Initial diagnostic questions

Scope of the initial diagnostic questions

- The diagnostic questions are referenced to the topics of the review units as follows

Unit code	Topic area
1.1	Numerical processing skills
1.2	Algebraic processing skills
2.1	Linear equations & straight line graphs
2.2	Quadratic equations and graphs of quadratic functions
2.3	Factor & remainder theorem
3.1	Properties of exponentials and logarithms
5.1	Basic trigonometry
5.2	Trigonometric equations
5.3	Further trigonometric identities, equations and graphs
9.1	Introduction to vectors
10.1	Introduction to complex numbers
11.1	Review of basic differentiation
11.5	Review of basic integration with applications

- Each of the 13 units has a group of between 3-5 questions. There are 50 questions in total. Each question is referenced to one or two learning objectives (LO) within the unit.

Attempting the diagnostic questions

- Do not revise before attempting the questions. The idea is to apply your current working knowledge and skills and subsequently use your results to self-evaluate, identifying the topic areas where your capabilities can be developed by further study of the unit(s).

- All final answers should be given in exact form where appropriate. You should not use a calculator on any of the questions except for those marked with a * symbol.

- You should refrain from referring to the solutions, formula books, notes or other sources of information when you attempt the questions.

- Attempt as many of the diagnostic questions as you can. There is no time limit and you do not need to do all questions in one go. You could say, do 10 questions per day over a 5-day period.

- If you cannot do a question, leave it. Do not look up the solution. Any questions that you cannot do are very good indicators of the topic areas that you need to do some preparation work on before starting your undergraduate degree programme.

- Set your work out neatly, writing down the steps in your working.

Self-evaluation after completing the questions

- Carefully review your solutions against those provided. Tick all of the questions that you found easy and answered correctly.

- Put a mark next to any questions which you could not do or where you made errors due to say, not knowing how to complete the solution process or not remembering an important piece of information that was necessary to solve the problem.

- Identify any **units** (indicated to the left of the question group) e.g unit 9.1 where you made a number of errors on the questions or you were perhaps unable to do some of the questions. You should also identify any units in which you did not feel confident or which you felt were particularly difficult. These are the units for which we recommend that you review ***all*** of the unit content.

- If you performed well on a particular unit and were confident, except for say one question, you should review the particular **unit learning objective** (LO) in the column to the right of the question.

- You are not expected to review more than 5 units over the summer. If you identify more than 5 units for review, choose the 5 which you need to do most work on to improve your skills, knowledge and confidence.

- You can review all of the units if you wish to do so. There is no time limit on your review work but you should aim to complete it before coming to the university in late September.

- Further diagnostic questions will be set when you have completed your review units and results forwarded to the department you are entering. You will also complete a feedback questionnaire.

Unit	Qu No.	Question	LO
1.1	1*	Calculate $1\,257\,386\,000\,000\ \div 0.000\,000\,000\,371$ giving your answer in the standard form $A\ \times 10^{n}$ where n is an integer and the value of $A: 1 \leq A < 10$ is correct to 3 significant figures.	1.1.4
	2*	A vehicle travels a distance of 27 km in 2 hours, 43 minutes. Calculate the average speed of the vehicle in metres per second, giving your answer correct to the nearest integer.	1.1.6
	3*	A solid metal sphere has a radius of $r = 5.5$ cm measured correct to 1 decimal place, so its actual radius could lie in the range $5.45\text{ cm} \leq r < 5.55\text{ cm}$. The mass of the sphere is $m = 6100$ g correct to the nearest 100 g, so its actual mass could lie in the range $6050\text{ g}\ \leq m\ < 6150\text{ g}$. By considering the rounding errors in both stated measurements, calculate the maximum possible density of the metal sphere, giving your answer in kilogrammes per cubic metre (kgm^{-3}) correct to 2 decimal places.	1.1.7
	4	Write $\sqrt{18} + \sqrt{32}$ in the form $a\sqrt{b}$ where a and b are positive integers.	1.1.9
	5	Rationalise the denominator of, $$\frac{\sqrt{3}+2}{\sqrt{3}-1}$$	1.1.10
1.2	6	Simplify the following expression $$\frac{a^{\frac{3}{2}}b^{-2}(2c^{2})^{-2}}{4b^{-\frac{1}{2}}a^{3}}$$	1.2.1
	7	Find a possible expression in terms of n, for the n^{th} term in the following sequence, $$1, 9, 25, 49, \ldots$$	1.2.7
	8	Rearrange to following formula to make g the subject, $$T = 2\pi\sqrt{\frac{L}{g}}$$	1.2.5
	9	A certain recurrence relation is defined by, $u_{n+1} = 3u_{n} - 2,\ u_{0} = 5$. Find the value of u_{3}.	1.2.8
	10	A 20-sided polygon is constructed in such a way that that starting from the shortest side, each consecutive side is 2 cm longer than the previous side. The longest side joins back onto the shortest side to close the polygon. Given the perimeter of the polygon is 480 cm. Find the length of the shortest side.	1.2.9

