FUTURE project

Simplified Modelling (WP4)

Outline of some previous/current relevant work + FUTURE plans

FUTURE Partners Meeting, 26th July 2021

Motivation

- LES expensive; gives very detailed information
- Need simpler, fast, reduced models
- Can be developed/calibrated/evaluated against LES/WT/Field data
- Can provide generic insight and simple applicable results/rules of thumb
- FUTURE will use/extend existing canopy and building wake models

The canopy double-averaging framework

- Remove spatial (horizontal) heterogeneity as well as temporal fluctuations
- Take horizontal averages over several streets: resolve vertical structure

Triple decomposition of instantaneous velocity field

Spatial fluctuation

$$u_i = U_i + \tilde{u}_i + u'_i.$$

Space-time mean   Temporal fluctuation

Eg spatial average of Reynolds-averaged Navier-Stokes equation

$$\frac{\partial \langle \overline{u_i} \rangle}{\partial t} + \langle \overline{u_j} \rangle \frac{\partial \langle \overline{u_i} \rangle}{\partial x_j} = -\frac{\partial \langle \overline{p} \rangle}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + D_i$$

$$\tau_{ij} = -\langle \overline{u'_i u'_j} \rangle - \langle \tilde{u}_i \tilde{u}_j \rangle + \nu \frac{\partial \langle \overline{u_i} \rangle}{\partial x_j},$$

Reynolds stress   Dispersive stress

$$D_i = \frac{1}{V} \int_S \overline{p}\, n_i \, dS - \frac{\nu}{V} \int_S \frac{\partial \overline{u}_i}{\partial n} \, dS.$$

Drag

Finnigan (2000), Coceal & Belcher (2004, 2006)

Produces extra terms: Reynolds and Dispersive fluxes and Drag – these need to be parametrized

Challenge is to incorporate correct physics for different regimes in the parametrizations

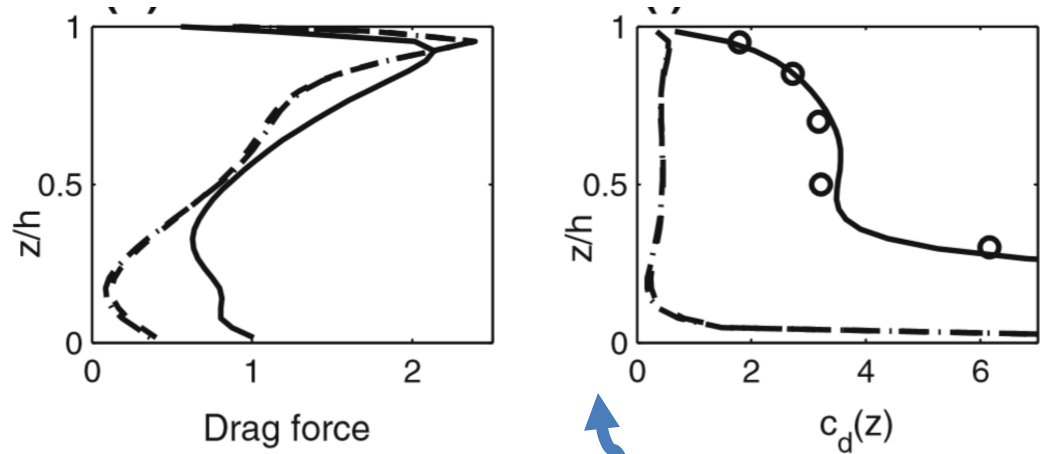
Distributed drag: parametrize $c_d(z)$ or $D(z)$?

Coceal & Belcher (2004)

$$D_i = \frac{1}{2}\frac{c_d(z)\lambda_f}{h(1-\beta)}|U|U_i = \frac{|U|U_i}{L_c},$$

