_{0,2}\}$, the solution to* (38) − (42) *satisfies the following weak inequality constraints:*

$$\begin{aligned} c_1+\beta c_2 &\geq c^{laffer}+\beta c^{laffer}, \text{ and} && (43)\\ c_2 &\geq c^{laffer}. && (44)\end{aligned}$$

This lemma provides necessary, but not sufficient, conditions to characterize the solution to (38) − (42). These conditions put a lower bound on the discounted sum of consumption from date 1 onward. (44) is clearly implied by (42) and stems from the lack of commitment at date 2.

Satisfaction of (43) stems from the lack of commitment at date 1. More specifically, (43) implies that it is not possible for $c_1 < c^{laffer}$ and $c_2 < c^{laffer}$; all future tax rates cannot be on the downward sloping portion of the Laffer curve. This is true even in the relaxed version of (38) – (42) which ignores (42). If it were the case that $c_1 < c^{laffer}$ and $c_2 < c^{laffer}$, then a deviation to $c_1 > c^{laffer}$ or $c_2 > c^{laffer}$ which raises the same revenue continues to satisfy the (relaxed) implementability condition (39) and strictly increases welfare.[11] In addition, note that (43) does not impose a clear lower bound on c_1; a solution to (38) – (42) could in principle admit $c_1 < c^{laffer}$, however this would require $c_2 > c^{laffer}$. Intuitively, a government at date 1 could choose a tax rate on the downward sloping portion of the Laffer curve at date 1 in order to increase short-term interest rates and buy back its outstanding long-term debt $b_{0,2}$ at a lower price, which would allow for a higher value of c_2.

Now consider the problem of the government at date 0. Using Lemma 7, we can consider the relaxed problem of the government at date 0:

$$\max_{c_0,c_1,c_2} \sum_{t=0,1,2} \beta^t \left(\log c_t - \eta c_t\right) \tag{45}$$

s.t.

$$\sum_{t=0,1,2} \beta^t \left(1 - \eta\left(c_t + g\right) - \frac{b_{-1,t+1}}{c_t}\right) = 0, \tag{46}$$

$$c_1 + \beta c_2 \geq c^{laffer} + \beta c^{laffer}, \text{ and} \tag{47}$$

$$c_2 \geq c^{laffer}. \tag{48}$$

This program corresponds to the date 0 program under commitment subject to additional constraints (47) and (48). Recall that the program under commitment admits a solution with $c_1 = c_2$. Note that this program which adds constraints (47) and (48) ignores the bounds on the date 0 government's debt issuance and only considers necessary, as opposed to sufficient, conditions for the values of c_1 and c_2 to be chosen by future governments. As such, welfare in the MPCE must be weakly below the solution to (45) – (48). In the Appendix, we characterize the solution to (45) – (48), and we verify that this solution corresponds to the unique MPCE in the three-period economy. This leads to the following main result of this section.

[11] More specifically, if the primary surplus at date 1 is below $b_{0,1}$, then a deviation to $c_1 > c^{laffer}$ relaxes (39) by reducing the short-term interest rate. If the primary surplus at date 1 exceeds $b_{0,1}$, then a deviation to $c_2 > c^{laffer}$ relaxes (39) by increasing the short-term interest rate.

Proposition 3 ***(transition to flat maturity)*** *The unique MPCE admits a sequence* $\{c_0, c_1, c_2\}$ *which satisfies* $(45)-(48)$ *and admits* $b_{0,1} = b_{0,2}$. *If the solution under commitment admits* $c_1 = c_2 < c^{laffer}$, *then the MPCE and commitment solution do not coincide, and the MPCE admits*

$$b_{0,1} = b_{0,2} = c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)\right). \tag{49}$$

This proposition states that the unique MPCE is characterized by the constrained date 0 problem $(45) - (48)$ and admits a flat maturity structure $b_{0,1} = b_{0,2}$, where these values of debt correspond to the natural debt limit characterized by (49) if the solution under commitment admits $c_1 = c_2 < c^{laffer}$.

In the case where the solution under commitment admits $c_1 = c_2 > c^{laffer}$ —so that (47) and (48) do not bind—this result is immediate and follows from the arguments in Lucas and Stokey (1983). Optimal tax rates under commitment mirror initial maturities, and are therefore stable beyond a particular horizon. This eventual stability is guaranteed with a transition to a flat maturity under no commitment, since otherwise the date 1 government would not choose $c_1 = c_2$.

In the cases where the commitment and no-commitment solutions do not coincide, the argument is more subtle. The date 0 government clearly desires a stable tax rate from date 1 onward given its initial maturities. However, if this desired tax rate exceeds the revenue-maximizing tax rate defining the peak of the Laffer curve, the date 0 government realizes that it cannot commit the date 1 and date 2 governments to its desired policy. Facing this binding upper bound on future tax rates captured by (47) and (48), the government chooses all future tax rates to equal the revenue-maximizing tax rate. To achieve this future outcome, it issues a flat maturity structure associated with the natural debt limit.

6.2 Robustness

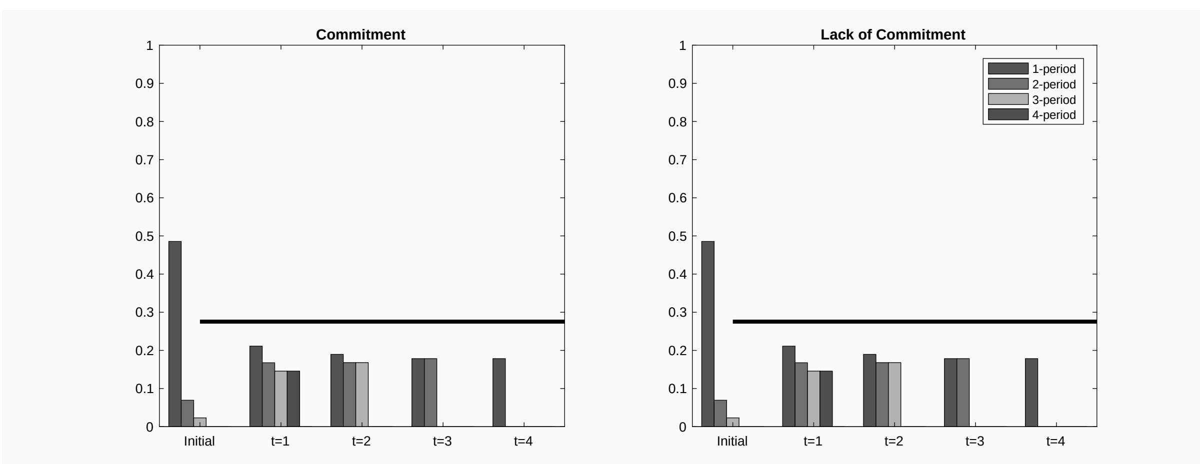
Extrapolating from Proposition 3 to other environments with more general preferences with $\gamma > 1$ and with a longer T-period horizon is challenging for two reasons. First, if $\gamma > 1$, the analog of Lemma 7 does not hold in a three-period environment, though one can derive non-linear, non-convex constraints which characterize the solution to the date 1 problem. The presence of these non-convex constraints makes it challenging to theoretically characterize the solution to the date 0 problem using backward induction. Second, even in the quasilinear setting with $\gamma = 1$, the analog of Lemma 7 cannot be established theoretically period by period in a longer T-period economy. In the three-period environment, the date 2 government's first order condition is sufficient to characterize the government's choice of consumption, even if the inherited debt $b_{1,1}$ is negative, and this facilitates backward induction. In a T-period economy, first order conditions are not sufficient to characterize allocations in other periods $t < T$ if the inherited debt is negative, since in these cases the implementability constraint at date t does not represent a convex constraint.

