Consolidation Questions

Key words: Complex number, imaginary number, real component, imaginary component, conjugate, Argand diagram, modulus, argument, principal argument.

Formulae

Given a complex number $z = x + iy$

The modulus, $|z| = \sqrt{x^2 + y^2}$

The conjugate, $z^* = x - iy$

The principal argument θ is given as follows,

$$\boldsymbol{\varphi = \tan^{-1}\left(\frac{|y|}{|x|}\right)}$$

Quadrant 2 $\boldsymbol{\theta = \pi - \varphi}$	Quadrant 1 $\boldsymbol{\theta = \varphi}$
Quadrant 3 $\boldsymbol{\theta = -(\pi - \varphi)}$	Quadrant 4 $\boldsymbol{\theta = -\varphi}$

1. Simplify these, giving your answer in the form $a + b\mathrm{i}$

a. $(9 + 6\mathrm{i}) - (8 + 10\mathrm{i})$

b. $7(1 - 3\mathrm{i})$

c. $\sqrt{-200}$

d. $\sqrt{-147}$

2. Simplify these, giving your answer in the form $a + b\mathrm{i}$

a. $(3 - 2\mathrm{i})(5 + \mathrm{i})(4 - 2\mathrm{i})$.

b. $(2 + 3\mathrm{i})^3$

c. $\mathrm{i}^5 + \mathrm{i}$

d. $(4\mathrm{i})^3 - 4\mathrm{i}^3$

3. Solve these equations

a. $x^2 - 6x + 11 = 0$

b. $x^2 + 5x + 25 = 0$

c. $x^2 - 2x + 25 = 0$

d. $x^2 + 3x + 5 = 0$

4. Write down complex conjugate z^* for

a. $z = 3 - 2i$

b. $z = 2 + 3i$

c. $z = \frac{2}{3} - \frac{1}{2}i$

d. $z = \sqrt{5} - 3i\sqrt{5}$

5. Find these in the form $a + b$i

a) $\frac{3-2i}{5+i}$

b) $\frac{2+3i}{3-2i}$

c) $\frac{2+i}{1+4i}$

d) $\frac{(3-4i)^2}{1+i}$

6. Show these numbers on an Argand diagram:

a. $z = 7 + 2\mathrm{i}$.

b. $z = 5 - 4\mathrm{i}$

c. $z = -6 - \mathrm{i}$

d. $z = -2 + 5\mathrm{i}$

7. Find the modulus and argument of each of the following complex numbers, giving your answers exactly where possible, and to two decimal places otherwise:

a) $2\sqrt{3} - i\sqrt{3}$
b) $-8 - 15i$
c) $-2+5\mathrm{i}$
d) $1 - 3\mathrm{i}$