Consolidation Questions

Key words: accuracy, base dimensions, decimal places, irrational number, lower bound, rationalising the denominator, rational number, SI base units, prime factors, significant figures, square root, standard form, surd, upper bound.

You will need to use a calculator to answer some of these questions

LO 1.1.2 & 1.1.3: Round a number to a given number of decimal places/Significant figures

1. Round the following numbers to the stated number of decimal places,

a. 0.003550001 (2 dp)

b. 10.9062 (0 dp)

c. 3.099 (1 dp)

d. 1.0001 (3 dp)

2. Round the following numbers to the stated number of significant figures,

a. 1500.0002 (1 sf)

b. 0.0007888 (3 sf)

c. 1.00390 (2 sf)

d. 0.00105 (3 sf)

LO 1.1.4 Convert a number to standard index form and vice-versa

3. Working with standard form;

a. convert the following numbers to standard form,

(i) 723.5×10^{-5}

(ii) 0.0056×10^{5}

(iii) 2897.332

(iv) -0.000891

b. Evaluate the following, giving your answer in the form $A \times 10^n$, where A is correct to 3 sf.

(i) $(3.76 \times 10^{-2}) \times (0.00854) \div (3.76 \times 10^{4})$

(ii) $(2.49 \times 10^{-5})^{-3} \div (8.72 \times 10^{4})^{2}$

c. Convert the following numbers given in standard form, to ordinary numbers,

(i) 2.561×10^{-3}

(ii) 5.998871×10^{5}

(iii) $(2.5 \times 10^{3}) \div (5 \times 10^{2})$

(iv) $(2.5 \times 10^{-3}) \div (5 \times 10^{-5})$

LO 1.1.5 State the base dimensions and base units in mechanics

4. Base dimensions and units,

a. Write down the base dimensions (in terms of M, L, T) for each of the following quantities;

(i) *force = mass × acceleration,*

(ii) *work done = force × distance,*

(iii) *pressure = force/area,*

(iv) *power = work done/time*

(v) *velocity = displacement/time*

b. By considering the base dimensions, which of the following quantities have incorrect SI base units? Write the correct units in cases where the units are incorrectly stated.

(i) Volume (m^3)

(ii) Area (m^2)

(iii) Velocity (ms^{-1})

(iv) Acceleration (m^2s)

(v) Force ($kg\ m\ s^{-2}$)

(vi) Pressure (kg s m^{-1})

(vii) Power (kg m^{-1} s^{-3})

LO 1.1.6 Standardise units in calculations

5. Conversions between units and mensuration with units,

a. Convert 1.238 litres to a capacity in (i) ml and (ii) μ l, giving your answers in standard form.

b. Convert 2.8302×10^7 g, to a mass in (i) kg, (ii) tons

c. A vehicle has an average speed of 120 kmh^{-1}. What is the average speed in ms^{-1}?

d. Convert 8.5×10^{-5} m to a length in millimetres.

e. A rectangular metal plate has a length of 654 mm and width of 1206 mm. What is the area of the plate, in square metres?

f. A container in the shape of a cuboid has a width of 835mm, height of 14 cm and length of 1.204 metres. The container is completely filled with water. Calculate the capacity of the container in litres and the mass of water in kg (given that 1 litre of water has a mass of 1kg.)

g. A solid metal sphere has a radius of $r = 485$ mm. It is made of metal with a density of $\rho = 5486$ kg m^{-3}. Calculate the mass of the sphere in kg. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ and mass = ρV.

LO 1.1.7 Carry out calculations within limits of accuracy

6. Accuracy – upper and lower bounds on measurements.

a. Harry has a height of $h = 1.77$ m, to 2 dp. What is the upper and lower bound for Harry's height? Write this as an inequality for the range of values within which h lies.

b. In an athletics competition, Aidana completed the 100 m sprint in a time of $t = 11.2$ seconds. Express the range of values in which t must lie, as an inequality.

c. A room in one of the student's apartments has a width of 11 m and a length of 18 m, measured to the nearest metre. Write as an inequality, the range of values within which the floor area A, must lie.

d. A circular floor mat has a radius of 1.5 m, to 1dp. Calculate the upper and lower bounds of the circumference and the area of the mat (use the value of π stored in your calculator).

e. A cylindrical can has a radius of 14.3 cm and a height of 22.7 cm, both measured correct to 1dp. Calculate the upper and lower bounds on the volume of the can, in cm^3.

f. A solid metal cube of side length 3.0 cm has a mass of 700 g. The length of the side is correct to 1dp and the mass is correct to the nearest gram. Write an inequality for the upper and lower bounds within which the density ρ of the metal must lie (density = mass ÷ volume).

LO 1.1.8 State the difference between rational and irrational numbers

7. Rational and irrational numbers - indicate if the following numbers are rational or irrational,

(i) $\frac{\sqrt{75}}{\sqrt{5}}$

(ii) $\sqrt{4}$

(iii) π^2

(iv) $6 - \sqrt{4} + \pi$

(v) $0.\dot{2}$

(vi) $\left(\sqrt{5} \times \sqrt{12}\right)^2$

LO 1.1.9 Carry out operations of ASMD with simple surds

8. Rational and irrational numbers – arithmetical operations with surds. Calculate the following and where possible express the answer in the form $q\sqrt{p}$ where q is rational and p is a prime number.