Given these challenges, we numerically simulate the transition path of a finite horizon economy with five periods ($t = 0, ..., 4$), where we solve for the government's strategy at each date using backward induction. We choose the following parameters:

$$\left\{\beta = .9808^{10}, g = .25, \eta = 0.8, \gamma = 2\right\}.$$

The value of β allows us to interpret each period as a 10 year interval and it implies a stable annual real interest rate of about 2 percent. We choose g so that in this economy government spending is 20 percent of GDP, in line with U.S. federal outlays since 1950. Our value of γ implies a Frisch elasticity of 1.[12]

Figure 1: Debt Maturity Structure with Low Initial Indebtedness



Notes: The figure displays the optimal maturity structure derived from the commitment solution as in Lucas and Stokey (1983) (left panel) and under lack of commitment (right panel). The solid line indicates the maximum attainable government primary surplus.

We consider the path of government debt maturity under two scenarios, one with a

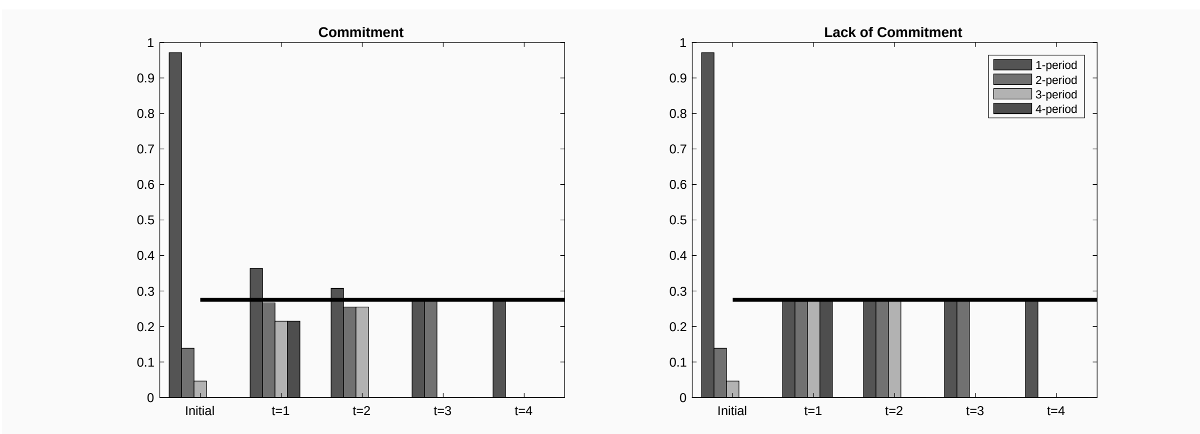
[12]The parameter η only scales the level of output and does not affect the results. The current parametrization implies that the first-best level of consumption is normalized to one.

low level of indebtedness and one with a high level of indebtedness. In both cases, 84 percent of the face value of debt is due at date 0, 12 percent is due at date 1, and 4 percent is due at date 3, which is roughly in line with the distribution of debt maturities in the U.S. economy.

Figure 1 depicts the path of debt in the first scenario of low initial indebtedness and shows that the debt converges to a flat maturity. Moreover, we can verify numerically that the solution under lack of commitment coincides with that under commitment.

Figure 2 depicts the path of debt in the second scenario of high initial indebtedness, where the market value of inherited debt from the perspective of date 0 is twice as much as in the first scenario. This figure shows that the debt converges to a flat maturity, and we can verify that this flat maturity coincides with the natural debt limit. In this case, the solution under lack of commitment does not coincide with that under commitment. These numerical results are consistent with the results in Proposition 3.

Figure 2: Debt Maturity Structure with High Initial Indebtedness



Notes: The figure displays the optimal maturity structure derived from the commitment solution as in Lucas and Stokey (1983) (left panel) and under lack of commitment (right panel). The solid line indicates the maximum attainable government primary surplus.

7 Concluding Remarks

We have analyzed optimal fiscal policy under lack of commitment and established that such a policy may not coincide with that under commitment, even if the government can issue a rich maturity structure of debt. Motivated by this finding—which contrasts with previous results in the literature—we have analyzed optimal fiscal policy under lack of commitment in a framework which encompasses the cases in which the commitment and no-commitment solutions do not coincide. We have shown that the unique stable maturity structure of government debt is flat, with the government owing the same amount of resources to the private sector at all future dates. In addition, we provide examples in which the optimal maturity structure under lack of commitment converges to this flat distribution. In the examples where the commitment and no-commitment solutions do not coincide, debt converges to the natural debt limit.

Our analysis thus provides a theoretical argument for the use of consols in debt management based on the sequential optimization of fiscal policy by the government. The use of consols has been pursued historically, most notably by the British government during the Industrial Revolution, when consols were the largest component of the British government's debt (see Mokyr, 2011). Moreover, the introduction of consols has been discussed as a potential option in the management of U.S. government debt (e.g. Cochrane, 2015).

Our analysis leaves several interesting avenues for future research. We have considered a situation in which a government's objective in its debt issuance strategy is to minimize its financing costs. In practice, government debt management offices also pursue other objectives, such as supporting financial stability. For example, this can be achieved either by providing liquidity to segments of the market which lack it or through the bond auction process which itself may serve a purpose of aggregating financial market information. How these factors matter for the optimal maturity management of government debt is an interesting question for future research.

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Appendix

A.1 Proofs for Section 3

Proof of Lemma 1

The necessity of these conditions is proved in the text. To prove sufficiency, let the government choose the associated level of debt $\left\{\{b_{t,k}\}_{k=1}^{\infty}\right\}_{t=0}^{\infty}$ which satisfies (9) and a tax sequence $\{\tau_t\}_{t=0}^{\infty}$ which satisfies (8). Let bond prices satisfy (8). (9) given (1) implies that (3) and (4) are satisfied. Therefore household optimality holds and all dynamic budget constraints are satisfied along with the market clearing, so the equilibrium is competitive.■

B.2 Proofs for Section 4

B.1 Proof of Lemma 2

We prove this lemma in four steps.

