Discussion Papers in Economics

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DP 03/22

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ISSN: 1749-5075

Identification through the Forecast Error Variance Decomposition: an Application to Uncertainty*

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March 11, 2022

Abstract

We develop a novel approach to achieve point identification in a Structural Vector Autoregression, based on imposing constraints on the forecast error variance decomposition. We characterize the properties of this approach and provide Bayesian algorithms for estimation and inference. We use the approach to study the effects of uncertainty shocks, allowing for the possibility that uncertainty is an endogenous variable, and distinguishing macroeconomic from financial uncertainty. Using US data we find that macroeconomic uncertainty is mostly endogenous, and that overlooking this fact can lead to distortions on the estimates of its effects. We show that the distinction between macroeconomic and financial uncertainty is empirically relevant. Finally, we study the relation between uncertainty shocks and pure financial shocks, showing that the latter can have attenuated effects if one does not take into account the endogeneity of uncertainty.

Keywords: Causality, Financial Conditions, Identification, Structural Vector Autoregression, Uncertainty.

JEL: C11, C32, E32, E37, E44.

*This paper previously circulated with the title "Identification of Uncertainty Shocks" and "The Identifying Information in the Forecast Error Variance: an Application to Endogenous and Heterogeneous Uncertainty and its Relationship with Financial Shocks". We thank Ana Galvão, Valentina Corradi, Luca Fanelli and Stepana Lazarova for insightful suggestions. We are also grateful to participants to 2021 RES conference, 2021 AMES conference, 2021 IAAE conference, 7^{th} RCEA Time Series workshop, 2021 EEA-ESEM Conference, Workshop in Macroeconometrics, 2021 EWMES, and 2021 CEF for beneficial discussion.

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1 Introduction and Related Literature

Since the influential paper of Bloom (2009), the business cycle relationship between uncertainty and macroeconomic variables and the underlying transmission mechanism have received extensive consideration.[1] Several measures of uncertainty have been proposed, and many scholars have analyzed the macroeconomic effects of uncertainty shocks.[2]

Three challenges come to the fore. First, most works usually employ structural vector autoregressions (SVARs) with some recursive identification scheme. The common assumption is that uncertainty is exogenous, i.e. it does not respond contemporaneously to economic variables, whereas economic variables react contemporaneously to uncertainty.[3] Recursive schemes are widespread due to the simplicity of implementation and interpretation, but for uncertainty it is extremely challenging to defend them as convincing identification strategies.

In fact, the current evidence makes researchers unable to take up a position on the direction of the causality between uncertainty and economic variables: there is no uncontroversially accepted, theoretically grounded belief to assert that a specific recursive or sign restriction scheme is credible for identification of uncertainty shocks. On the contrary, both directions of causality are conceivable and macroeconomic theory is also ambiguous about the possible sign of the effects of uncertainty on the economy.

Uncertainty can affect the economy through firms' behavior, which is influenced by uncertainty because of (i) the real option argument (Bernanke, 1983; McDonald & Siegel, 1986); (ii) the delay of hiring and investment decisions (Bloom, 2009; Bloom et al., 2018; Leduc & Liu, 2016); (iii) the interaction with financial frictions that impact on firms' decisions (Arellano et al., 2018; Gilchrist et al., 2014; Alfaro et al., 2018). The uncertainty can influence the economy also through precautionary savings (Basu & Bundick, 2017; Fernández-Villaverde et al., 2011). On the other hand, some scholars have pointed out that bad economic and/or credit conditions are likely to cause a rise uncertainty (Van Nieuwerburgh & Veldkamp, 2006; Bachmann & Moscarini, 2011; Fajgelbaum et al.,

[1] Bloom (2014) provides an excellent survey.

[2] A partial list of works consists of Bloom (2009), Bachmann et al. (2013), Caggiano et al. (2014), Jurado et al. (2015), Rossi and Sekhposyan (2015), Caldara et al. (2016), Baker et al. (2016), Basu and Bundick (2017), Cesa-Bianchi et al. (2018), Shin and Zhong (2020), Carriero et al. (2018b), Bloom et al. (2018), Angelini et al. (2019), Ludvigson et al. (2021), and Carriero et al. (2021).