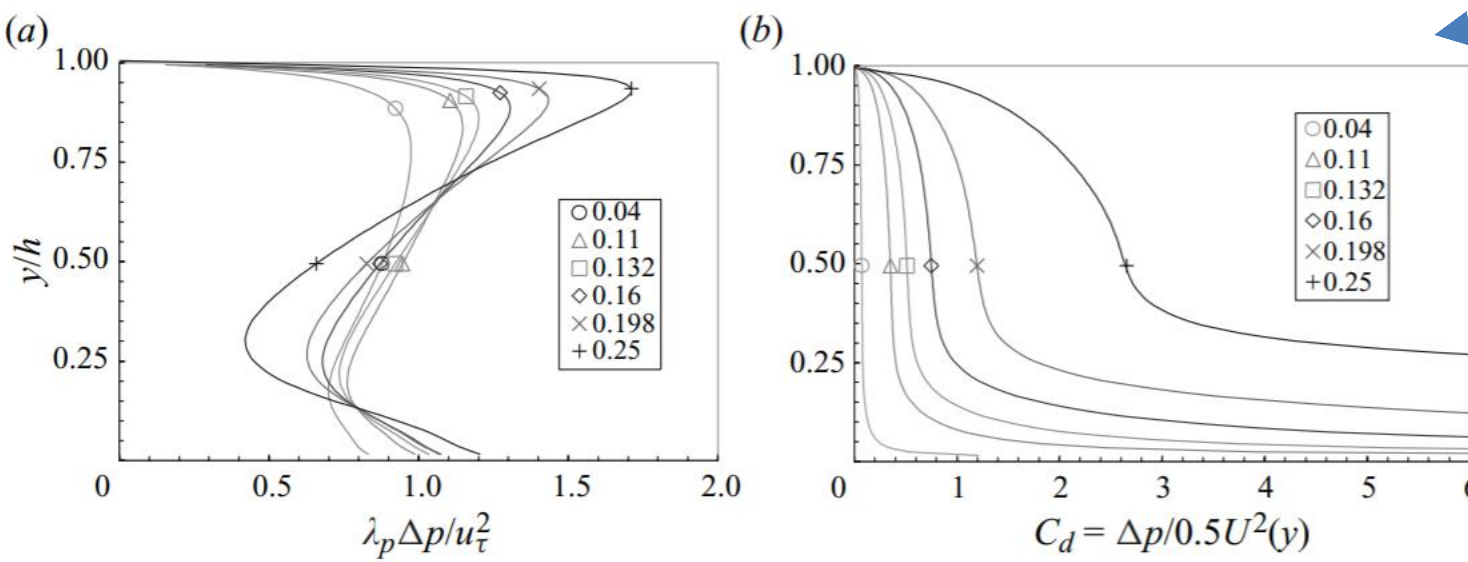
$$L_c = \frac{2h}{c_d(z)}\frac{(1-\beta)}{\lambda_f}.$$

Coceal et al. (2006)



Effects of building layout and density

Leonardi & Castro (2010)



Turbulence parametrization – K(z) or l_m (z)?

$$\frac{d\tau}{dz} = D \qquad\qquad D(z) = \alpha(z)\ U^{\delta}(z) \qquad \delta\ \in\ \{0,1,2\}$$

K-based

$$\tau(z) = K(z)\ U'(z) \qquad\qquad U''(z) + \frac{K'(z)}{K(z)}\ U'(z) = \frac{\alpha(z)}{K(z)}\ U^{\delta}(z)$$

$$\text{If } \delta = 0 \qquad\qquad KU'' + K'U' = \alpha$$

$$U(z) = \int^{z} \frac{1}{K(y)} \int^{y} \alpha(s)\ ds\ dy$$

l_m - based

$$K = l_m^2\ |U'| \qquad\qquad U''(z) + \frac{l_m'(z)}{l_m(z)}\ U'(z) = \frac{\alpha(z)}{2l_m^2(z)}\ \frac{U^{\delta}(z)}{U'(z)}$$

Coceal et al. (2021), in prep.

Mixing-length: an old model vs. reality (DNS)