Step 1. We first establish that the problem is concave and the solution unique. Consider the relaxed problem in which (18) is replaced with

$$1 - \frac{b_{-1,1}}{c_0} - \eta\left(c_0 + g\right)^{\gamma} + \sum_{t=1}^{\infty} \beta^t \left(1 - \eta\left(c_t + g\right)^{\gamma}\right) \geq 0. \tag{B.1}$$

We can establish that $(B.1)$ holds as an equality in the relaxed problem, implying that the relaxed and constrained problems are equivalent. If instead $(B.1)$ held as an inequality in the relaxed problem, the solution to the relaxed problem would admit $c_t = c^{fb}\ \forall t$. Given (15), c^{fb} satisfies $\eta c^{fb}\left(c^{fb} + g\right)^{\gamma-1} = 1$, and substitution of $c_t = c^{fb}$ into $(B.1)$ yields

$$\frac{1}{c^{fb}}\left(-b_{-1,1} - \frac{1}{1-\beta}g\right) \geq 0$$

which is a contradiction since $b_{-1,1} > 0$. Therefore, $(B.1)$ holds as an equality in the solution to the relaxed problem and the solutions to the relaxed and constrained problems coincide. Since the left hand side of $(B.1)$ is strictly concave in c_t given that $b_{-1,1} > 0$ and since the objective (17) is strictly concave, it follows that the solution is unique.

Step 2. We now establish the first two parts of the lemma. Letting $\mu_0 > 0$ correspond to the Lagrange multiplier on $(B.1)$, the first order condition for c_0 is (19). The first order condition for c_t for all $t \geq 1$ is

$$\frac{1}{c_t} - \eta\left(c_t + g\right)^{\gamma-1} + \mu_0\left(-\eta\gamma\left(c_t + g\right)^{\gamma-1}\right) = 0. \tag{B.2}$$

Since the left hand side of $(B.2)$ is strictly decreasing in c_t, it follows that the solution to $(B.2)$ is unique with $c_t = c_1$ $\forall t \geq 1$, where (20) defines c_1. It follows from the fact that the program is strictly concave and constraint set convex that satisfaction of $(19) - (21)$ is necessary and sufficient for optimality for a given $\mu_0 > 0$. We are left to verify that $c_0 > c_1$. Note that the left hand side of (19) is strictly increasing in $b_{-1,1}$ and strictly decreasing in c_0 for a given $\mu_0 > 0$. Therefore, c_0 is strictly increasing in $b_{-1,1}$ for a given $\mu_0 > 0$, where $c_0 = c_1$ if $b_{-1,1} = 0$. It follows then that since $b_{-1,1} > 0$, $c_0 > c_1$.

Step 3. We now establish the last part of the lemma. We first show that the solution to the system in $(19)-(21)$ admits c_1 which is strictly decreasing in $b_{-1,1}$. Let $F^0\left(c_0, \mu_0, b_{-1,1}\right)$ correspond to the function on the left hand side of (19), let $F^1\left(c_1, \mu_0\right)$ correspond to the function on the left hand side of (20), and let $I\left(c_0, c_1, b_{-1,1}\right)$ correspond to the function on the left hand side of (21). Since the solution to this system of equations is unique, we can apply the Implicit Function Theorem. Implicit differentiation yields

$$\frac{dc_1}{db_{-1,1}} = \frac{-F^0_{c_0} I_{b_{-1,1}} + F^0_{b_0} I_{c_0}}{F^0_{c_0} I_{c_1} + \frac{F^0_{\mu_0} F^1_{c_1} I_{c_0}}{F^1_{\mu_0}}}. \tag{B.3}$$

From the second order condition for (19) and (20), $F^0_{c_0} < 0$ and $F^1_{c_1} < 0$. Moreover, by inspection, $I_{c_1} < 0$ and $F^1_{\mu_0} < 0$. Finally, note that $F^0_{\mu_0} I_{c_0} = \left[I_{c_0}\right]^2 > 0$. This establishes that the denominator in $(B.3)$ is positive. To determine the sign of the numerator, let us expand the numerator by substituting in for the functions. By some algebra, the numerator is equal to

$$\frac{1}{c_0}\left(-\frac{1}{c_0^2} - \eta\left(\gamma - 1\right)\left(c_0 + g\right)^{\gamma-2}\right) + \mu_0\left[-\frac{b_{-1,1}}{c_0^4} - \frac{1}{c_0}\eta\gamma\left(\gamma-1\right)\left(c_0+g\right)^{\gamma-2} - \frac{1}{c_0^2}\eta\gamma\left(c_0+g\right)^{\gamma-1}\right] < 0.$$

This establishes that c_1 is strictly decreasing in $b_{-1,1}$.

Step 4. Given step 3, we complete the proof of the last part of the lemma by establishing that there exists $b^*_{-1,1} > 0$ for which the solution to $(19) - (21)$ admits $c_1 = c^{laffer}$. We first establish that $b^*_{-1,1}$ exceeds 0. Note that if $b_{-1,1} = 0$ then the solution admits $c_1 > c^{laffer}$. This is because $(19) - (21)$ imply that the solution admits $c_0 = c_1$. Substitution into (21) yields

$$\frac{c_1\left(1-\eta\left(c_1+g\right)^{\gamma}\right)}{1-\beta}=0. \tag{B.4}$$

This equation admits two solutions: $c_1 = 0$ and $c_1 = \eta^{-1/\gamma} - g$, and the optimal policy satisfies $c_1 = \eta^{-1/\gamma} - g$, since welfare is arbitrarily low otherwise. Given the definition of c^{laffer} in (16) and the strict concavity of the objective in (16), it follows that c^{laffer} must be between 0 and $\eta^{-1/\gamma} - g$, which means that $c_1 > c^{laffer}$.

Now let us consider the system of equations (19)$-$(21) for $b_{-1,1} = b^*_{-1,1}$ and $c_1 = c^{laffer}$. To see that this solution exists, note that $\frac{1}{c^{laffer}} - \eta\left(c^{laffer}+g\right)^{\gamma-1} > 0$ since $c^{laffer} < c^{fb}$. Therefore, a value of $\mu_0 > 0$ which satisfies (20) exists. Multiply (19) by c_0 and substitute (21) into (19) to achieve

$$1-\eta c_0\left(c_0+g\right)^{\gamma-1}+\mu_0\left(1-\eta\left(c_0\left(1+\gamma\right)+g\right)\left(c_0+g\right)^{\gamma-1}+\frac{\beta}{1-\beta}\left(1-\eta\left(c^{laffer}+g\right)^{\gamma}\right)\right)=0. \tag{B.5}$$

Note that given the value of $\mu_0 > 0$ satisfying (20) for $c_1 = c^{laffer}$, a solution to $(B.5)$ which admits $c_0 > 0$ exists. This is because the left hand side of $(B.5)$ goes to

$$1+\mu_0\left(1-\eta g^{\gamma}+\frac{\beta}{1-\beta}\left(1-\eta\left(c^{laffer}+g\right)^{\gamma}\right)\right)>0$$

as c_0 goes to 0, where we have used the fact that $g < \left(\frac{1}{\eta}\right)^{1/\gamma}$. As c_0 goes to infinity, the left hand side of $(B.5)$ becomes arbitrarily negative. Therefore a solution to $(B.5)$ for $c_0 > 0$ exists. Given that $b_{-1,1}$ enters linearly in (21), it follows that a value of $b_{-1,1}$ which satisfies the system also exists. This establishes the last part of the lemma.■

B.2 Proof of Proposition 1

We consider each case separately.