(i) $\sqrt{75}$

(ii) $\sqrt{48}$

(iii) $\sqrt{27}$

(iv) $\sqrt{2500}$

(v) $\sqrt{196}$

(vi) $\sqrt{65}$

(vii) $\sqrt{132}$

(viii) $\sqrt{345}$

(ix) $\sqrt{0.09}$

(x) $\sqrt{\frac{49}{81}}$

(xi) $\left(2\sqrt{5} \times \sqrt{12}\right)^2$

(xii) $\left(3\sqrt{5} + 2\sqrt{12}\right)^2$

(xiii) $\sqrt{63} - \sqrt{28}$

(xiv) $\sqrt{0.\dot{2}}$

LO 1.1.10 Rationalise the denominators of fractions containing surds

9. Rationalisation of fractions with surd denominators - rationalise the following and where possible, give your answer in the form $a + b\sqrt{c}$ where c is prime, a and b are rational.

a. $\frac{1}{1-\sqrt{5}}$

b. $\frac{1}{1+\sqrt{7}}$

c. $\frac{1}{\sqrt{12}}$

d. $\frac{2+\sqrt{3}}{\sqrt{3}-2}$

e. $\frac{5-\sqrt{50}}{2\sqrt{5}-5}$

f. $\frac{12+2\sqrt{18}}{3\sqrt{6}-5\sqrt{8}}$

g. $\frac{2\sqrt{24}-2\sqrt{12}}{3\sqrt{12}-2\sqrt{72}}$

Challenge and research based questions

1. A hollow, spherically shaped metal container has an internal radius of $r = 0.83$ m, correct to 2 dp. The container is filled with a liquid that has a density of $\rho = 1.25$ g cm^{-3}, also correct to 2dp. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ and mass = ρV.

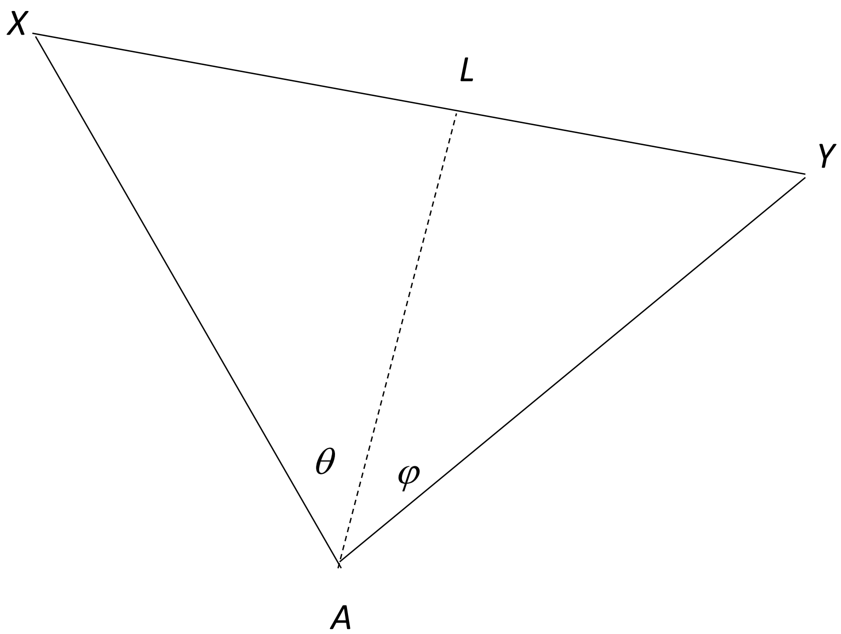
    a. Calculate the maximum and minimum mass of the fluid inside of the container.

    b. For transportation, regulations require that the actual mass of liquid must be correct to the nearest 1kg. There is no facility for measuring the mass of the liquid directly. What would be the minimum accuracy on the measurement of the radius and the density to compute the mass of the fluid correct to the nearest 1kg?

2. An observer on a boat sees two masts on rocks, at points X and Y. The points X and Y are known to be 2000 m apart, measured to the nearest metre. A lighthouse is located on another rock, at a point L, on the line joining XY. The two angles, θ and φ shown in the diagram, are measured when the boat is at a point A, such that the angle ALX is 90°. The distance from the boat to the lighthouse, AL is given by,

$$AL = \frac{2000}{\tan\theta + \tan\varphi}$$

Given that there may be an error of up to $\pm 2^{\circ}$ in the measurement of each angle and these are recorded as $\theta = 57^{\circ}, \varphi = 21^{\circ}$ find the upper and lower bound on the distance AL correct to the nearest metre.



3. The production cost, C (in 100's of euros) of operating a machine that produces components at a rate r (1000's of units per hour), is given by $C = r^2$, where $2 \leq r \leq 5$. The production rate r cannot be predicted or measured exactly, there is always some uncertainty in its measurement. The company has two models for the measurement error. Model 1 involves a constant error in the production of $\pm$200 units per hour. Model 2 involves an error that is proportional to $\pm$ 5% of the production rate, r. For accounting purposes, the company needs to estimate the maximum and minimum possible cost at the lowest and highest production rates. Use MS Excel to represent the cost, $C = r^2$ graphically. Also graph the upper and lower error bounds for the two models on the same axes. Advise the company about which error model is suitable for the lowest and highest production rates.