Key words: Gradient, gradient function, tangent, differentiation, derivative, derived function, rate of change, equation of the tangent, stationary point, local maximum, local minimum, global maximum, global minimum, point of inflection, stationary point of inflection, turning point, second derivative test, composite function, chain rule, product rule, quotient rule, function of a function.

Formulae

Chain rule

If $y = f(u)$ where $u = g(x)$, then $y' = f'(u)\, u'(x)$

Product rule

If u and v are functions of x, and $y = uv$, then

$$y' = uv' + u'v$$

Quotient rule

Given the quotient $y = \dfrac{u}{v}$ where u and v are functions of x then

$$y' = \frac{u'v - uv'}{v^2}$$

Table of standard derivatives

$f(x)$	$f'(x)$
ax^n	anx^{n-1}
$\ln x$	$\dfrac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$

1. Differentiate:

   a. $e^{t^4+3t^2} + 2e^{2t}$

   b. $4\ln 2x - \ln(3x+1)$

   c. $\sqrt{e^\theta + e^{2\theta}}$

   d. $\ln\left(\sqrt{\dfrac{1-x}{1+x}}\right)$

2. Find the function $f'(x)$ where $f(x)$ is

   a. $\sin\dfrac{2x}{5} - e^{\sqrt{x}}$

   b. $\tan^4\left(\dfrac{x}{2}\right)$

   c. $3\cos^2 2x$

   d. $\sec^2 x - \tan^2 x$

3. Find $\dfrac{dy}{dx}$ if

   a. $y = \sin(\cos 2x)$

   b. $y = \ln\left(\sec\dfrac{x}{2}\right)$

   c. $y = \sin^4(x^2)$

4. Differentiate:

   a. $y = 4x \ln x^2$

   b. $f(t) = \sin^3 t \cos t$

   c. $g(\theta) = e^\theta(\tan\theta - 1)$

   d. $p(x) = \dfrac{\sin 3x}{e^x}$

   e. $q(x) = \dfrac{\sec^2 x}{x}$

5. For the given curve, find the value of $\dfrac{dy}{dx}$ at the specified point:

   a. $y = xe^{-x}; \left(1, \dfrac{1}{e}\right)$

   b. $y = \cot^3(\pi - x); \left(\dfrac{3\pi}{4}, 1\right)$

6. Let $f(x) = cx + \ln(\cos(x))$. For what value of c is $f'\left(\dfrac{\pi}{4}\right) = 6$?

7. Find the equation of the tangent to the curve $y = (2x+3)e^{2x}$ at $x = 0$.

8. Find the equation of the normal to the curve $y = \cos(2x)$ at $x = \dfrac{\pi}{4}$.

9. Suppose that $f(x) = M \sin x + N \cos x$ for some constants M and N. If $f\left(\frac{\pi}{4}\right) = 3$ and $f'\left(\frac{\pi}{4}\right) = 1$,

    find an equation of the tangent line to $y = f(x)$ at $x = \frac{3\pi}{4}$.

10. Given $f(x) = e^{2x}(2 - x^2)$

    a. Use calculus to find the coordinates of the stationary points on the graph of $y = f(x)$.

    b. Show that $f''(x) = -2e^{2x}(2x^2 + 4x - 3)$.

    c. Hence, or otherwise, determine which stationary point is a maximum and which is a minimum.

11. The diagram shows part of the curve with equation $y = f(x)$ where
$$f(x) = x(1 + x) \ln x \qquad \{x > 0\}$$
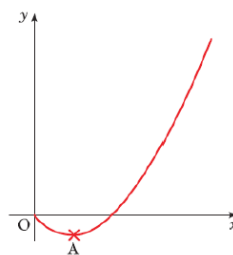
The point A is the minimum point of the curve.

a. Find $f'(x)$.

b. Hence show that the x-coordinate of A is the solution of the equation $x = g(x)$, where
$$g(x) = e^{-\frac{1+x}{1+2x}}$$

References:

Some of the questions on this worksheet were reproduced from the following sources;

Attwood, G., Macpherson, A., Moran, B., Petran, J., Pledger, K., Staley, G. and Wilkins, D. (2008), Edexcel AS and A-Level Modular Mathematics series C1-C4, Pearson, Harlow, UK.

Graham, D., Graham, C and Whitcombe, A (1995) Letts Study Guide, A-Level Mathematics, Letts Educational, UK. ISBN: 1 85758 338 8