Consolidation Questions

Key words: vector, scalar, distance, direction, displacement, speed, velocity, triangle law, zero vector, magnitude, modulus, unit vector, position vector, column vector, horizontal component, vertical component.

1. indicate which of the following quantities are scalars and which are vectors,

a. Power

b. Force

c. Speed

d. Velocity

2. In the diagram, $\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{QS} = \mathbf{b}$, $\overrightarrow{SR} = \mathbf{c}$ and $\overrightarrow{PT} = \mathbf{d}$.

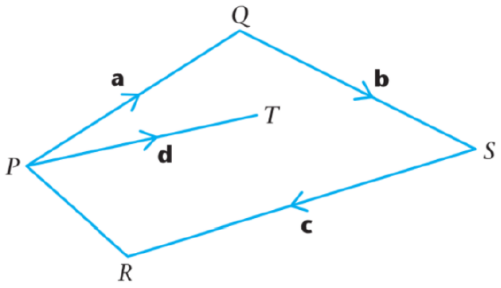
Find in the terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$

a. $\overrightarrow{QT}$

b. $\overrightarrow{PR}$

c. $\overrightarrow{TS}$

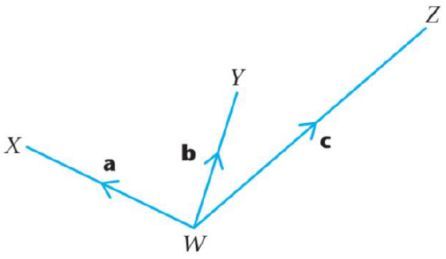
d. $\overrightarrow{TR}$



3. In the diagram, $\overrightarrow{WX} = \mathbf{a}$, $\overrightarrow{WY} = \mathbf{b}$, $\overrightarrow{WZ} = \mathbf{c}$.

It is given that $\overrightarrow{XY} = \overrightarrow{YZ}$.

Prove that $\mathbf{a} + \mathbf{c} = 2\mathbf{b}$ ($2\mathbf{b}$ **is** equivalent to $\mathbf{b} + \mathbf{b}$).



4. Find $|\mathbf{a} + \mathbf{b}|$ if

a. The vector $\mathbf{a}$ is directed due north and $|\mathbf{a}| = 24$. The vector $\mathbf{b}$ is directed due west and $|\mathbf{b}| = 7$.

b. The vector $\mathbf{a}$ is directed north-east and $|\mathbf{a}| = 20$. The vector $\mathbf{b}$ is directed due south-east and $|\mathbf{b}| = 13$.

5. Find in terms of **a** and **b**

a. $\overrightarrow{PR}$, $\overrightarrow{PM}$ and $\overrightarrow{QM}$ if in the triangle PQR $\overrightarrow{PQ} = 2\mathbf{a}$ and $\overrightarrow{QR} = 2\mathbf{b}$. The mid-point of PR is M.

b. $\overrightarrow{AM}$,$\overrightarrow{BD}$, $\overrightarrow{MB}$and $\overrightarrow{DA}$ if in the trapezium $ABCD$, AB is parallel to DC, DC=3AB, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$. The mid-point of DC is M.

6. Given that $\mathbf{a} = 4\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = 5\mathbf{i} - 12\mathbf{j}$, $\mathbf{c} = -7\mathbf{i} + 24\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 3\mathbf{j}$, find a unit vector in the direction of **a**, **b**, **c** and **d**.

7. In the diagram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and C divides AB in the ratio 5:1.

a. Write down in terms of **a** and **b**, expressions for $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{OC}$.

Given that $\overrightarrow{OE} = \gamma\mathbf{b}$, where γ is a scalar:

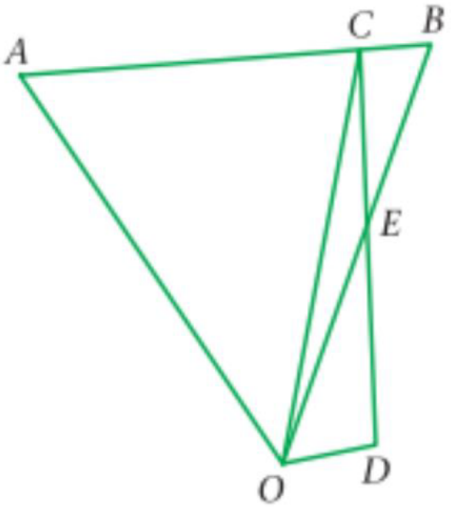
b. Write down in terms of **a, b** and γ, **an** expressions for $\overrightarrow{CE}$.

Given that $\overrightarrow{OD} = \mu(\mathbf{b} - \mathbf{a})$, where μ is a scalar:

c. Write down in terms of **a, b,** γ and μ, an expressions for $\overrightarrow{ED}$.

Given also that E is midpoint of CD:

d. Deduce the values of γ and μ.



8. Find the distance between A and B when they have the following coordinates:

a. $A(3, 0, 5)$ and $B(1, -1, 8)$

b. $A(8, 11, 8)$ and $B(-3, 1, 6)$

c. $A(3, 5, -2)$ and $B(3, 10, 3)$

d. $A(-1, -2, 5)$ and $B(4, -1, 3)$

9. Find the possible values of k, given that

a. The coordinates of A and B are $(7, -1, 2)$ and $(k, 0, 4)$ respectively and distance from A to B is 3 units.

b. The coordinates of A and B are $(5, 3, -8)$ and $(1,\ k, -3)$ respectively and distance from A to B is $3\sqrt{10}$ units.

10. The points A and B have position vectors $\begin{pmatrix} 2t+1 \\ t+1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} t+1 \\ 5 \\ 2 \end{pmatrix}$ respectively. Find

a. $\overrightarrow{AB}$

b. In terms of t, $|\overrightarrow{AB}|$

c. The value of t, that makes $|\overrightarrow{AB}|$ a minimum.

d. The minimum value of $|\overrightarrow{AB}|$.

References:

Some of the questions on this worksheet were reproduced from the following sources;

Attwood, G., Macpherson, A., Moran, B., Petran, J., Pledger, K., Staley, G. and Wilkins, D. (2008), Edexcel AS and A-Level Modular Mathematics series C1-C4, Pearson, Harlow, UK.

Graham, D., Graham, C and Whitcombe, A (1995) Letts Study Guide, A-Level Mathematics, Letts Educational, UK. ISBN: 1 85758 338 8