Key words and terminology: axis of symmetry, bi-quadratic, coefficients, coincident roots, completing the square, complex conjugate roots, constant, discriminant, distinct roots, equation, exponent, factorisation, function, fourth-order, horizontal asymptote, infinity, maximum, minimum, polynomial, quartic, real roots, repeated roots, tangent, vertex, vertical asymptote.

Formulae

The quadratic formula: You need to remember this formula. It will not be given to you in your assessments. You need to remember this formula. The general quadratic function is $f(x) = ax^2 + bx + c$, where, a, b, c are real coefficients. The solutions to the quadratic equation, $f(x) = 0$ are given by,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

LO 2.2.1 Solve a quadratic equation by factorisation

1. Solve the following equations by factorisation,
   a. $2x^2 - 5x = 0$
   b. $x^2 - 9x + 20 = 0$
   c. $4x^2 + 17x = 6x - 2x^2$
   d. $9x^2 - 4 = 0$

LO 2.2.2 Complete the square on a quadratic function

2. Complete the square for the following expressions, writing them in the form $p(x + q)^2 + r$,
   a. $x^2 + 6x + 8$
   b. $3x^2 - 12x + 7$
   c. $x^2 - 2mx + n$
3. Solve the following equations by completing the square (give your answers in exact rational form),
   a. $9x^2 + 18x = 16$
   b. $5x^2 - 3x + \frac{2}{5} = 0$

LO 2.2.3 Solve a quadratic equation using the quadratic formula

4. Solve the following equations using the quadratic formula,
   a. $4x^2 = 1 - 4x$ give your answers in exact form as surds
   b. $7x^2 - \frac{5}{6}x - \frac{1}{6} = 0$ using a calculator, give your answers correct to 2 decimal places

LO 2.2.4 Apply the quadratic discriminant in problem solving

5. Solve the following problems using properties of the discriminant,
   a. The equation $x^2 + kx + 9 = 0$ has real and distinct roots. Find the range of values of k.
   b. Find the values of k for which $x^2 + kx + 4 = 0$ has equal roots.
   c. The equation $ax^2 + 7x + \frac{1}{4} = 0$ has equal roots. Find the value of a.
   d. Find the set of values of k for which the equation $x^2 + kx + 2k - 3 = 0$ has no real roots.

6. Show that the equation $x(x - 2p) = q(x - p)$ has real roots for all real values of p and q. If q = 3, find a non-zero value for p, so that the roots are rational.

7. For what values of the real number a does the quadratic equation $ax^2 + 8x + a = 0$ have real roots?

8. For what values of the real number c does the quadratic equation $3x^2 + 6x + c = 0$ have no real roots?

LO 2.2.5 Sketch the graph of a quadratic function

9. Find the vertex points on the parabolas generated by the following functions, indicating whether the vertex is a maximum or minimum. State also the equation of the axis of symmetry of the parabola.
   a. $f(x) = 3x^2 - 6x - 7$
   b. $y = 8 - 2x - x^2$

10. Sketch the graphs of the following functions. Your sketch must include (1) y-intercept, (2) any x-intercepts, (3) axis of symmetry, (4) coordinates of the vertex point.

    a. $f(x) = 2x^2 - 17x + 33$

    b. $y = -17x^2 - 170x - 412$

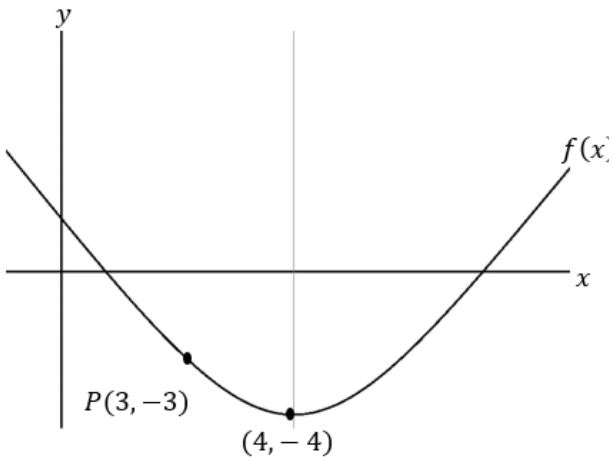
11. Using Microsoft Excel, investigate the effect of changing the coefficients in the general quadratic function $f(x) = ax^2 + bx + c$ on the shape and location of the graph of the function, relative to the coordinate axes.