[3] We use the terms exogenous (endogenous) as shorthand for predetermined (not predetermined) within the period.

2017; Brunnermeier & Sannikov, 2014; Atkinson et al., 2021; Plante et al., 2018). Empirical contributions that have allowed for both directions of causality include Carriero et al. (2021), Ludvigson et al. (2021), and Angelini et al. (2019). All these contributions have shown that the direction of causality might depend on the uncertainty typology and measure of choice. Additional literature pointed out that uncertainty can stimulate economic activity (growth options theory): a mean-preserving spread in risk originated from an unbounded upside combined with a limited downside can lead firms to invest and hire, since the rise in mean preserving risk raises expected profits.[4]

A separate challenge is about the origins of uncertainty. Standard theories claim that uncertainty originates from macroeconomic fundamentals, e.g., productivity, and that such real economic uncertainty, when interacted with market frictions, decreases real activity. However, it has been argued that uncertainty depresses the economy via its impact on financial markets (Gilchrist et al., 2014), or through sources of uncertainty specific to financial markets (Bollerslev et al., 2009). Furthermore, Ng and Wright (2013) discuss that financial uncertainty –as distinct from macroeconomic uncertainty– could have a pivotal role in recessions after 1982, both as a cause and as a propagation channel. The challenge also arises because the theoretical literature has focused on volatility coming from fundamentals, while empirical efforts have usually tested those frameworks employing uncertainty proxies that are strongly correlated with financial market variables. This naturally leads to wonder whether it is macroeconomic uncertainty or financial uncertainty (or both) to drive business cycle fluctuations. The current literature does not disentangle the contributions of macroeconomic versus financial uncertainty to business cycle fluctuations, nor it allows feedback between macroeconomic and financial uncertainty. Exceptions are the small-scale models in Ludvigson et al. (2021) and Angelini et al. (2019) and the contribution in Shin and Zhong (2020).

The final challenge is that there is high degree of comovement between indicators of financial distress such as credit spreads and uncertainty proxies as both variables are "fast moving", as pointed out in several studies including Caldara et al. (2016), Brianti (2021), Caggiano et al. (2021). It is therefore difficult to impose plausible zero contemporaneous restrictions to identify these two disturbances. It is also difficult to impose sign restrictions

[4]For instance, see Oi (1961), Hartman (1972), Abel (1983), Bar-Ilan and Strange (1996), Pástor and Veronesi (2006), Kraft et al. (2018), Segal et al. (2015), and Fernández-Villaverde and Guerrón-Quintana (2020).

as uncertainty and financial shocks could have theoretically the same qualitative effects on both prices and quantities.

This paper proposes a new approach to identification which allows to deal with the three issues above. The approach allows for endogeneity of uncertainty, i.e. for a causal transmission channel going from uncertainty to the economic variables as well as the opposite causal mechanism going from the economic variables to uncertainty. It also allows to separately identify different sources of uncertainty, and to disentangle uncertainty shocks from pure financial shocks. To our knowledge, this is the first paper to tackle these issues in a unified framework. While we focus on uncertainty and financial disturbances, the identification and estimation toolkit developed in this paper is general, and can be applied in any SVAR where standard ordering and sign restrictions are not desirable or sufficient to identify all of the shocks of interest (as discussed in Section 2.3).

The proposed identification scheme is based on constraints on the Forecast Error Variance (FEV) decomposition of a Structural Vector Autoregression (SVAR). The key idea is that the structural shock to a variable in the system must be the main responsible for the variation in that variable. For example, consider the task of identifying a macroeconomic uncertainty shock, a financial uncertainty shock, and a credit supply shock:[5] the identifying assumption is that the structural shock to macroeconomic uncertainty is such that: i) it explains (in the short run) the unexpected movements of macroeconomic uncertainty variable more than it explains the fluctuations of financial uncertainty or the credit spread, and ii) it maximizes its contribution to (a function of) the total variation of macroeconomic uncertainty. The first requirement identifies a set of plausible structural models, while the second requirement point-identifies a single model. The same approach is used to simultaneously identify also financial uncertainty and credit supply shocks.