Case 1. Suppose that $b_{-1,1} < b^*_{-1,1}$. From Lemma 2, the date 0 solution admits $c_t = c_1 > c^{laffer}$ $\forall t \geq 1$. To show that this solution is time-consistent, suppose that the date 0 government chooses $\{b_{0,k}\}_{k=1}^{\infty}$ satisfying

$$b_{0,k} = c_1\left(1-\eta\left(c_1+g\right)^{\gamma}\right) > 0 \; \forall k \geq 1 \tag{B.6}$$

for c_1 defined in (19) $-$ (21). The fact that $b_{0,k} > 0$ follows from the fact that the highest value of $c_1 > c^{laffer}$ is below that associated with $b_{-1,1} = 0$ which satisfies $(B.4)$. Now consider the solution to (22) $-$ (23). Analogous arguments as those in steps 1 and 2 of the proof of Lemma 2 imply that the unique solution satisfies (23) and (24) for some $\mu_1 > 0$. Therefore, to check that the date 1 solution admits $c_t = c_1 \ \forall t \geq 1$ for c_1 which satisfies (20), it is sufficient to check that there exists some $\mu_1 > 0$ satisfying (24). Using $(B.6)$ to substitute in for $b_{0,k}$ in (24), we find that

$$\mu_1 = -\frac{1 - \eta c_1 (c_1 + g)^{\gamma - 1}}{1 - \eta (c_1 + g)^{\gamma} - \eta \gamma c_1 (c_1 + g)^{\gamma - 1}} > 0, \tag{B.7}$$

where we have appealed to the fact that $c_1 < c^{fb}$ (from (20)) to assign a positive sign to the numerator in $(B.7)$ and the fact that $c_1 > c^{laffer}$ to assign a negative sign to the denominator in $(B.7)$. This establishes that the date 0 solution is time-consistent.

Case 2. Suppose by contradiction that the optimal date 0 policy is time-consistent. This would require (24) to hold for $c_t = c_1 \ \forall t \geq 1$ for $c_1 < c^{laffer}$ which satisfies (20). For a given μ_1, satisfaction of (24) thus requires that $b_{0,k} = b_{0,1} \ \forall k \geq 1$. Equation (23) thus implies that $(B.6)$ for $b_{0,k} > 0$ holds, and substitution of $(B.6)$ into (24) implies that

$$\mu_1 = -\frac{1 - \eta c_1 (c_1 + g)^{\gamma - 1}}{1 - \eta (c_1 + g)^{\gamma} - \eta \gamma c_1 (c_1 + g)^{\gamma - 1}} < 0, \tag{B.8}$$

where we have appealed to the fact that $c_1 < c^{fb}$ (from (20)) to assign a positive sign to the numerator and the fact that $c_1 < c^{laffer}$ to assign a positive sign to the denominator. However, conditional on $\{b_{0,k}\}_{k=1}^{\infty}$ for $b_{0,k} = b_{0,1} > 0 \ \forall k \geq 1$, the solution to $(22) - (23)$ must admit a positive multiplier $\mu_1 > 0$, and this follows by analogous arguments as those in step 1 in the proof of Lemma 2, which contradicts $(B.8)$. Therefore, the date 1 solution does not coincide with the date 0 solution.■

C.3 Proofs for Section 5

Proof of Lemma 3

Note that if $b_k = b \ \forall k$, then from Assumption 1, the solution under commitment admits $\{c_t, n_t\} = \{c^* (b), n^* (b)\} \ \forall t$, and this solution can be implemented with $b'_k = b$ given (30)–(31). Since the MPCE satisfies the same constraints of the problem under commitment plus additional constraints regarding sequential optimality, it follows that

$$W(\mathbf{B}) = \frac{u(c^*(b), n^*(b))}{1 - \beta} \geq V(\mathbf{B}) \tag{C.9}$$

if $b_k = b \, \forall k$. Now consider optimal policy under the MPCE in $(12) - (14)$ given $b_k = b \, \forall k$. A government has the option of choosing $c = c^* (b)$ and $n = n^* (b)$ together with $b'_k = b$ $\forall k$. This satisfies the resource constraint (13) and the implementability constraint (14). Therefore, it follows that

$$V (\mathbf{B}) \geq u (c^* (b), n^* (b)) + \beta V (\mathbf{B}) . \tag{C.10}$$

Equations $(C.9)$ and $(C.10)$ imply that

$$V (\mathbf{B}) = W (\mathbf{B}) . \tag{C.11}$$

By Assumption 1, $W(\mathbf{B})$ is uniquely characterized by $\{c_k, n_k\} = \{c^* (b), n^* (b)\} \; \forall k$. Therefore, it follows that any solution to $(12) - (14)$ given $b_k = b \; \forall k$ admits $c = c^* (b)$ and $n = n^* (b)$.■

Proof of Lemma 4

Conditional on $\mathbf{B}$, if a solution admits $b'_k = b_k$, then this means that $\mathbf{B}$ is an absorbing state with $\mathbf{B} = \mathbf{B}'$ and consumption and labor are constant and equal to some $\{c, n\}$ from that period onward. Therefore, $h^k (\mathbf{B}') = \beta^k u_c (c, n) \; \forall k \geq 1$ for $h^k (\mathbf{B}')$ defined in (11). As such, (14) can be rewritten as

$$u_c (c, n) c + u_n (c, n) n - u_c (c, n) b_1 + u_c (c, n) \sum_{k=1}^{\infty} \beta^k (b'_k - b_{k+1}) = 0 \tag{C.12}$$

which combined with (33) and the fact that $b'_k = b_k$ implies that

$$u_c (c, n) c + u_n (c, n) n = u_c (c, n) \widehat{b}. \tag{C.13}$$

Now consider the solution to the following problem given $\widehat{b}$:

$$\max_{c,n} \frac{u (c, n)}{1 - \beta} \text{ s.t. } c + g = n \text{ and } (C.13) . \tag{C.14}$$

It is necessary that $V (\mathbf{B})$ be weakly below the value of $(C.14)$. This is because the solution to $V (\mathbf{B})$ also admits a constant consumption and labor (as in the program in $(C.14)$) and since the constraint set in $(C.14)$ is slacker, since the program ignores the role of government debt in changing future policies. Note furthermore that the value of

$(C.14)$ equals $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k}$, where this follows from Assumption 1. Therefore,

$$V\left(\mathbf{B}\right) \leq W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k}. \tag{C.15}$$