12. Show that the coordinates of the vertex on the graph of the general quadratic function $f(x) = ax^2 + bx + c$ are given by $\left(-\frac{b}{2a}, \left\{\frac{4ac-b^2}{4a}\right\}\right)$

13. Consider the quadratic function $f(x) = 2x^2 - 4x + 10$. By completing the square on $f(x)$ find the values of m, n and p in the expression $f(x) = m(x+n)^2 + p$ and choose which corresponding description best matches the graph of $f(x)$.

    a. $m = 2,\ n = 1,\ p = 6,$ the axis of symmetry is $x = -1,\ f(x)$ has a minimum.
    b. $m = 2,\ n = -1,\ p = 8,$ the axis of symmetry is $x = 1,\ f(x)$ has a minimum.
    c. $m = 2,\ n = -1,\ p = 8,$ the axis of symmetry is $x = -1,\ f(x)$ has a maximum.
    d. $m = 1,\ n = 1,\ p = 8,$ the axis of symmetry is $x = 1,\ f(x)$ has a minimum.
    e. $m = 2,\ n = 1,\ p = 8,$ the axis of symmetry is $x = -1,\ f(x)$ has a minimum.

14. The graph shows a function of the form $f(x) = ax^2 + bx + c$. The vertex is at $(4, -4)$ and the point $P(3, -3)$ lies on the curve. Find the values of x where the curve cuts the x-axis.



LO 2.2.6 Solve a bi-quadratic equation

15. Solve (where real solutions exist) the following bi-quadratic (quartic) equations by making a substitution of the form $x^2 = ky$.

    a. $x^4 - 13x^2 + 36 = 0$

    b. $4x^4 - 24x^2 + 27 = 0$

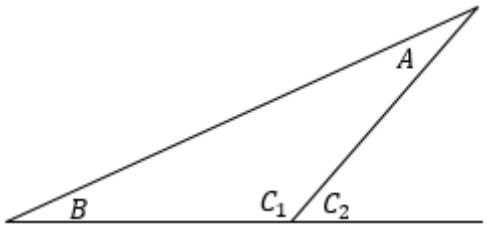
    c. $x^4 + 3x^2 - 40 = 0$

    d. $x^4 + 12x^2 + 35 = 0$

16. The diagram below shows the triangle ΔABC. The value of three angles $A, B,$ and C_2 are represented by the following expressions, where A, B, C_1 and C_2 are measured in degrees, and have positive values, less than 180 degrees:

$$\angle A = x^2 - 10x, \quad \angle B = -20x - 10, \text{ and } \angle C_2 = 60 + 3x.$$

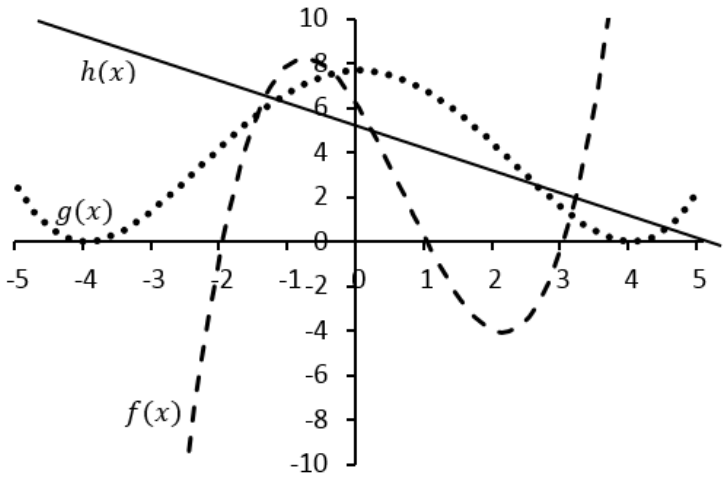
(Where the symbol $\angle A$ means the angle A)



Find the value of x.

LO 2.2.7 Interpret graphically the nature of the roots of a polynomial equation

17. The graph shows three functions, all with real coefficients; a cubic, $f(x)$, a biquadratic (quartic), $g(x)$ and a line $h(x)$. All three functions have the domain $x \in \mathbb{R}$. The function $g(x)$ has repeated (double) roots located at $(\pm 4, 0)$. Which one of the following sets of statements is correct.



a. The equation, $h(x) = 0$ has one real solution, $g(x) - h(x) = 0$ has two real solutions, $f(x) - h(x) = 0$ has three real solutions.