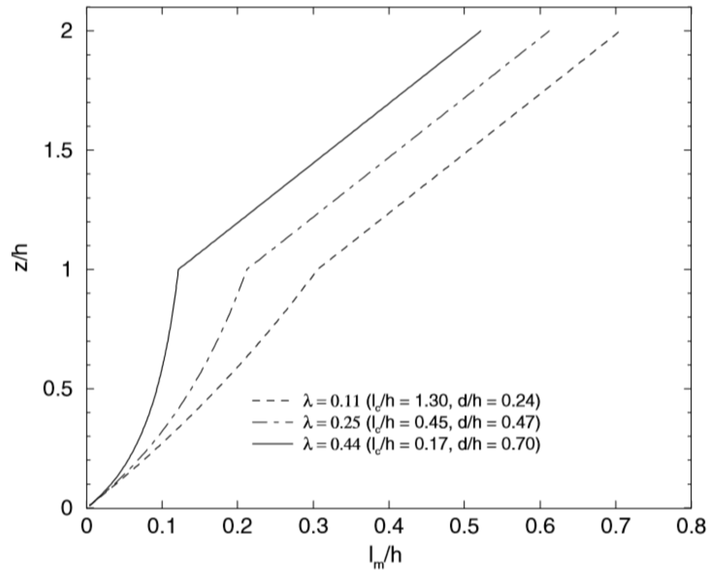
Coceal & Belcher (2004): l_m model

$$l_{\mathrm{m}} = \kappa(z-d)$$

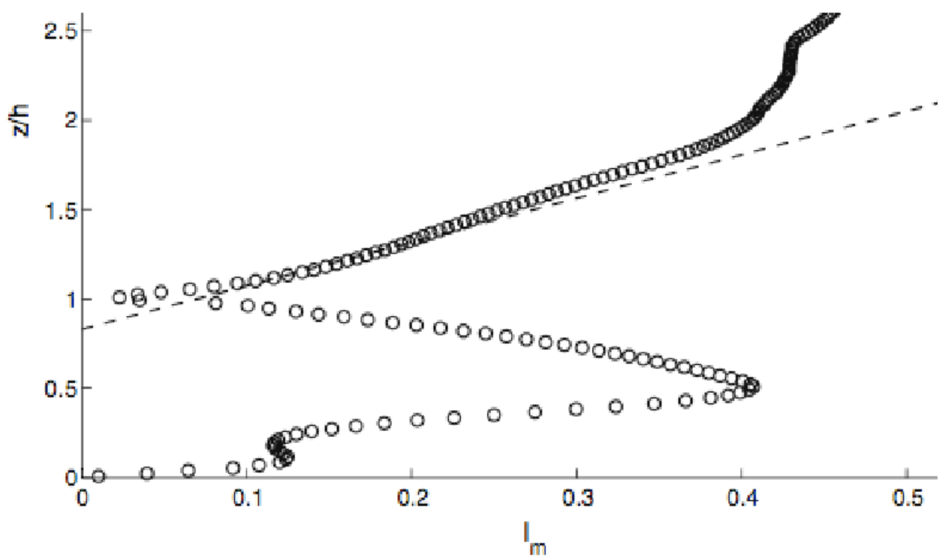
Above canopy

$$\frac{1}{l_{\mathrm{m}}} = \frac{1}{\kappa z} + \frac{1}{l_{\mathrm{c}}}.$$

Within canopy



Coceal et al. (2006): typical profile from DNS

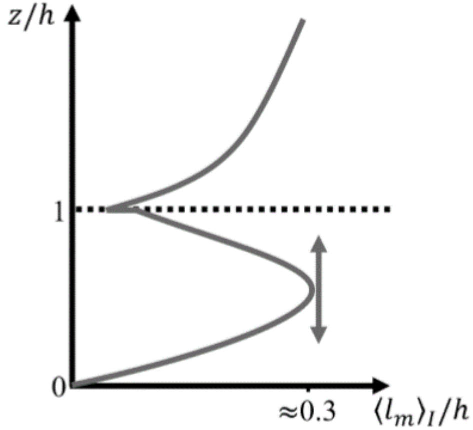


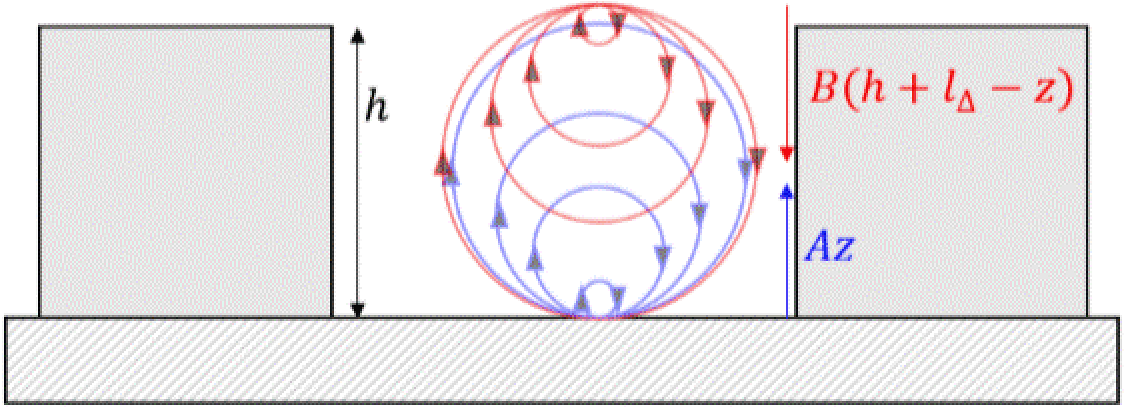
Blocking of eddies by canopy-top shear layer

The scheme gets l_m wrong in the canopy

We need a new scheme!