2.1	11	A straight line passes through the points $A(2,5)$ and $B(7,15)$. Find the equation of the line in the form $ax+by+c=0$ where a,b,c are integers.	2.1.2
	12	Find in the form $y=mx+c$ the equation of the line which is the perpendicular bisector of the line segment PQ with end points $P(-2,-5)$ and $Q(4,7)$.	2.1.3
	13	Find the distance between the points $C(a,2)$ and $D(a+2,6)$	2.1.4
2.2	14	Given the quadratic equation $2x^2-(2+k)x+5=0$, where k is a constant, find the values of k such that the x-axis is tangential to the minimum point on the graph of the corresponding quadratic function.	2.2.4
	15	The graph of a parabola crosses the x-axis at the point $A(2,0)$ and at a second point, B. The vertex (minimum point) of the parabola is at $C(4,-4)$. Find the equation of the corresponding quadratic function in the form $f(x)=ax^2+bx+c$.	2.2.5
	16	By making a suitable substitution or otherwise, solve the bi-quadratic equation, $t^4-13t^2+36=0$	2.2.6
2.3	17	Determine if $x=3$ is a root of $p(x)=\ 2x^3-4x^2+7x-12$.	2.3.4
	18	The polynomial $p(x)=x^3+ax^2+bx-3$ has a remainder of 15 when divided by $(x-2)$. Given also that $p(-1)=0$, find the integer values of a and b.	2.3.3
3.1	19	Write $10^3=1000$ in logarithmic form.	3.1.3
	20	Rewrite the following as a single logarithm: $\ln(xy)^3+2\ln x^{\frac{3}{2}}-\frac{1}{2}\ln x^3y^2$	3.1.4
	21	Solve the exponential equation: $e^{2x}-7e^x+10=0$	3.1.5
5.1	22	Write down $\sin(-135°)$ as an angle in radians, giving your answer in terms of π.	5.1.1 5.1.6
	23	Sketch the graph of $y=2\sin 2x\,,-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ indicating the coordinates of the crossing points on the axes and any maximum/minimum points.	5.1.8 5.1.9
	24	Sketch the graph of $x=\cos\left(2\theta-\frac{\pi}{4}\right),-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$ indicating the coordinates of the crossing points on the axes and any maximum/minimum points.	5.1.8 5.1.9
	25	Sketch the graph of $\theta=\tan\left(\frac{t}{2}-\frac{\pi}{4}\right),-\pi\le t\le\pi$ indicating the coordinates of the crossing points on the axes and any maximum/minimum points.	5.1.8 5.1.9

5.2	26	Solve the equation $\sin t = \sqrt{2}\cos t$ on the interval $-\pi \le t \le \pi$	5.2.2
	27	Solve the equation $\sin\left(2x + \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}$ on the interval $-\pi \le x \le \pi$	5.2.4
	28	Solve the equation $2\sin^2\theta + \sin\theta - 1 = 0$ on the interval $-\pi \le \theta \le \pi$	5.2.5
	29	Solve $\frac{1}{2}\sin 2k - \frac{1}{2}\cos k + \frac{\sqrt{3}}{2}\sin k - \frac{\sqrt{3}}{4} = 0$ on the interval $-\pi \le k \le \pi$	5.2.5
5.3	30	Sketch the graph of $y = -\text{cosec}\,2\theta$ on the domain $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	5.3.2
	31	Solve the equation $2\cos x = \cot x$ on the domain $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	5.3.3
	32	Sketch the graph of $y = \tan^{-1}x$ where $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ and find the two solutions to the equation $\tan^{-1}\lvert x\rvert = \frac{\pi}{3}$	5.3.5
9.1	33	Given the two vectors **a** = 2**i** – 3**j** and **b** = 4**i** + 5**j**, find the vector 3**a** – 2**b**	9.1.8
	34	Find a unit vector parallel to **a** = 4**i** – 3**j**	9.1.4
	35	Given the two vectors $\overrightarrow{OA} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ find the distance $\lvert\overrightarrow{AB}\rvert$	9.1.9
10.1	36	Given the two complex numbers $z_1 = 2 + 4i$ and $z_2 = -5 + 3i$ find $Re(z_1) - Im(z_2)$	10.1.1
	37	Using the same two complex numbers as in question 36, evaluate $z_1^* z_2$	10.1.2 10.1.4
	38	Using the same two complex numbers as in question 36, evaluate $\frac{z_1}{z_2}$	10.1.5
	39	Find the complex roots of the quadratic equation $z^2 - 4z + 8 = 0$ indicate the roots on an Argand diagram. What is the connection between the two roots? State the name of the theorem that defines this connection.	10.1.3 10.1.6
	40	Find the modulus and argument of the complex number $z = 2 - i\sqrt{2}$	10.1.7 10.1.8
11.1	41	Differentiate the following function and find the gradient at the point where $x = \frac{\pi}{2}$. $$f(x) = 3x^3 - \cos x^2 \quad x \in \mathbb{R}$$	11.1.1
	42	Given $\theta = \sin\left(e^{-x} - \sqrt[3]{x}\right)$, find θ'	11.1.1

	43	Find the equation of the tangent to the curve $x = e^{2t}\tan t$ at the point where $t = \frac{\pi}{4}$	11.1.2
	44	Find any stationary points on the curve $y = xe^{x^2-x}$	11.1.2
	45	Use the second derivative test to determine the nature of the stationary point on the curve $$x = \frac{\ln 2t}{t^2} \quad t > 0$$	11.1.3
11.5	46	Find the following indefinite integral, $$\int 3x^3 + \frac{7}{x^2} - \sqrt[3]{x^4} + \ln 2 \ dx$$	11.5.1
	47	Find the total area enclosed between the curve $f(x) = x^3 - 4x$ and the x-axis on the interval $-2 \le x \le 2$.	11.5.2
	48	Find the following indefinite integral, $$\int \sin t \cos t - \frac{4}{t} + \frac{2}{e^t} \ dt$$	11.5.3
	49	Evaluate the following definite integral, $$\int_{\pi/6}^{\pi/3} \cot x \ dx$$	11.5.3
	50	A variable p changes with respect to an angle θ according to the relationship $p' = -\cos\theta + 1$. Given that $p\left(\frac{\pi}{6}\right) = \frac{1}{2}$ find an expression for p in terms of θ.	11.5.4