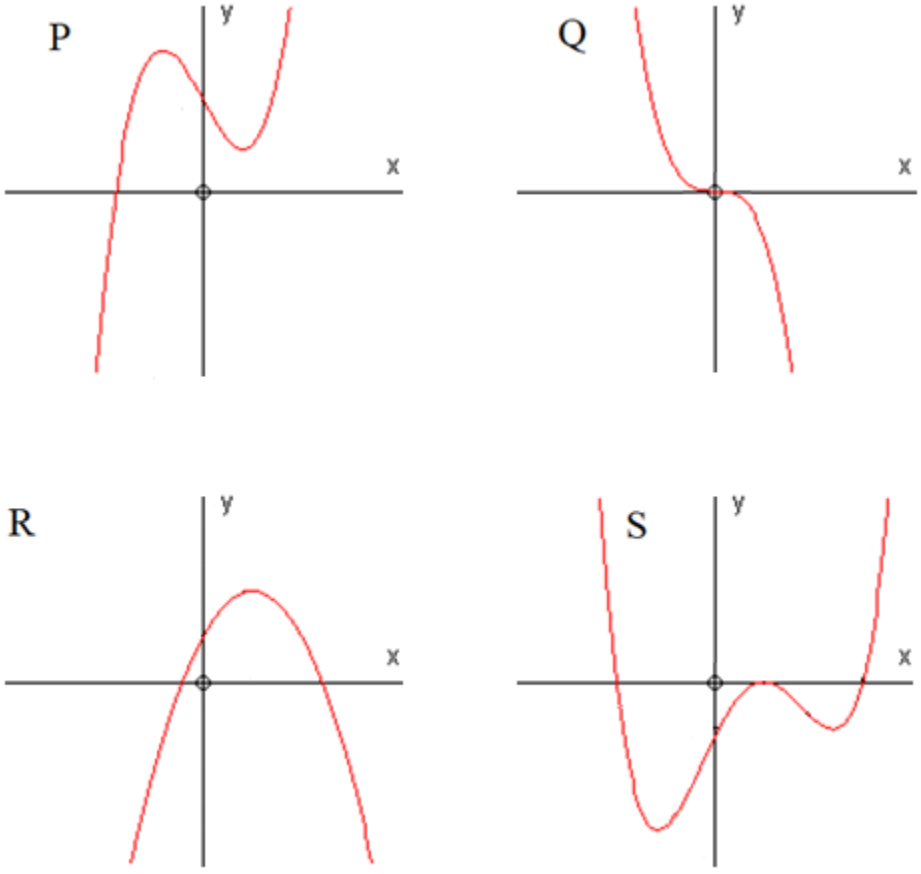
b. $h(0)$ is less than both $f(0)$ and $g(0)$. The equation $g(x) - 2 = 0$ has four complex roots.

c. $h(x) < 0$ when $x > 10$. The function $g(x) = k(x-4)^2(x+4)^2$, where $-1 < k < 0$.

d. The function $g(x) = k(x-4)^2(x+4)^2$, where $k > 1$. The equation $g(x) + 2 = 0$ has four complex roots.

e. The function $g(x) = k(x-4)^2(x+4)^2$, where $0 < k < 1$. The equation $g(x) + 2 = 0$ has four complex roots.

18. Which one of the following statements is correct?



a) Graphs P and Q represent a function of the form $y = ax^3 + bx^2 + cx + d$, where $a > 0$. Graphs R and S both represent quadratic functions.

b) Graph Q represents a function of the form $y = ax^3 + bx^2 + cx + d$, where $a < 0$. Graph R represents a function of the form $y = bx^2 + cx + d$, where $b < 0$. The function for graph P has three real roots.

c) The function for graph P has one real root. Graph Q represents a function of the form $y = ax^3 + bx^2 + cx + d$, where $a < 0$.

d) Graph R represents a function of the form $y = bx^2 + cx + d$, where $b > 0$. Graph S represents a function of the form $y = ax^3 + bx^2 + cx + d$, where $a > 0$.

e) All graphs are of functions which must have one real root and one complex root.