Mixing-length: a new, better model

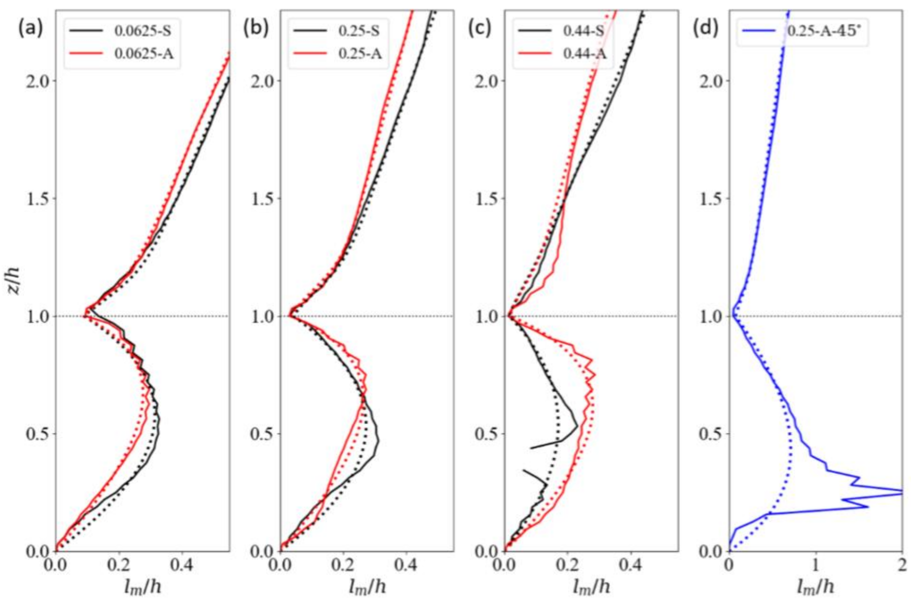




Blunn et al. (2021): schematic of l_m profile

$$\frac{1}{l_m} = \frac{1}{Az} + \frac{1}{B(h + l_\Delta - z)}$$

- Can find A, B and l_Δ empirically
- Let $l_m(h)$, z_m, $l_m(z_m)$ ~ f(λ)
- Gives 3 algebraic equns
- Solve simultaneously
- Extended for variable building heights



Blunn et al. (2021), in prep.

Simple analytical models of the mean wind profile – how correct are they?

$$\frac{\partial}{\partial z}\langle\overline{u'w'}\rangle = D_x$$

$$D_x = U^2/L_c$$

$$\langle\overline{u'w'}\rangle = l_m^2(\partial U/\partial z)^2$$

$$L_c = \frac{2h}{c_d(z)}\frac{(1-\beta)}{\lambda_f}.$$

Assuming both l_m (= l_c) and L_c (i.e. c_d) are constant

$$\frac{\partial}{\partial z}\left(\frac{\partial U}{\partial z}\right)^2 = \frac{U^2}{l_c^2 L_c}$$

$$U = U_h\,\mathrm{e}^{(z-h)/l_s} \qquad l_s = (2l_c^2 L_c)^{1/3}$$

Cionco (1965)
Macdonald (2000)

Exponential solution is simple, convenient and popular

But it suffers from theoretical weaknesses (e.g. wrong limiting behaviour near ground and for sparse canopies) and questionable empirical support about assumptions (e.g. crude assumptions about l_m and c_d) and comparisons with LES and DNS data (Castro, 2017)*

* 'Are urban-canopy velocity profiles exponential?' – No!

Constructing approximate analytical solutions respecting correct physics

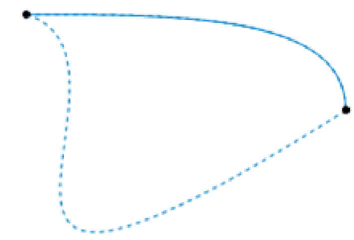
- Can always solve numerically. But not as insightful as an equation, e.g. cannot see the parameter dependence.
- Also, analytical solutions are very easy to implement in larger models and useful for quick estimates.
- Perturbation approach: approximate the maths instead of the physics.
- Can we use exact solutions to simpler, solvable problems?

