Consolidation Questions

Key words and terminology: degree, divisor, factor theorem, greatest common factor (GCF), identity, irreducible quadratic, long division, polynomial equation, quotient, remainder, remainder theorem.

Formulae

There are no specific formulae that you need to remember for this topic. However, you should know the procedures for long division, factorisation and applying both the factor and remainder theorems.

LO 2.3.1 Factorise polynomial expressions

1. Factorise the following expressions completely,

   a. $6x^3 + 3x$

   b. $6x^3 - 54x$

   c. $x^3 - 3x^2y + 3xy^2 - y^3$

   d. $x^3 - 12x^2 + 48x - 64$

   e. $4x^4 - 5x^2 + 1$

LO 2.3.2 Divide a polynomial by a linear or quadratic factor

2. Use algebraic long division to divide the following polynomials,

   a. $2x^3 + 3x^2 - x + 2 \;\; by \; x + 2$

   b. $x^3 - 4x^2 - x + 1 \; by \; 2x - 1$

   c. $-3x^4 + 4x^3 - x + 1 \; by - 3x - 2$

   d. $x^5 + 2x^4 - 2x^2 + 1 \; by - x^2 + 1$

   e. $-3x^5 + x^4 - 3x^3 - x^2 + x - 1 \; by - 2x^2 + 2x + 1$

LO 2.3.3/2.3.4 Apply the remainder/factor theorems

3. Use the remainder theorem to find the remainder of

   $f(x) = x^4 + 5x^3 - 2x^2 + 3x + 2$, when it is divided by $x + 1$.

   a. -3
   b. 9
   c. 7
   d. -7
   e. 10

4. Use the factor theorem to find which one of the following linear expressions is a factor of polynomial $f(x) = 2x^4 - 12x^3 + 22x^2 - 7x - 15$.

   a. $x + 4$
   b. $x - 3$
   c. $x + 3$
   d. $x - 4$
   e. $x - 10$

5. Given that $f(x) = x^5 - 2x^4 + 3x^3 - 6x^2 - 4x + 8$. Find the remainder of $\frac{f(x)}{x+1}$.

   a. 0
   b. 4
   c. 8
   d. 2
   e. 1

6. The function $f(x) = x^3 + ax^2 + bx - 12$ has a factor of $(x - 3)$ and leaves a remainder of – 18 when divided by $(x - 2)$. Find the values of the coefficients a and b.

   a. $a = 2$, $b = -11$
   b. $a = -2$, $b = 11$
   c. $a = 1$, $b = -2$
   d. $a = -1$, $b = 2$
   e. $a = 2$, $b = -1$

7. The polynomial $ax^3 + x^2 + bx + 6$ is divisible by $(x - 2)$ and leaves a remainder of -4 when divided by $(x - 1)$. Find a and b. Hence express the polynomial as the product of three linear factors.

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8. Given that $f(x) \equiv x^3 - 2x^2 - 5x + 6$. Show that $(x - 3)$ is a factor of $f(x)$. Hence find the other two factors. State the set of values of x for which $f(x) \geq 0$

9. Given that $ax^3 + bx^2 - 11x + 6$ is divisible by $(x - 2)$ and leaves a remainder of 9 when divided by $(x + 1)$. Find the constants a and b.

10. Given that $x = -3$ is a solution of $f(x) \equiv 6x^3 + 17x^2 - 5x - 6$, express $f(x)$ as a product of 3 linear factors. Hence find the other two solutions.

11. Show that $(x - 2)$ is a factor of $f(x) = x^4 - 2x^3 - 8x^2 + 13x + 6$. Hence, or otherwise, find the exact solutions of the equation $f(x) = 0$.

12. Given that $f(x) = 2x^4 + 5x^3 - 6x^2 - 20x - 8$, show that $(2x + 1)$ is a factor of $f(x)$. Hence factorise $f(x)$ completely and find the values of x for which $f(x) = 0$.