Now consider the welfare of the government in the MPCE if, instead of choosing $b'_k = b_k$ $\forall k$, it instead chooses $b'_k = \widehat{b}\ \forall k$ with $c = c^*(\widehat{b})$ and $n = n^*(\widehat{b})$. It follows from Lemma 3 that under this perturbation, $h^k(\mathbf{B}') = \beta^k u_c(c^*(\widehat{b}), n^*(\widehat{b}))\ \forall k \geq 1$, which implies that the resource constraint (13) and implementability constraint (14) are satisfied under this deviation. Because the continuation value associated with this deviation is $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k}$, it follows that for this deviation to be weakly dominated:

$$W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k} \leq V\left(\mathbf{B}\right). \tag{C.16}$$

Given $(C.15)$ and $(C.16)$, it follows that $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k} = V\left(\mathbf{B}\right)$. Therefore, given $\mathbf{B}$, there exists another solution to $(12)-(14)$ with $b'_k = \widehat{b}\ \forall k$ which achieves the same welfare.■

Proof of Lemma 5

Before proving this lemma, define c^{laffer} analogously as in Section 4:

$$c^{laffer} = \arg\max_{c}\left\{c + \frac{u_n(c, c+g)}{u_c(c, c+g)}(c+g)\right\}, \tag{C.17}$$

and let b^{laffer} correspond to the value of the maximized objective in $(C.17)$. It follows that a solution to $(27)-(29)$ exists if $b_k = b\ \forall k \geq 1$ if $b \leq b^{laffer}$.

Given this definition, we can proceed to prove this lemma by contradiction. By Lemma 4,

$$V\left(\mathbf{B}\right) = W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widehat{b}\ \forall k} = \frac{u(c^*(\widehat{b}), n^*(\widehat{b}))}{1-\beta} \tag{C.18}$$

for $\widehat{b}$ defined in (33). Now suppose that $b_1 \neq \widehat{b}$. Given the definition of $\widehat{b}$, this means that $\widehat{b} \in (\underline{b}, \bar{b})$ and that $\widehat{b} \leq b^{laffer}$. We consider two cases separately.

Case 1. Suppose that $\widehat{b} < b^{laffer}$, and suppose that the government locally deviates to $b'_k = \widetilde{b} \neq \widehat{b}\ \forall k$ so that from tomorrow onward, consumption is $c^*(\widetilde{b})$ and labor is $n^*(\widetilde{b})$, where this follows from Lemma 3. This means that $h^k(\widetilde{\mathbf{B}}) = \beta^k u_c(c^*(\widetilde{b}), n^*(\widetilde{b}))$ under the deviation. In order to satisfy the resource constraint and implementability condition, let the government deviate today to a consumption and labor allocation $\{\widetilde{c}, \widetilde{n}\}$ which satisfies

$$\widetilde{c} + g = \widetilde{n} \tag{C.19}$$

and

$$\begin{aligned} u_c(\widetilde{c}, \widetilde{n})\widetilde{c} + u_n(\widetilde{c}, \widetilde{n})\widetilde{n} - (u_c(\widetilde{c}, \widetilde{n}) - u_c(c^*(\widetilde{b}), n^*(\widetilde{b})))b_1 = \\ u_c(c^*(\widetilde{b}), n^*(\widetilde{b}))\left(\widehat{b} + \frac{\beta}{1-\beta}(\widehat{b} - \widetilde{b})\right) \end{aligned} \tag{C.20}$$

where we have appealed to the definition of $\widehat{b}$ in (33). For such a deviation to be weakly dominated, it must be that

$$V(\mathbf{B}) \geq u(\widetilde{c}, \widetilde{n}) + \beta W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k = \widetilde{b}\ \forall k}. \tag{C.21}$$

Clearly, the value of the right hand side of $(C.21)$ equals $V(\mathbf{B})$ if $\widetilde{b} = \widehat{b}$. Therefore, it must be that $\widetilde{b} = \widehat{b}$ with $\{\widetilde{c}, \widetilde{n}\} = \{c^*(\widehat{b}), n^*(\widehat{b})\}$ maximizes the right hand side of $(C.21)$ subject to $(C.19)$, and $(C.20)$. More specifically, we can consider the solution to the following program

$$\max_{\widetilde{c}, \widetilde{n}, \widetilde{b}} u(\widetilde{c}, \widetilde{n}) + \beta W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k = \widetilde{b}\ \forall k} \text{ s.t. } (C.19) \text{ and } (C.20). \tag{C.22}$$

For the deviation to not strictly increase welfare, $\widetilde{b} = \widehat{b}$ must be a solution to $(C.22)$. By Assumption 1, $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k = \widetilde{b}\ \forall k} = u(c^*, n^*)/(1-\beta)$ where $\{c^*, n^*\} = \{c^*(\widetilde{b}), n^*(\widetilde{b})\}$ are the unique levels of consumption and labor which maximize welfare given $\widetilde{b}$ and are defined in (30) and (31). Letting μ_1 represent the Lagrange multiplier on the implementability condition for the program defining $W\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k = \widetilde{b}\ \forall k}$ in $(27) - (29)$, it follows from first order conditions that

$$\begin{aligned} & u_c(c^*, n^*) + u_n(c^*, n^*) + \\ \mu_1 & \begin{pmatrix} u_c(c^*, n^*) + u_n(c^*, n^*) \\ +u_{cc}(c^*, b^*)(c^* - \widetilde{b}) + u_{cn}(c^*, n^*)(c^* - \widetilde{b} + n^*) + u_{nn}(c^*, n^*)n^* \end{pmatrix} = 0. \end{aligned} \tag{C.23}$$

Since $\{c^*, n^*\} \neq \{c^{fb}, n^{fb}\}$ by the statement of the lemma, $(C.23)$ implies that $\mu_1 \neq 0$. Using this observation, implicit differentiation of (30) and (31) taking $(C.23)$ into account implies

$$c^{*\prime}(\widetilde{b}) = n^{*\prime}(\widetilde{b}) = -\mu_1 \frac{u_c(c^*, n^*)}{u_c(c^*, n^*) + u_n(c^*, n^*)}. \tag{C.24}$$

Finally, by the Envelope condition,

$$\frac{dW\left(\{b_k\}_{k=1}^{\infty}\right)|_{b_k=\widetilde{b}\ \forall k}}{d\widetilde{b}} = -\mu_1 \frac{u_c\left(c^*, n^*\right)}{1-\beta}. \tag{C.25}$$

Now consider the solution to $(C.22)$. Let μ_0 correspond to the Lagrange multiplier on $(C.20)$. First order conditions with respect to $\widetilde{c}$ and $\widetilde{n}$ imply

$$u_c(\widetilde{c},\widetilde{n}) + u_n(\widetilde{c},\widetilde{n}) + \tag{C.26}$$

$$\mu_0 \left(\begin{array}{c} u_c(\widetilde{c},\widetilde{n}) + u_n(\widetilde{c},\widetilde{n}) \\ +u_{cc}(\widetilde{c},\widetilde{n})(\widetilde{c}-b_1) + u_{cn}(\widetilde{c},\widetilde{n})(\widetilde{c}-b_1+\widetilde{n}) + u_{nn}(\widetilde{c},\widetilde{n})\widetilde{n} \end{array} \right) = 0.$$

