Diagnostic questions 2

Instructions and information

- These 20 questions are designed to take about 1 hour in total to complete.

- Refrain from referring to any notes or other sources of information when you attempt the questions although you can refer to a formula book and use a calculator as necessary.

- You will need some paper and a pen or pencil for working out.

- Enter your final answers all questions on the corresponding answer sheet.

- Questions 1-10 are multiple choice. Select the one correct answer from the 5 options available indicating your choice of A, B, C, D or E with an X on the provided answer sheet.

- After completing your solutions to questions 11-20 the final answer should go into the corresponding box on the provided answer sheet. Do not write any working out on the answer sheet.

- When you have finished, you may optionally submit the answer sheet to us for marking. You will receive your result in August/September depending on when you do the questions. The result will also be forwarded to the department you are entering. It does not count as credit towards any assessment or any other component of work on your degree programme.

Questions 1-10: are multiple choice. Choose the one correct answer A, B, C, D or E for each question and indicate your choice with an X on the answer sheet.

1. A sphere has a radius of 0.55 metres, correct to 2 decimal places. Find, in ***cubic centimetres***, the difference between the maximum and minimum possible volumes of the sphere, corresponding to maximum and minimum radius the sphere could have. The formula for the volume of a sphere is $V_{sphere} = \frac{4}{3}\pi r^3$. Choose the best answer.

   a. $380\ cm^3$

   b. $40000\ cm^3$

   c. $3.8 \times 10^4\ cm^3$

   d. $3800\ cm^3$

   e. $0.040\ m^3$

2. Given the following three expressions,

$$y_1 = a^3x^2b$$

$$y_2 = (ab)^3x$$

$$y_3 = ax^{-2}b^{-1}$$

   an expression for $\frac{y_1\,y_2}{y_3}$ can be written as,

   a. 1

   b. $a^5x^3b^5$

   c. $a\,x^{-2}b^5$

   d. $a^5x\,b^5$

   e. $(a\,b\,x)^5$

3. A circle is drawn such that both the x and the y axes are tangential to the circle. The centre of the circle has coordinates (a, b) where $a, b > 0$. The radius of the circle is $r = 5$. In the same Cartesian plane, a line segment is drawn from the points $P(2,6)$ to $Q(4,8)$. Find the exact distance from the centre of the circle to the midpoint of the line segment PQ.

a. $\sqrt{2}$

b. $2\sqrt{2}$

c. 2

d. $\frac{1}{2}$

e. None of the above answers are correct

4. The equation, $\sin 2\theta - \cos^2\theta = 0$, has two solutions, θ_1, θ_2 on the interval, $0 \leq \theta \leq \frac{\pi}{2}$. The sum of the two solutions is,

a. $\theta_1 + \theta_2 = \frac{\pi}{2} + \tan^{-1}\left(-\frac{1}{2}\right)$

b. $\theta_1 + \theta_2 = \tan^{-1}(\,2\,)$

c. $\theta_1 + \theta_2 = \frac{\pi}{2}$

d. $\theta_1 + \theta_2 = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{2}\right)$

e. $\theta_1 + \theta_2 = 0$

5. The graphs of three functions on the domain $-10 \leq x \leq 10$ are shown in figure 1; $f(x)$ (a quadratic), $g(x)$ (a cubic) and $h(x)$ (a quartic). Choose the most appropriate statement to describe the properties of the functions.