The homotopy perturbation method (HPM) - outline and example

$$N(u) \equiv u''(z) - u^2(z) = 0 \qquad u(0) = 0, u(1) = 1$$

He (1999)

$$L(u) \equiv u''(z) - 2a = 0$$



$$H(u,p) \equiv (1-p)\ L(u) + p\ N(u) = 0 \qquad p \in [0,1]$$

$$H(u,p) \equiv L(u) + p\ [N(u) - L(u)] = 0$$

$$u'' - 2a + p\ [2a - u^2] = 0 \qquad u(z) = \sum_{n=0}^{\infty} u_n\ p^n$$

$$(u_0'' - 2a) + (u_1'' + 2a - u_0^2)\ p + (u_2'' - 2u_0u_1)\ p^2 + \ldots = 0$$

$$p^0 : \qquad u_0'' = 2a$$

$$p^1 : \qquad u_1'' = u_0^2 - 2a$$

$$p^2 : \qquad u_2'' = 2u_0u_1$$

$$p = 1$$

$$u(z) = u_0 + u_1 + u_2 + \ldots$$

$$u_0(z) = (1-a)\ z + a\ z^2$$

$$u_1(z) = (-\tfrac{1}{12} + \tfrac{16}{15}a - \tfrac{1}{60}a^2)\ z - a\ z^2 + \tfrac{1}{12}(1-a)^2\ z^4 + \tfrac{1}{10}a(1-a)\ z^5 + \tfrac{1}{30}a^2\ z^6$$

The parameter *a* can be optimised in various ways (e.g. method of weighted residuals)

Application of the HPM to a realistic urban canopy model

$$D_i = \frac{1}{2}\frac{c_d(z)\lambda_f}{h(1-\beta)}|U|U_i = \frac{|U|U_i}{L_c},$$

$$L_c = \frac{2h}{c_d(z)}\frac{(1-\beta)}{\lambda_f}.$$

$$\frac{1}{l_m} = \frac{1}{\kappa z} + \frac{1}{l_c}.$$

Coceal & Belcher, 2004 (CB04)

$$z\,U'' + \frac{1}{(z+1)}\,U' - \alpha\frac{(z+1)^2}{z}\frac{U^2}{U'} = 0$$

A number of simpler models can be employed as the unperturbed problem

Close to the ground this looks like the simpler (but still non-linear) equation:

$$z\,U'' + U' - \beta\,U^2 = 0.$$

HPM using solution of a linearised equation

$$z\,U'' + \frac{1}{(z+1)}\,U' - \alpha\frac{(z+1)^2}{z}\frac{U^2}{U'} = 0$$

$$\Downarrow$$

$$z\,U'' + U' - \beta\,U^2 = 0.$$

$$\Downarrow$$

$$z\,U_0'' + U_0' - \gamma\,U_0 = 0$$

$$U(z) = C_1\ I_0(2\sqrt{\gamma z}) + C_2\ K_0(2\sqrt{\gamma z})$$

Bessel function solution to the linearized equation fits *at zeroth order* simply by matching the HPM parameter at one point for dense as well as sparse canopies

Hence, the fully nonlinear CB04 model can 'borrow' the Bessel function solution across the full range of canopy densities!

Can obtain $\gamma(\alpha)$ 'empirically' (e.g. using method of weighted residuals)

