

Key words and terminology: exponential function, base, exponent, exponential growth, exponential decay, logarithm function, logarithm identities, logarithm of a sum, logarithm of a quotient, logarithm of a power function, change-of-base function.

Formulae: You need to remember these formulae. They will not be given to you in your assessments.

$$b^{\log_b a} = a$$

$$\log_b b^a = a$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

$$a \log_b x = \log_b x^a$$

LO 3.1.3 Write an expression in index form or logarithmic form

1. Write as a logarithm;

a. $4^4 = 256$

b. $3^{-2} = \frac{1}{9}$

c. $10^6 = 1000000$

d. $11^1 = 11$

2. Write in index form;

a. $\log_2 16 = 4$

b. $\log_{10} 1000 = 3$

c. $\log_5 625 = 4$

d. $\log_2 \frac{1}{16} = -4$

LO 3.1.4 Apply the laws of logarithms

3. Express the following in terms of $\log a$ and/or $\log b$:

a. $\log(a^2 b)$

b. $\log\left(\frac{1}{a^2}\right)$

c. $\log\left(\frac{a^2}{b^3}\right)$

d. $\log(\sqrt[6]{a^2 b})$

4. Write the following expressions as a single logarithm, then simplify your answer;

- a. $\log_2 40 - \log_2 5$
- b. $\log_6 4 + \log_6 9$
- c. $2\log_{12} 3 + 4\log_{12} 2$
- d. $\log_8 25 + \log_8 10 - 3\log_8 5$
- e. $2\log_{10} 20 - (\log_{10} 5 + \log_{10} 8)$

5. Write the following expressions in terms of $\log_a x$, $\log_a y$ and $\log_a z$;

- a. $\log_a (x^3 y^4 z)$
- b. $\log_a \frac{x^5}{y^2}$
- c. $\log_a (a^2 x^2)$
- d. $\log_a \frac{x\sqrt{y}}{z}$
- e. $\log_a \sqrt{ax}$

LO 3.1.5 Solve exponential equations

6. Solve the following equations, giving your answer correct to three decimal places;

- a. $9^{x+5} = 50$
- b. $7^{2x-1} = 23$
- c. $2^{2x+3} = 3^{3x+2}$
- d. $8^{3-x} = 10^x$
- e. $3^{4-3x} = 4^{x+5}$

7. Solve the following equations giving your answer correct to three decimal places;

- a. $3^{2x} + 3^{x+1} - 10 = 0$
- b. $7^{2x} + 12 = 7^{x+1}$
- c. $2^{2x} + 3(2^x) - 4 = 0$
- d. $3^{2x+1} - 26(3^x) - 9 = 0$

e. $4(3^{2x+1}) + 17(3^x) - 7 = 0$

8. Solve the quadratic equation, $e^{2x} + 3e^x - 4 = 0$

9. Solve the equation, $e^0 - 15e^{-x} + 50e^{-2x} = 0$

LO 3.1.1 Sketch graphs of exponential functions (including $y = e^x$)

10. Sketch the following graphs;

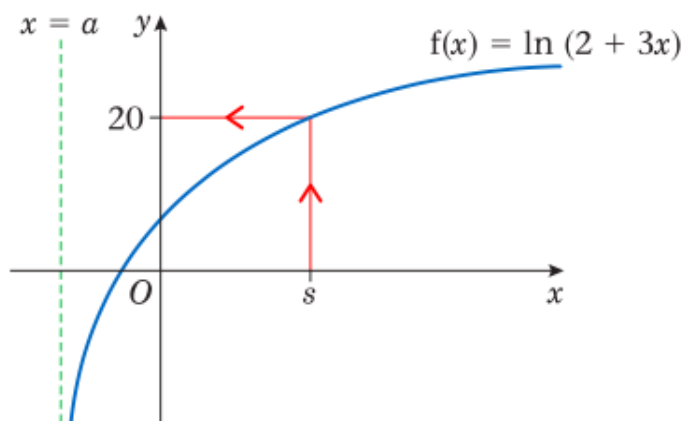
a. $y = e^x + 1$

b. $y = 4e^{-2x}$

c. $y = 2e^x - 3$

LO 3.1.2 Sketch the graph of a natural logarithmic function

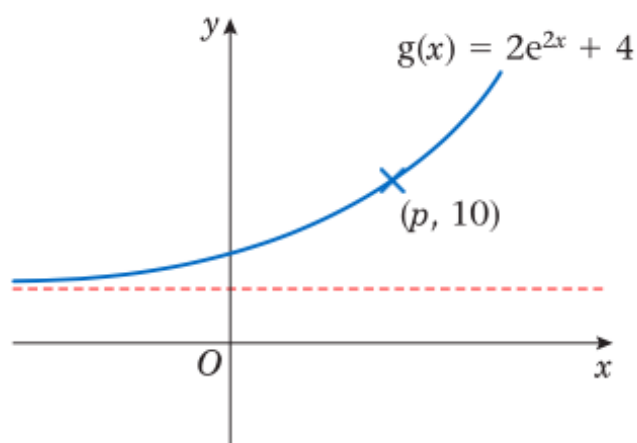
11. The following graph is of the function $f(x) = \ln(2 + 3x)$ where X a real number greater than a



a. State the value of a .

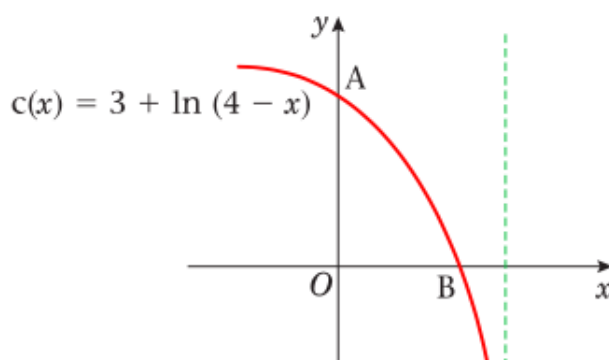
b. Find the value of s for which $f(s) = 20$.

12. The following graph is of the function $g(x) = 2e^{2x} + 4$ where X is any real number



- Find the range of the function.
- Find the value of p to two decimal places.

13. The function $c(x) = 3 + \ln(4 - x)$ is shown below



- Find the exact coordinates of point A .
- Calculate the exact coordinates of point B .

14. By first applying the laws of logarithms, sketch the graph of $y = \ln \left\{ \frac{e^{-4}}{(x+3)^2} \right\}$, $x \neq -3$

References:

Some of the questions on this worksheet were reproduced from the following sources;

Attwood, G., Macpherson, A., Moran, B., Petran, J., Pledger, K., Staley, G. and Wilkins, D. (2008), Edexcel AS and A-Level Modular Mathematics series C1-C4, Pearson, Harlow, UK.

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