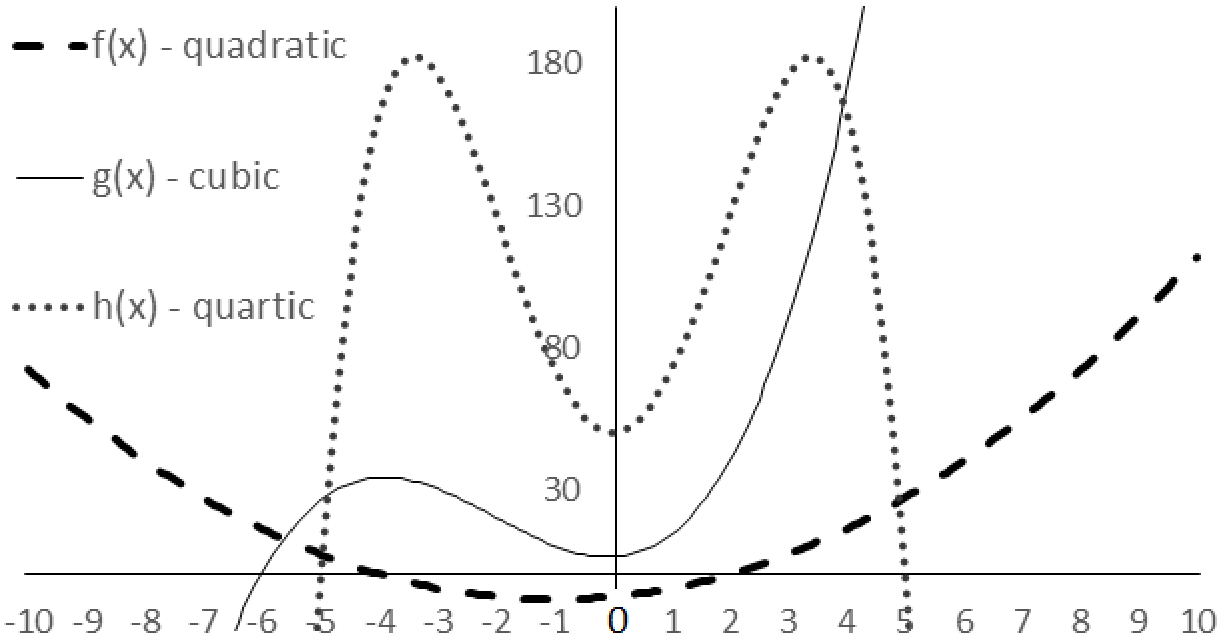

Figure 1

a. The coefficient of x^4 in $h(x)$ is negative, the discriminant of the quadratic, $f(x)$ is less than zero, the equation $g(x) = 0$ has one real root.

b. The coefficient of x^3 in $g(x)$ is positive, the coefficient of x^2 in $f(x)$ is positive, the discriminant of the quadratic, $f(x)$ is zero.

c. The coefficient of x^4 in $h(x)$ is negative, the discriminant of the quadratic, $f(x)$ is greater than zero, the equation $g(x) = 0$ has one real root, the equation $h(x) = 0$ has two real roots.

d. The coefficient of x^4 in $h(x)$ is negative, the discriminant of the quadratic, $f(x)$ is greater than zero, $f(x) + g(x) + h(x)$ must form a new function containing x^5.

e. The equation $h(x) = g(x)$ must have at least two real roots, the coefficient of x^4 in $h(x)$ is positive, the discriminant of the quadratic, $f(x)$ is less than zero, the equation $g(x) = 0$ has one real root.

6. Find an equivalent expression for $\sqrt{\frac{(1-\sin x)}{(1+\sin x)}}$ where $-\frac{\pi}{2} < x < 0$.

a. $\sec x - \tan x$

b. $\tan x + \sec x$

c. $\operatorname{cosec} x + \tan x$

d. $\operatorname{cosec} x - \tan x$

e. $\sec x - \cot x$

7. Given $\tan\alpha = 1/5$ and $\tan(\alpha + \beta) = 5$ where α and β are acute, find the value of $\tan\beta$.

a. 12/5

b. 6/5

c. 1

d. $25/\sqrt{5}$

e. $\sqrt{5}$

8. A parabolic antenna shown in figure 2 is aligned to point vertically upward with respect to a local, $x - y$ coordinate system, with a fixed origin O. The parabolic arc AB can be defined by a quadratic function of the form $f(x) = \frac{1}{16}x^2 + px + q$. The line SS' at $x = 4$ metres is the vertical axis of symmetry of the arc AB. The point A on the parabola of $f(x)$ is located at $(-4,\ 40)$ metres, relative to O. Find the coordinates of the vertex V.