Since $\{\widetilde{c},\widetilde{n}\} \neq \{c^{fb}, n^{fb}\}$ by the statement of the lemma, $(C.26)$ implies that $\mu_0 \neq 0$. Since the solution admits $\widetilde{b} = \widehat{b} \in \left(\underline{b}, \overline{b}\right)$, then we can ignore the bounds on government debt, and first order conditions with respect to $\widetilde{b}$ taking into account $(C.24)$ and $(C.25)$ yields

$$\mu_0\mu_1 \frac{u_{cc}(c^*,n^*) + u_{cn}(c^*,n^*)}{u_c(c^*,n^*) + u_n(c^*,n^*)} \left(\widehat{b} - b_1 + \frac{\beta}{1-\beta}(\widehat{b} - \widetilde{b})\right) + \frac{\beta}{1-\beta}(\mu_0 - \mu_1) = 0. \tag{C.27}$$

Note that $(C.23)$ and $(C.26)$ imply that

$$\frac{\beta}{1-\beta}(\mu_0 - \mu_1) = \frac{\beta}{1-\beta}\mu_0\mu_1 \frac{u_{cc}(c^*,n^*) + u_{cn}(c^*,n^*)}{u_c(c^*,n^*) + u_n(c^*,n^*)}(\widetilde{b} - b_1) \tag{C.28}$$

Now consider if $\widetilde{b} = \widehat{b}$ so that $\{\widetilde{c},\widetilde{n}\} = \{c^*, n^*\}$. In that case, use $(C.28)$ to substitute into $(C.27)$ to achieve:

$$\mu_0\mu_1 \frac{u_{cc}(c^*,n^*) + u_{cn}(c^*,n^*)}{u_c(c^*,n^*) + u_n(c^*,n^*)}(\widehat{b} - b_1) = 0. \tag{C.29}$$

If it were the case that $\widehat{b} \neq b_1$, then $(C.29)$ would require that $u_{cc}(c^*,n^*) + u_{cn}(c^*,n^*) = 0$, which contradicts the fact that $u_{cc}(c^*,n^*) + u_{cn}(c^*,n^*) < 0$. Therefore, $\widehat{b} = b_1$.

Case 2. Suppose that $\widehat{b} = b^{laffer}$. In this case, consider an analogous perturbation as in case 1 which reduces $\widehat{b}$ locally. For such a perturbation to be weakly dominated, the analog of $(C.29)$ requires

$$\mu_0\mu_1 \frac{u_{cc}(c^*,n^*) + u_{cn}(c^*,n^*)}{u_c(c^*,n^*) + u_n(c^*,n^*)}(\widehat{b} - b_1) \geq 0 \tag{C.30}$$

It follows from $(C.25)$ that $\mu_1 > 0$ since any reduction in inherited debt can facilitate higher consumption and higher welfare. Since $u_{cc}(c^*, n^*) + u_{cn}(c^*, n^*) < 0$, satisfaction of $(C.30)$ requires

$$\mu_0(\widehat{b} - b_1) \leq 0. \tag{C.31}$$

Given that $\{\widetilde{c}, \widetilde{n}\} = \{c^*, n^*\} = \{c^{laffer}, c^{laffer} + g\}$, it can be verified from $(C.17)$ that if $b_1 < (>)\ \widehat{b} = b^{laffer}$, then $(C.26)$ implies that $\mu_0 > (<)\ 0$. This follows from the fact that $c^{laffer} < c^{fb}$ and the term in parentheses multiplying μ_0 in equation $(C.26)$ is equal to 0 if $b_1 = b^{laffer}$ and is increasing in b_1. Therefore, $(C.31)$ cannot hold unless $\widehat{b} = b_1$.$\blacksquare$

Proof of Lemma 6

Suppose that $b_l = \widehat{b}\ \forall l \leq m$. Given $\mathbf{B}$, let $\widehat{\mathbf{B}}(1)$ represent the portfolio which sets $\widehat{b}_k = b_{k+1}$ so that no retrading takes place. Note that in such a portfolio, $\widehat{b}_1 = b_2$. Define $\widehat{\mathbf{B}}(2)$ analogously as the portfolio involving no retrading at the next date, so that $\widehat{b}_k = b_{k+2}$ under $\widehat{\mathbf{B}}(2)$, and define $\widehat{\mathbf{B}}(l)\ \forall l \leq m$ analogously. In any MPCE for which $b_1 = \widehat{b}$, a possible deviation sets $\{c, n\} = \{c^*(\widehat{b}), n^*(\widehat{b})\}$ and $b'_k = b_{k+1}$ so that no retrading takes place, where this deviation satisfies the resource constraint and implementability condition given $(30) - (31)$. For such a deviation to be weakly dominated, it is necessary that:

$$V(\mathbf{B}) \geq u(c^*(\widehat{b}), n^*(\widehat{b})) + \beta V(\widehat{\mathbf{B}}(1)). \tag{C.32}$$

Forward induction on this argument implies that

$$V(\mathbf{B}) \geq \sum_{l=0}^{m-1} \beta^l u(c^*(\widehat{b}), n^*(\widehat{b})) + \beta^m V(\widehat{\mathbf{B}}(m)). \tag{C.33}$$

Combining $(C.18)$ with $(C.33)$, we achieve

$$V(\mathbf{B}) \geq V(\widehat{\mathbf{B}}(m)). \tag{C.34}$$

Now consider optimal policy starting from $\widehat{\mathbf{B}}(m)$. Note that since $b_l = \widehat{b}\ \forall l \leq m$, then following the same arguments as in the proof of Lemma 4, a feasible strategy starting from $\widehat{\mathbf{B}}(m)$ is to issue a flat debt maturity with all bonds equal to $\widehat{b}$. Such a strategy ensures a constant consumption and labor allocation forever equal to $\{c^*(\widehat{b}), n^*(\widehat{b})\}$. As such, it follows that $(C.34)$ holds with equality and that choosing a flat maturity structure going forward is optimal.

Now we prove by contradiction that $b_{m+1} = \widehat{b}$. Suppose it were the case that $b_{m+1} \neq \widehat{b}$. This means that starting from $\widehat{\mathbf{B}}(m)$, the immediate debt which is owed by the government does not equal $\widehat{b}$. If this is the case, then the same arguments as those in the proof of Lemma 5 imply that there exists a deviation from the government's equilibrium strategy at $\widehat{\mathbf{B}}(m)$ which can strictly increase the government's welfare. However, if this is the case, $(C.34)$ which holds with equality is violated. Therefore, it must be that $b_{m+1} = \widehat{b}$.■

Proof of Proposition 2 and Corollaries 1 and 2

The proof of Proposition 2 follows directly by induction after appealing to Lemmas 5 and 6.