Our identification strategy involves the solution a constrained maximization problem, where the objective function is an equally weighted linear combination of the FEV of the variables of interest and the constraints are the inequality restrictions on the FEV. We show that this corresponds to a quadratic optimization problem on the columns of the rotation matrix transforming reduced-form residuals into structural shocks. We provide a flexible toolkit and establish mild conditions under which the solution of the optimization problem exists and is unique. We develop simple algorithms to perform

[5]A credit supply shock is defined as a shock to credit supply and measured through the credit spreads.

Bayesian estimation and inference, even though of course the identification result and properties do hold also in a frequentist setting.

The FEV decomposition has been already used as an identification device. A major example is the method put forward by Uhlig (2004) in which the FEV decomposition is optimized under a set of constraints given by sign restrictions, which can be individually verified for each FEV decomposition of each variable. In our approach instead the constraints are inequalities restrictions that have to hold across the FEV decompositions of different variables: hence they need to be verified for all the shocks simultaneously. This is computationally more challenging, but has the advantage of (i) identifying simultaneously a multiplicity of shocks, and (ii) being well-suited (even) when sign restrictions are unavailable, or cannot help distinguish competing shocks: uncertainty disturbances are a natural example. Furthermore, the fact that inequality restrictions on the FEV decomposition correspond to quadratic constraints on the rotation matrix has some effects on estimation and inference with respect to the case of sign restrictions.

The approach also differs from Amir-Ahmadi and Drautzburg (2021) and Volpicella (2021). Amir-Ahmadi and Drautzburg (2021) employed set-identification through ranking restrictions on the impulse response functions, combined with standard sign restrictions. Volpicella (2021) puts sign restrictions and bounds on the FEV to set-identify a single shock; on the contrary in this paper we point-identify shocks, do not place bounds, and allow identification of a multiplicity of shocks.

Turning to the empirical application, we start with a simulation exercise in which we show that our approach recovers the impulse response functions in different Data Generating Processes (DGPs), with exogenous or endogenous uncertainty; our identification strategy successfully captures the effect of uncertainty on the economy regardless the exogeneity extent of the uncertainty disturbances in the DGP.

We apply the proposed identification scheme to a SVAR model estimated with US data. We find that both macroeconomic and financial uncertainty shocks act as negative demand shocks, i.e. decrease the real activity and trigger a deflationary pressure. The responses to the two shocks are quantitatively substantially different: macroeconomic uncertainty has a stronger and more persistent effect on the real activity variables. We also find evidence that separating macroeconomic from financial uncertainty is important, and not doing so can dramatically distort the impulse responses.

We find evidence that uncertainty is endogenous to some extent. In particular, dismissing the feedback effect from the macroeconomy to macroeconomic uncertainty changes the estimated responses in non trivial ways. This suggests that naive schemes such as sign restrictions and recursive ordering are too restrictive. These results are in line with Ludvigson et al. (2021) and Carriero et al. (2021). Angelini et al. (2019) instead found that both macro and financial uncertainty are exogenous.

In closing we turn our interest on the relation between financial conditions and uncertainty, motivated by several recent theoretical contributions have emphasized the pivotal role that financial conditions might have in amplifying and propagating the effects of uncertainty to real economy (Arellano et al., 2018; Christiano et al., 2014; Gilchrist et al., 2014; Brunnermeier & Sannikov, 2014; Alfaro et al., 2018). We find that this channel is crucial for the transmission of financial uncertainty shocks but negligible for the transmission of macroeconomic uncertainty shocks. We also find that financial shocks are recessionary and ignoring the endogenous role of uncertainty leads to under-estimating their effects on the economy.