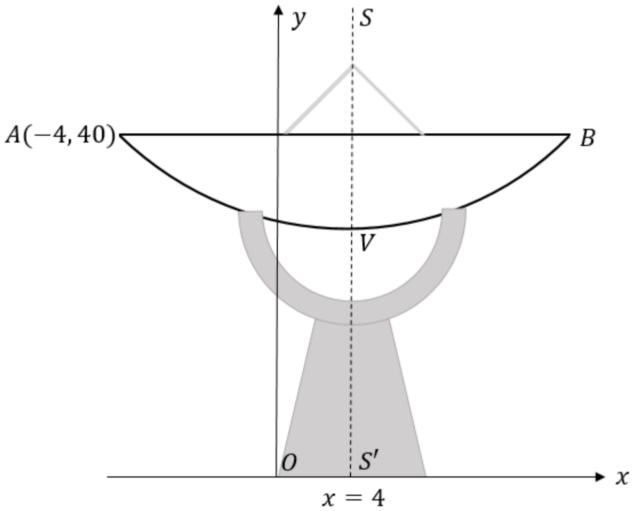

Figure 2

a. $(4, 36)$

b. $(4, 35)$

c. $(4, 38)$

d. $(4, 37)$

e. $(4, 34)$

9. A sequence comprising the terms $\{U_n\}$ $n \geq 1$ is defined by the recurrence relation $U_{n+2} = kU_{n+1} + U_n$ where k is a real constant. Given $U_1 = 5$, $U_2 = 2$ and $U_4 = 0$ find the possible value(s) of k.

a. 2

b. $2\ or -0.5$

c. $-2\ or\ 0.5$

d. $2\ or\ 0.5$

e. $-2\ or -0.5$

10. Figure 3 below shows a parallelogram OABC, where the position vectors $\overrightarrow{OA}$ = 3**q** + **p** and $\overrightarrow{OB}$ = 8**q** + 5**p**. Given that D is the point such that $\overrightarrow{CA} = \frac{1}{2}\overrightarrow{CD}$, find the position vector $\overrightarrow{OD}$ in terms of **q** and **p**.

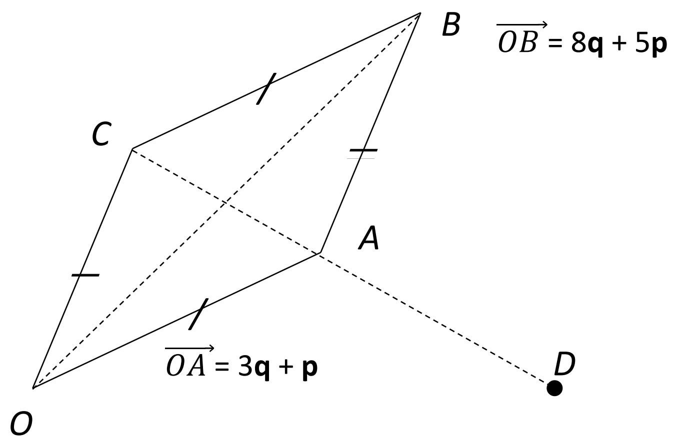

Figure 3

a. $\overrightarrow{OD}$ = -9**q** - 10**p**

b. $\overrightarrow{OD}$ = 15**q** - 15**p**

c. None of the solutions are correct

d. $\overrightarrow{OD}$ = **q** - 2**p**

e. $\overrightarrow{OD}$ = -5**q** - 10**p**

Questions 11-20: have a single final answer. Write your final answer in the space provided for each question on the answer sheet.

11. By making a suitable substitution find all the *real solutions* to the equation $4x^4 - 16x^2 - 9 = 0$.

12. Consider the polynomial $f(x) = 2x^4 + ax^3 + bx^2 - 20x + 12$. Given that $f(x)$ is exactly divisible by $(x + 2)$ and that on division by that on division by $(x + 1)$ the remainder is 18, find the value of a and b.

13. Solve the equation $15e^{-4t} - 8e^{-2t} = -1$, giving your answer(s) in exact, logarithmic form.

14. Convert the complex number $z = \frac{1-i}{2+i}$ to the form $z = x + yi$, where $x, y \in \mathbb{R}$.

15. Differentiate the function $f(\theta) = \frac{3\cos\theta}{\theta}$ $(\theta \neq 0)$ and find the gradient of the curve for $f(\theta)$ at the point where $\theta = \frac{\pi}{2}$.

16. Find the coordinates of the local *minimum* point on the graph of $y = x^3 - 27x + 9$.

17. It is known that the derivative of a function y is $\frac{dy}{dx} = \frac{2+\cos x}{\sin x + 2x + 2}$ and that a point on the graph of $y = f(x)$ is $(0, \ln 2)$. Find y in terms of x $(x \geq 0)$.

18. Find the following indefinite integral,

$$10\int \cos(2\theta)\sin^4(2\theta)d\theta$$

19. Evaluate the following definite integral,

$$\int_0^1 \frac{dx}{2x+1}$$

20. Find the area contained between the graph of $f(x) = \sqrt{x+2}$ and the x-axis between $x = -2$ and $x = 2$.

END OF QUESTIONS