Review Questions

Key words: Stationary point, local maximum, local minimum, second derivative, chain rule, rate of change.

Formulae

The chain rule : $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

1. Differentiate the following using appropriate notation for the derivative:

$a.\quad y = 2\sqrt{x}$ $\quad b.\quad y = \frac{3}{x^2}$ $\quad c.\quad y = \frac{1}{3x^3}$

$d.\quad f(x) = \frac{1}{3}x^3(x-2)$ $\quad$ e. $y = \frac{2}{x^3} + \sqrt{x}$ $\quad f.\quad y = \sqrt[3]{x} + \frac{1}{2x}$

$g.\quad y = \frac{2x+3}{x}$ $\quad h.\quad y = \frac{3x^2-6}{x}$ $\quad$ i. $y = \frac{2x^3+3x}{\sqrt{x}}$

$j.\quad f(x) = x(x^2 - x + 2)$ $\quad k.\quad f(x) = 3x^2(x^2 + 2x)$ $\quad l.\quad y = (3x-2)\left(4x + \frac{1}{x}\right)$

2. Find the gradient of the curve with equation y = f(x) at the point A where:

a. $f(x) = x(x+1)$ and A is at (0, 0)

b. $f(x) = \frac{2x-6}{x^2}$ and A is at (3, 0)

c. $f(x) = \frac{1}{\sqrt{x}}$ and A is at (1/4, 2)

d. $f(x) = 3x - \frac{4}{x^2}$ and A is at (2, 5)

3. Find the values of x for which f(x) is an increasing function, given that f(x) equals:

$a.\quad 3x^2 + 8x + 2$ $\quad b.\quad 4x - 3x^2$

$c.\quad 5 - 8x - 2x^2$ $\quad d.\quad 2x^3 - 15x^2 + 36x$

4. Find the values of x for which f(x) is an increasing function, given that f(x) equals:

$a.\quad 1 - 27x + x^3$ $\quad b.\quad x + \frac{25}{x}$

$c.\quad x^{1/2} + 9x^{-1/2}$ $\quad d.\quad x^2(x+3)$

5. Find the first and second derivatives where y equals:

$a.$ $12x^2 + 3x + 8$ $b.$ $15x + 6 + \frac{3}{x}$

$c.$ $9\sqrt{x} - \frac{3}{x^2}$ $d.$ $(5x + 4)(3x - 2)$ $e.$ $\frac{3x+8}{x^2}$

6. Find the equation of the tangent to the curve where:

a. $y = x^2 - 7x + 10$ at the point (2, 0)

b. $y = x + \frac{1}{x}$ at the point (2, 2.5)

c. $y = 4\sqrt{x}$ at the point (9, 12)

d. $y = \frac{2x-1}{x}$ at the point (1, 1)

e. $y = 2x^3 + 6x + 10$ at the point (–1, 2)

f. $y = x^2 + \frac{-7}{x^2}$ at the point (1, –6)

7. Find the coordinates of the stationary points on the curves of the given equations,

a. $y = x(x^2 - 4x - 3)$

b. $y = x + \frac{1}{x}$

c. $y = x^2 + \frac{54}{x}$

d. $y = x - 3\sqrt{x}$ Take $\sqrt{x}$ to be $+\sqrt{x}$ only

e. $y = x^{1/2}\,(x - 6)$ Take $\sqrt{x}$ to be $+\sqrt{x}$ only

f. $y = x^4 - 12x^2$

8. Establish whether the stationary points in question 1 are local maxima or local minima

9. Use suitable substitutions and the chain rule to differentiate the following with respect to x,

g. $y = (5x + 3)^6$

h. $y = \frac{1}{5x+3}$

i. $y = (1 - 4x)^{-3}$

j. $y = (2x^2 + 3)^6$

k. $y = \left(\sqrt{x^2 - 1} + 1\right)^6$