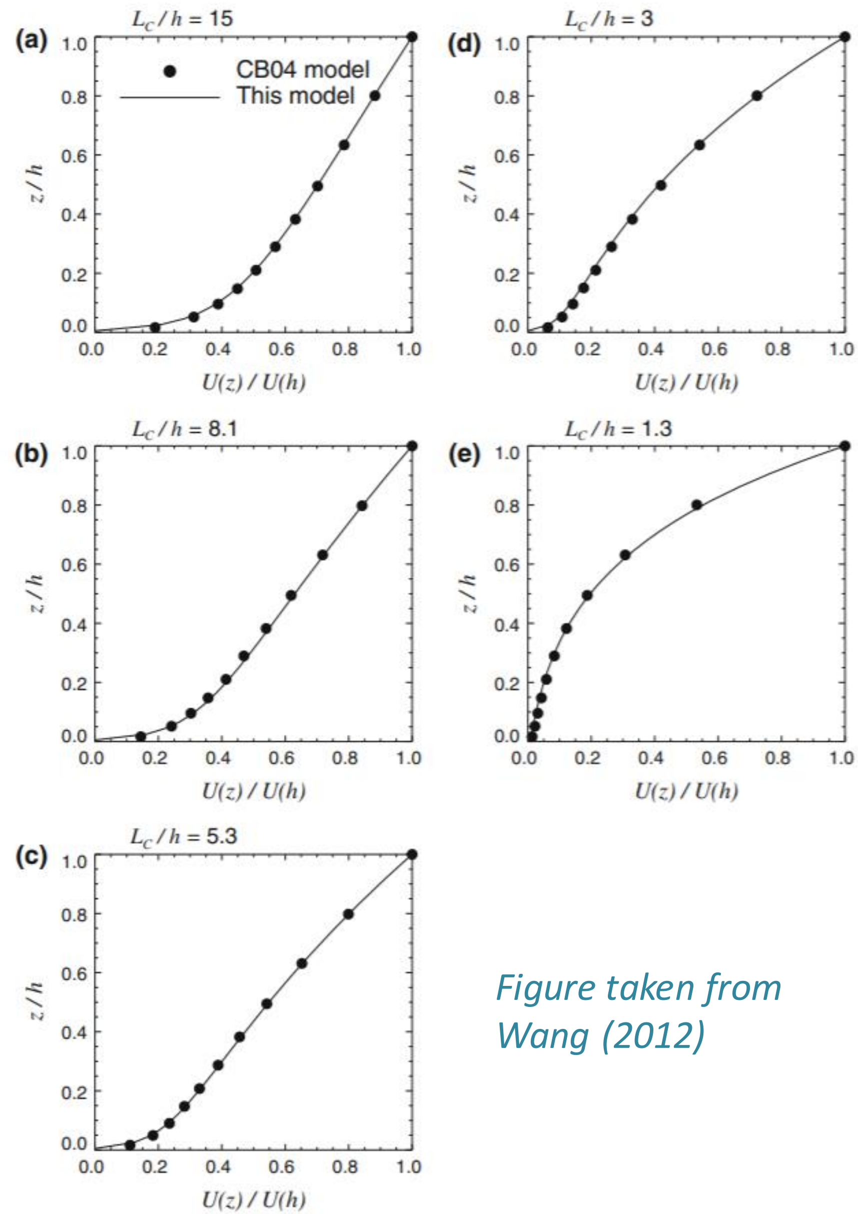
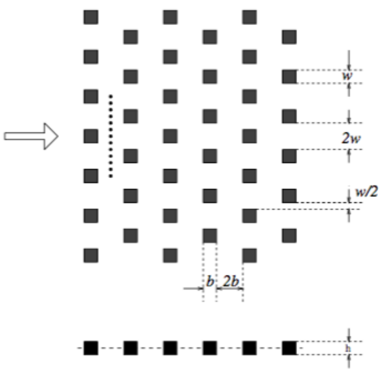
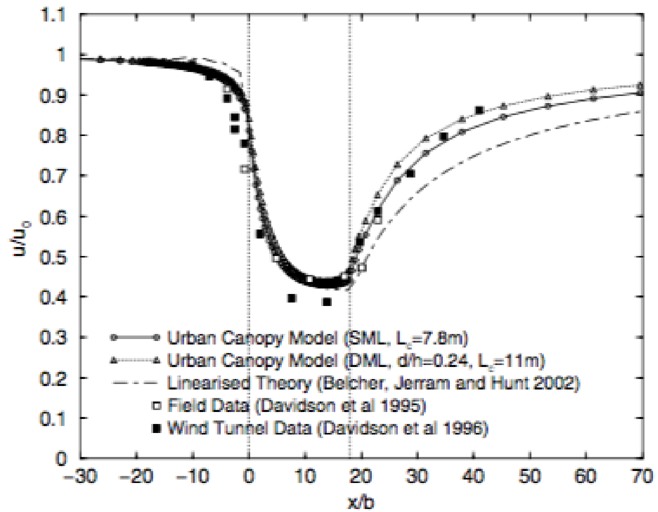


Figure taken from Wang (2012)

Heterogeneity - modelling flow adjustment



Coceal & Belcher (2004)