To prove the first corollary, note that for the statement of Proposition 2 to be false, it is necessary that $\{c, n\} = \{c^{fb}, n^{fb}\}$. However, if this is the case, then $(C.12)$ implies that

$$c^{fb} + \frac{u_n(c^{fb}, n^{fb})}{u_c(c^{fb}, n^{fb})} n^{fb} = -g = \sum_{k=1}^{\infty} \beta^{k-1} (1-\beta) b_k \geq \underline{b} \tag{C.35}$$

which contradicts $b_k > -g$.

To prove the second corollary, note that from Lemma 3, it is necessary that the continuation equilibrium starting from a flat government debt maturity entail consumption and labor equal to $\{c^*(b), n^*(b)\}$ forever. The arguments in the proof of Lemmas 5 and 6 imply that if the government were to choose a non-flat maturity structure going forward, future governments would not choose $\{c^*(b), n^*(b)\}$ forever. Therefore, all solutions to $(12) - (14)$ admit $b'_k = b\ \forall k$.■

D.4 Proofs of Section 6

D.1 Proof of Lemma 7

(44) is implied by (42). Let us consider a relaxed representation of $(38) - (42)$ in which we ignore (41) and we replace (39) with a weak inequality constraint:

$$\sum_{t=1,2} \beta^{t-1} \left(1 - \eta\,(c_t + g) - \frac{b_{0,t}}{c_t}\right) \geq 0. \tag{D.36}$$

We establish that the solution to the relaxed problem satisfies (43) and we verify that (39) and (41) are satisfied under this solution.

Step 1. Analogous arguments as in step 1 of Lemma 2 imply that since $b_{0,1} \geq -g$ and $b_{0,2} \geq -g$, $(D.36)$ holds as an equality in the relaxed problem. Note furthermore that if $b_{0,1} \geq 0$ and $b_{0,2} \geq 0$, the constraint set is convex so that the solution is unique.

Step 2. The choice of c_1 is characterized by the following first order condition for $\mu_1 \geq 0$ which represents the Lagrange multiplier on $(D.36)$:

$$\frac{1}{c_1} - \eta = \mu_1 \left(\eta - \frac{b_{0,1}}{c_1^2} \right). \tag{D.37}$$

We can show that

$$\eta - \frac{b_{0,1}}{c_1^2} \geq 0. \tag{D.38}$$

Suppose that $(D.38)$ does not hold. In that case, $b_{0,1} > 0 > -g$, which means that $(D.36)$ is a binding constraint with $\mu_1 > 0$, implying that the right hand side of $(D.37)$ is negative. For the left hand side of $(D.37)$ to be negative, this requires that $c_1 > c^{fb}$. Substituting this fact back into the right hand side of $(D.37)$, this means that

$$b_{0,1} > \eta c_1^2 > c^{fb},$$

which is a contradiction since $b_{0,1} \leq c^{fb}$. Therefore, $(D.38)$ is necessary.

Step 3. The choice of c_2 is characterized by the following first order condition for $\kappa \geq 0$ which represents the Lagrange multiplier on (42):

$$\frac{1}{c_2} - \eta + \kappa = \mu_1 \left(\eta - \frac{b_{0,2}}{c_2^2} \right). \tag{D.39}$$

Analogous arguments using this condition as in step 2 imply that

$$\eta - \frac{b_{0,2}}{c_2^2} \geq 0. \tag{D.40}$$

Combination of $(D.38)$ and $(D.40)$ taking into account the definition of c^{laffer} in (36) and $(D.36)$ which binds and implies (43).

Step 4. We are left to check that (41) is satisfied. There are two cases to consider.

Case 1. Suppose that (42) holds as an equality in the solution to the relaxed problem. In that case, $c_2 = c^{laffer}$ and (40) implies that

$$b_{1,1} = c^{laffer} \left(1 - \eta \left(c^{laffer} + g\right)\right) \in \left(-g, c^{fb}\right)$$

so that (41) is satisfied.

Case 2. Suppose that (42) holds as a strict inequality in the solution to the relaxed problem. In this case, the solution is characterized by $(D.37)$ and $(D.39)$ for $\kappa = 0$. Given $(D.38)$ and $(D.40)$, it follows that $(D.37)$ and $(D.39)$ can only be satisfied for $c_1 \leq c^{fb}$

and $c_2 \leq c^{fb}$. Since $c_2 > c^{laffer}$, (40) implies that

$$b_{1,1} < c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)\right) < c^{fb}.$$

To check that $b_{1,1} \geq -g$, suppose it were the case that the solution to the relaxed problem admitted $b_{1,1} < -g$. In this case, satisfaction of (40) would require that $c_2 > c^{fb}$ which contradicts the fact that $c_2 < c^{fb}$. Therefore, (41) is satisfied.■

D.2 Proof of Proposition 3

We proceed by characterizing the solution to the relaxed problem in (45) – (48). We then complete the proof by showing that the solution to this problem is the unique MPCE which admits $b_{0,1} = b_{0,2}$.

Step 1. Let us consider the relaxed version of (45) – (48) which ignores (48) and which replaces (46) with a relaxed constraint

$$\sum_{t=0,1,2} \beta^t \left(1 - \eta\left(c_t + g\right) - \frac{b_{-1,t+1}}{c_t}\right) \geq 0. \tag{D.41}$$

Analogous arguments as in step 1 of Lemma 2 imply that since $b_{-1,t} \geq 0\ \forall t$, $(D.41)$ is a binding constraint in the relaxed problem and admits a positive Lagrange multiplier $\mu_0 > 0$. Let us consider the solution to this relaxed problem and then verify that constraint (48) is satisfied.

Step 2. Analogous arguments as in step 2 of Lemma 2 imply that since $b_{-1,t} \geq 0$, the constraint set in the relaxed problem is convex, which means that first order conditions with respect to c_t are necessary and sufficient to characterize the unique optimum. Letting $\beta\phi$ correspond to the Lagrange multiplier on constraint (47), first order conditions are:

$$\frac{1}{c_0} - \eta = \mu_0\left(\eta - \frac{b_{-1,1}}{c_0^2}\right), \text{ and} \tag{D.42}$$

$$\frac{1}{c_t} - \eta + \phi = \mu_0\left(\eta - \frac{b_{-1,t+1}}{c_0^2}\right) \text{ for } t = 1, 2. \tag{D.43}$$

$(D.42) - (D.43)$ imply that since $b_{-1,2} = b_{-1,3}$, the solution admits $c_1 = c_2$. This means that (48) is satisfied, since if $\phi > 0$, then $c_1 = c_2 = c^{laffer}$, otherwise $c_1 = c_2 \geq c^{laffer}$. Note furthermore that since $\mu_0 > 0$, $c_t < c^{fb}$ with

$$\eta - \frac{b_{-1,t+1}}{c_t^2} \geq 0 \tag{D.44}$$

for $t = 0, 1, 2$ where this follows from the fact that $b_{-1,t} \leq c^{fb}$ using analogous arguments as in step 2 in the proof of Lemma 7.