The paper is organized as follows. Section 2 introduces the identification strategy; Section 3 illustrates the effectiveness of our approach via a simulation; Section 4 presents the empirical application; Section 5 concludes. Appendix A and Appendix B provide proofs and robustness checks, respectively.

2 Theoretical framework

Consider a SVAR(p) model

$$\boldsymbol{A}_0\boldsymbol{y}_t = \boldsymbol{a} + \sum_{j=1}^{p} \boldsymbol{A}_j\boldsymbol{y}_{t-j} + \boldsymbol{\epsilon}_t \tag{2.1}$$

for $t = 1, \dots, T$, where $\boldsymbol{y}_t$ is an $n \times 1$ vector of endogenous variables, $\boldsymbol{\epsilon}_t$ an $n \times 1$ vector white noise process, normally distributed with mean zero and variance-covariance matrix $\boldsymbol{I}_n$, $\boldsymbol{A}_j$ is an $n \times n$ matrix of structural coefficient for $j = 0, \dots, p$. The disturbances $\boldsymbol{\epsilon}_t$ are mutually uncorrelated, and are therefore interpretable as structural shocks. The initial conditions $\boldsymbol{y}_1, \dots, \boldsymbol{y}_p$ are given. Let $\boldsymbol{\theta} = (\boldsymbol{A}_0, \boldsymbol{A}_+)$ collect the structural parameters, where $\boldsymbol{A}_+ = (\boldsymbol{a}, \boldsymbol{A}_j)$ for $j = 1, \dots, p$.

The reduced-form representation is a Vector Autoregression (VAR):

$$\boldsymbol{y}_t = \boldsymbol{b} + \sum_{j=1}^{p} \boldsymbol{B}_j \boldsymbol{y}_{t-j} + \boldsymbol{u}_t, \tag{2.2}$$

where $\boldsymbol{b} = \boldsymbol{A}_0^{-1}\boldsymbol{a}$ is an $n \times 1$ vector of constants, $\boldsymbol{B}_j = \boldsymbol{A}_0^{-1}\boldsymbol{A}_j$, $\boldsymbol{u}_t = \boldsymbol{A}_0^{-1}\boldsymbol{\epsilon}_t$ denotes the $n \times 1$ vector of reduced-form errors. $var(\boldsymbol{u}_t) = E(\boldsymbol{u}_t\boldsymbol{u}_t') = \boldsymbol{\Sigma} = \boldsymbol{A}_0^{-1}(\boldsymbol{A}_0^{-1})'$ is the $n \times n$ variance-covariance matrix of reduced-form errors. Let $\boldsymbol{\phi} = (\boldsymbol{B}, \boldsymbol{\Sigma}) \in \boldsymbol{\Phi}$ collect the reduced-form parameters, where $\boldsymbol{B} \equiv [\boldsymbol{b}, \boldsymbol{B}_1, \dots, \boldsymbol{B}_p]$, $\boldsymbol{\Phi} \subset \mathcal{R}^{n+n^2p} \times \boldsymbol{\Xi}$, and $\boldsymbol{\Xi}$ is the space of symmetric positive semidefinite matrices.

We define the $n \times n$ matrix

$$\boldsymbol{IR}^h = \boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{A}_0^{-1} \tag{2.3}$$

as the impulse response at h-th horizon for $h = 0, 1, \dots,$ where $\boldsymbol{C}_h(\boldsymbol{B})$ is the h-th coefficient matrix of $(\boldsymbol{I}_n - \sum_{h=1}^{p} \boldsymbol{B}_h L^h)^{-1}$. Its (i, j)-element denotes the effect on the i-th variable in $\boldsymbol{y}_{t+h}$ of a unit shock to the j-th element of $\boldsymbol{\epsilon}_t$. As is well known there are several observationally equivalent $\boldsymbol{A}_0$ matrices, and expression (2.3) actually involves a set of impulse responses.