Full nonlinear model vs linearised analytical solutions *(Belcher et al., 2003)*

$$x_0 \sim L_c \ln K$$

$$K = (U_h/u_*)(h/L_c)$$

$$L_c/h = \frac{2}{c_d}\left(\frac{1-\lambda}{\lambda}\right)$$

$$x_0 = 3L_c \ln K$$

$$N_c = \frac{2}{c_d}\left(\frac{1}{\sqrt{\lambda}} - \sqrt{\lambda}\right)$$

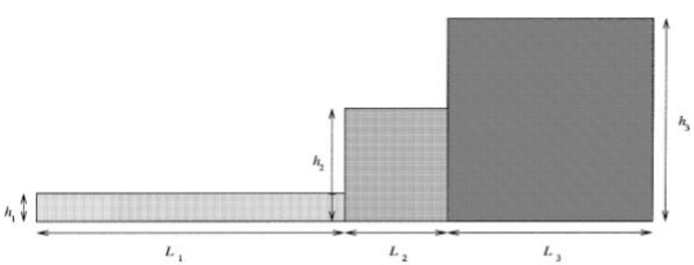


Figure 6. Representation of a selected section of Los Angeles in a two-dimensional simulation as a series of three canopies. The relevant dimensions are as follows: $h_1 = 6.4$ m, $h_2 = 24.5$ m, $h_3 = 45.0$ m and $L_1 = 800$ m, $L_2 = 400$ m, $L_3 = 1200$ m.

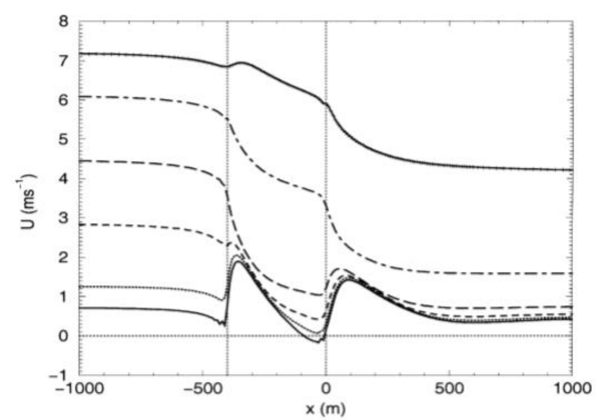


Figure 7. Variation of mean horizontal wind speed with fetch at different heights through the selected region of Los Angeles. Solid line: $z = h_1/4$; dotted line: $z = h_1/2$; dashed line: $z = h_1$; long-dashed line: $z = 2h_1$; dot-dashed: $z = 4h_1$; plus symbols: $z = 8h_1$. See text for description.

Coceal & Belcher (2005)

FUTURE work plan

- Extend to include effects of tall buildings
- Investigation of different geometrical setups: e.g. isolated tall building in a canopy of lower buildings; small building cluster; canopy of tall buildings
- Develop and validate with LES and WT/Field data
- Extension of model to 2D & 3D; variable averaging area; downwind modelling
- Effect of non-neutral stratification
- Wake models for dispersion applications
- Coupled wake-canopy modelling
- Simplified formulae and rules of thumb for providing quick estimates

Extra slides

A linearized analytical model valid for sparse canopies

$$F_{\mathrm{d}} = C_{\mathrm{d}} a(z) U(z) \left| U(z) \right| \quad \Longrightarrow \quad F_{\mathrm{d}} = C_{\mathrm{L}} a_0 U(z) \left| U_{\mathrm{h}} \right|$$

$$\tau(z) = -\left\langle \overline{u'w'} \right\rangle = K \frac{\partial U}{\partial z} \qquad K = l_{\mathrm{m}} u_{*} \qquad l_{\mathrm{m}} = \kappa z s$$

$$-\frac{\partial}{\partial z}\left(\kappa z s_{\mathrm{h}} u_{*} \frac{\partial U(z)}{\partial z}\right) = -\frac{1}{\rho}\frac{\partial P}{\partial x} - C_{\mathrm{L}} a_0 U(z) U_{\mathrm{h}}$$

$$U(z) = C_1 I_0\left(g(z)\right) + C_2 K_0\left(g(z)\right) + U_{\mathrm{p}}$$

I_0 and K_0 are Bessel functions

$$g(z) = 2\sqrt{A \frac{z}{h}}$$

Some assumptions made specifically to allow exact analytical solution!

Wang (2012)

Expected to work fairly well for sparse canopies