Step 3. By Lemma 7, an MPCE cannot achieve higher welfare than the solution to (45) – (48). We now establish that the solution to (45) – (48) is supported by an MPCE with $b_{0,1} = b_{0,2} \in \left[0, c^{fb}\right]$. Suppose that the date 0 government selects $b_{0,1} = b_{0,2}$ which satisfy

$$b_{0,1} = c_1 \left(1 - \eta \left(c_1 + g\right)\right) \tag{D.45}$$

where c_1 corresponds to the values of consumption satisfying $(D.43)$. Let us assume and later verify that this choice of debt satisfies

$$b_{0,1} \in \left[0, c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)\right)\right] \tag{D.46}$$

so that it is feasible (since $c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)\right) < c^{fb}$). Consider the continuation equilibrium from date 1 onward characterized by the solution to (38) – (42) and analyzed in the proof of Lemma 7. By step 1 in the proof of that lemma, if $b_{0,1} = b_{0,2} \geq 0$, the solution is unique and characterized by the date 1 implementability condition (39) and the date 1 first order conditions $(D.37)$ and $(D.39)$ for some positive μ_1. $c_1 = c_2$ satisfying $(D.43)$ clearly satisfy (39). To check that $(D.37)$ and $(D.39)$ are satisfied for positive μ_1 under these values, set $\kappa = 0$ (so that $c_1 = c_2$ from $(D.37)$ and $(D.39)$), and let us verify that the implied μ_1 is positive. Substituting $(D.45)$ into $(D.37)$, we achieve

$$\mu_1 = -\frac{1 - \eta c_1}{1 - \eta\left(c_1 + g\right) - \eta c_1} > 0$$

where we have appealed to the fact that $c_1 \in [c^{laffer}, c^{fb})$. Therefore, the date 1 government selects the same allocation as the date 0 government. We now verify that $(D.46)$ holds. There are two cases to consider.

Case 1. Suppose that the date 0 solution admits $c_1 = c_2 = c^{laffer}$. In that case $b_{0,1} = b_{0,2} = c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)\right)$ so that debt issuance is feasible.

Case 2. Suppose that the date 0 solution admits $c_1 = c_2 > c^{laffer}$. Since the right hand side of $(D.45)$ is strictly below $c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)\right) < c^{fb}$, it follows that $b_{0,1} = b_{0,2} < c^{fb}$. To verify that $b_{0,1} = b_{0,2} \geq 0$, note that (46) given $c_1 = c_2$ and $b_{-1,2} = b_{-1,3}$ can be rewritten as

$$1 - \eta\left(c_0 + g\right) - \frac{b_{-1,1}}{c_0} + \beta\left(1 + \beta\right)\left(1 - \eta\left(c_1 + g\right) - \frac{b_{-1,2}}{c_1}\right) = 0. \tag{D.47}$$

From $(D.44)$, the left hand side of $(D.47)$ is decreasing in c_0 and c_1. Following similar arguments to step 2 in the proof of Lemma 2, equations $(D.42) - (D.43)$, imply that $c_0 \geq c_1 = c_2$ since $b_{-1,1} \geq b_{-1,2} = b_{-1,3} \geq 0$. Substituting these inequalities into $(D.47)$, which is declining in c_0 and c_1, yields

$$1 - \eta (c_1 + g) \geq 0. \tag{D.48}$$

Therefore, $b_{0,1}$ satisfying $(D.45)$ is weakly positive. Therefore, the solution to (45) – (48) represents an MPCE in which the date 0 government issues a flat maturity with $b_{0,1} = b_{0,2}$.

Step 4. We complete the proof by showing that there does not exist an MPCE which does not achieve the same welfare as (45) – (48) and that $b_{0,1} = b_{0,2}$ is necessary in the MPCE. If there existed an MPCE which did not provide welfare characterized by (45) – (48), then the MPCE would provide strictly lower welfare than the solution to (45) – (48), where this follows from Lemma 7. However, in this situation, the government at date 0 could deviate to choose $b_{0,1} = b_{0,2}$ which satisfy $(D.45)$ for $c_1 = c_2$ which correspond to the values of consumption satisfying $(D.43)$. By step 3, this choice would lead the date 0 government to achieve the same welfare as (45) – (48) and make it strictly better off. Therefore, any MPCE coincides with the solution to (45) – (48). We complete this step then by proving that $b_{0,1} = b_{0,2}$ is necessary to induce the date 1 government to pursue the same policy which satisfies (45) – (48). There are two cases to consider analogous to the cases considered in step 3.

Case 1. Suppose that the date 0 solution admits $c_1 = c_2 = c^{laffer}$. Suppose by contradiction that some value $b_{0,1} \neq b_{0,2}$ could induce the date 1 government solving (38) – (42) to choose $c_1 = c_2 = c^{laffer}$, where satisfaction of (39) requires

$$b_{0,1} + \beta b_{0,2} = (1+\beta)\, c^{laffer} \left(1 - \eta \left(c^{laffer} + g\right)\right). \tag{D.49}$$

For $(D.37)$ and $(D.39)$ to be satisfied, this would require $\kappa > 0$, since otherwise $(D.37)$ and $(D.39)$ would imply $c_1 \neq c_2$ since $b_{0,1} \neq b_{0,2}$ in the date 1 problem. Analogous arguments as in step 2 and 3 of the proof of Lemma 7 imply that $(D.38)$ and $(D.40)$ must hold. However, note that

$$\eta - \frac{b_{0,t}}{c_t^2} = 0 \tag{D.50}$$

if $c_t = c^{laffer}$ and $b_{0,t} = c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)\right)$, where we have appealed to the definition of c^{laffer} in (36). Since $(D.49)$ implies that either $b_{0,1} > c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)\right)$ or $b_{0,2} > c^{laffer}\left(1 - \eta\left(c^{laffer} + g\right)\right)$ and since the left hand side of $(D.50)$ is strictly decreasing in $b_{0,t}$, it follows that $(D.38)$ and $(D.40)$ cannot simultaneously hold, which is a contradiction. Therefore, $b_{0,1} = b_{0,2}$ uniquely guarantee that $c_1 = c_2$ in this case.

Case 2. Suppose that the date 0 solution admits $c_1 = c_2 > c^{laffer}$. Suppose by contradiction that some value $b_{0,1} \neq b_{0,2}$ could induce the date 1 government solving $(38) - (42)$ to choose $c_1 = c_2 > c^{laffer}$. In this case, $(D.37)$ and $(D.39)$ would need to be satisfied with $\kappa = 0$. However, this is not possible since $b_{0,1} \neq b_{0,2}$ implies $c_1 \neq c_2$.■