To formalize this fact we follow Uhlig (2005) and define the set of all IRFs through an $n \times n$ orthonormal matrix $\boldsymbol{Q} \in \boldsymbol{\Theta}(n)$, where $\boldsymbol{\Theta}(n)$ characterizes the set of all orthonormal $n \times n$ matrices. Uhlig (2005) showed that $\{\boldsymbol{A}_0 = \boldsymbol{Q}'\boldsymbol{\Sigma}_{tr}^{-1} : \boldsymbol{Q} \in \boldsymbol{\Theta}(n)\}$ is the set of observationally equivalent $\boldsymbol{A}_0$'s consistent with reduced-form parameters, where $\boldsymbol{\Sigma}$ relates to $\boldsymbol{A}_0$ by $\boldsymbol{\Sigma} = \boldsymbol{A}_0^{-1}(\boldsymbol{A}_0^{-1})'$, $\boldsymbol{\Sigma}_{tr}$ denotes the lower triangular Cholesky matrix with non-negative diagonal coefficients of $\boldsymbol{\Sigma}$. The likelihood function depends on $\boldsymbol{\phi}$ and does not contain any information about $\boldsymbol{Q}$, leading to ambiguity in decomposing $\boldsymbol{\Sigma}$. The identification problem arises because there is a multiplicity of $\boldsymbol{Q}$'s which deliver $\boldsymbol{A}_0$ given $\boldsymbol{\phi}$. Specifically, the impulse response of variable i to shock j at horizon h, i.e., (i, j)-element of $\boldsymbol{IR}^h$, can be expressed as $\boldsymbol{e}_i'\boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}\boldsymbol{Q}\boldsymbol{e}_j \equiv \boldsymbol{c}_{ih}'(\boldsymbol{\phi})\boldsymbol{q}_j$, where $\boldsymbol{e}_i$ is the i-th column vector of $\boldsymbol{I}_n$, $\boldsymbol{q}_j$ is the j-th column of $\boldsymbol{Q}$ and $\boldsymbol{c}_{ih}'(\boldsymbol{\phi})$ represents the i-th row vector of $\boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}$. Alternative identification schemes can be achieved by placing a set of restrictions on $\boldsymbol{Q}$. For example, imposing $\boldsymbol{Q} = \boldsymbol{I}_n$ implies a recursive ordering identification, i.e., the Cholesky decomposition, whereas sign restrictions specify a set of admissible $\boldsymbol{Q}$'s.

2.1 Identification strategy

Our identification scheme identifies $k \leq n$ shocks $j \in 1, \ldots, k$, denoted by $\boldsymbol{q}_j = \boldsymbol{Q}\boldsymbol{e}_j$, where $\boldsymbol{q}_j'\boldsymbol{q}_{\tilde{j}} = 0$ for $j \neq \tilde{j}$ is the standard orthogonality condition.

In our empirical application we will set $k = 3$ and shocks of interest are those to macroeconomic uncertainty, financial uncertainty, and credit supply. Consider the goal of identifying the structural shock to macroeconomic uncertainty. Identification is achieved by verifying that a given candidate model satisfies two requirements. The first requirement is that such shock must explain the unexpected movements of macroeconomic uncertainty variable more than it explains the fluctuations of financial uncertainty and credit spreads, upon impact. This reduces the identified set to a smaller set of candidates. The second requirement is that the shock maximizes its contribution to (a function of) the total variation of macroeconomic uncertainty. The same methodology applies to identify the financial uncertainty and credit supply shocks.

The second requirement implements the belief that movements in a given variable are significantly driven by structural shocks to the variable itself (this does not exclude endogeneity, though). Such an assumption seems reasonable in general, and in the specific example at hand has some empirical support in e.g. Caldara et al. (2016) and Brianti (2021), which show that movements in uncertainty are substantially driven by uncertainty disturbances, after controlling for other sources of uncertainty and financial conditions, while unanticipated deterioration in credit conditions are largely caused by an adverse financial shock, after controlling for uncertainty.

