Consolidation Questions

Key words and terminology: Pythagoras theorem, quadratic equation, Roots, trigonometric equation,

Formulae: You need to remember the following two identities as they will not be given to you in your assessements:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cos^2\theta + \sin^2 \equiv 1$$

1. Simplify each of the following expressions:

a. $1-\cos^2\left(\frac{1}{2}\theta\right)$

b. $5\sin^2(3\theta)+5\cos^2(3\theta)$

c. $\frac{\sin\theta}{\tan\theta}$

d. $\frac{\sqrt{1-\cos^2 x}}{\cos x}$

2. Given that $\sin x\cos y = 3\cos x\sin y$, express $\tan x$ in terms of $\tan y$.

3. Find, without using your calculator, the values of:

a. $\sin\theta$ and $\cos\theta$, given that $\tan\theta = \frac{5}{12}$ and θ is acute.

b. $\sin\theta$ and $\tan\theta$, given that $\cos\theta = -\frac{3}{5}$ and θ is obtuse.

c. $\cos\theta$ and $\tan\theta$, given that $\sin\theta = -\frac{7}{25}$ and $270^0 < \theta < 360^0$.

4. Solve the following equations for θ, in the interval $0^0 < \theta \leq 360^0$:

a. $\sqrt{3}\sin\theta = \cos\theta$

b. $\sin\theta + \cos\theta = 0$

c. $3\cos\theta = -2$

d. $(\sin\theta - 1)(5\cos\theta + 3) = 0$

5. Solve, in the intervals indicated, the following equations for θ, where θ is measured in radians. Give your answer in terms of π or two decimal places.

a. $\sin\theta = \frac{1}{\sqrt{2}}, \ -2\pi < \theta \leq 2\pi$

b. $\sin\theta = \tan\theta, \ 0 < \theta \leq 2\pi$

6. Find the values of θ, in the interval $0^0 \leq \theta \leq 360^0$, for which:

a. $\sin(-\theta) = \frac{1}{\sqrt{2}}$

b. $\tan(45^0 - \theta) = -1$

c. $\tan(\theta + 75^0) = \sqrt{3}$

d. $\cos(50^0 + 2\theta) = -1$

7. Solve, for θ, in the interval $0^0 \leq \theta \leq 360^0$, the following equations. Give your answer to three significant figures where they are not exact.

a. $4(\sin^2\theta - \cos\theta) = 3 - 2\cos\theta$

b. $2\sin^2\theta = 3(1 - \cos\theta)$

8. Given that angle B is reflex and $\tan B = \frac{\sqrt{21}}{2}$, find the exact value of:

a. $\sin B$

b. $\cos B$

9. a. Find the coordinates of the point where the graph of $y = 2\sin\left(2x + \frac{5}{6}\pi\right)$ crosses the y-axes.

b. Find the values of x, where $0 \leq x \leq 2\pi$, for which $y = \sqrt{2}$.

Additional questions

1. Sketch one complete period of the function:
$$f(t)=25\sin\left(\frac{\pi}{4}(t-2)\right)+55$$
2. Find the sinusoidal equation for the information: minimum value at $(9,25)$; maximum value at $(3,75)$; period 12 minutes.

3. Use the information given to write a sinusoidal equation, sketch is graph, and answer the questions posed.
The population of mosquitoes in a given area is primarily influenced by precipitation, humidity and temperature. In tropical regions, these tend to fluctuate sinusoidally in the course of a year. Using trap counts and statistical projections, fairly accurate estimates of a mosquito population can be obtained. Suppose the population in a certain region was modelled by the function
$$P(t)=50\cos\left(\frac{\pi}{26}t\right)+950$$
where $P(t)$ was the mosquito population (in thousands) in week t of the year. Use the model to:
   a. find the period of the function
   b. graph the function over one period
   c. find the maximum and minimum population values
   d. estimate the number of weeks the population is less than 915000

References:

Some of the questions on this worksheet were reproduced from the following sources;

Attwood, G., Macpherson, A., Moran, B., Petran, J., Pledger, K., Staley, G. and Wilkins, D. (2008), Edexcel AS and A-Level Modular Mathematics series C1-C4, Pearson, Harlow, UK.

Graham, D., Graham, C and Whitcombe, A (1995) Letts Study Guide, A-Level Mathematics, Letts Educational, UK. ISBN: 1 85758 338 8