Importantly, this approach does not require the researcher to take a stance in regards to the possible exogeneity or endogeneity of uncertainty: uncertainty can impact on macro variables, and vice-versa. In fact, Section 3 shows that our identification assumptions are consistent with DGPs regardless whether those frameworks consider endogenous or exogenous uncertainty, and successfully recover the impulse response functions of different DGPs.

2.1.1 Formal setup

In what follows we provide a formalization the strategy described above. Let $CFEV^i_j(\tilde{h})$ denote the FEV at horizon $\tilde{h}$ of variable i explained by the j-th structural shock:

$$CFEV^i_j(\tilde{h}) = \boldsymbol{q}'_j \boldsymbol{\Upsilon}^{\boldsymbol{i}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\boldsymbol{q}_j, \tag{2.4}$$

where $\boldsymbol{\Upsilon}^{\boldsymbol{i}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}(\boldsymbol{\phi})\boldsymbol{c}'_{ih}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{ih}(\boldsymbol{\phi})\boldsymbol{c}_{ih}(\boldsymbol{\phi})}$ is a $n \times n$ positive semidefinite matrix. Expression (2.4) describes the percent contribution - expressed with a number in the interval $[0, 1]$ - of the shock j to the unexpected fluctuations of variable i at horizon $\tilde{h}$.

Without loss of generality, suppose that (i) $j = 1$ is the first shock, $j = 2$ is the second shock, $j = 3$ is the third shock and so on; (ii) the n endogenous variables are ordered such that $i = 1$ is the macroeconomic uncertainty variable, $i = 2$ is the financial uncertainty variable, and $i = 3$ is the credit spreads. Define the following $I_{-j} = \{1, \dots, k\}/\{j\}$ as a subset of the shocks of interest. The identification of $\boldsymbol{Q}_{1:k} = [\boldsymbol{q}_1, \boldsymbol{q}_2, \dots, \boldsymbol{q}_k]$, with $k = 3$ and $j \in 1, 2, 3$, requires to solve the following constrained optimization problem:[6]

$$\boldsymbol{Q}^*_{1:k} = \arg\max_{\boldsymbol{Q}_{1:k}} \sum_{i=1}^{k} \boldsymbol{q}'_i \boldsymbol{\Upsilon}^{\boldsymbol{i}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\boldsymbol{q}_i \tag{2.5}$$

subject to

$$\boldsymbol{q}'_j \boldsymbol{\Upsilon}^{\boldsymbol{j}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\boldsymbol{q}_j \geq \boldsymbol{q}'_j \boldsymbol{\Upsilon}^{\boldsymbol{i}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\boldsymbol{q}_j \ for\ j = 1, \dots, k, \quad \forall i \in I_{-j} \tag{2.6}$$

and

$$\boldsymbol{Q}'_{1:k}\boldsymbol{Q}_{1:k} = \boldsymbol{I}_n. \tag{2.7}$$

In our application, we will set $\tilde{h} = 0$. For $j = 1$, we will identify the macroeconomic uncertainty shock as the innovation that maximizes its contribution to the FEV of the macroeconomic uncertainty variable subject to the following constraints. Restrictions (2.6) establish that (for $j = 1$) the contribution of the macroeconomic uncertainty shock to the FEV of the macroeconomic uncertainty variable must be higher than the contribution to the FEV of financial uncertainty variables and credit spreads (upon impact). Those restrictions are instrumental to separate macroeconomic uncertainty shocks from financial uncertainty and credit supply shocks. Restrictions (2.7) ensure that the identified shocks

[6] Once columns 1 to k are identified, we can always construct orthogonal columns $k + 1$ to n.

are mutually orthogonal.[7] Similarly for $j = 2, 3$, the problem (2.5)-(2.7) identifies the financial uncertainty and the credit supply shock, respectively. As a shorthand notation we will use $\mathbf{\Gamma}(\boldsymbol{\phi}, \boldsymbol{Q}) \geq \mathbf{0}$ to denote the whole set of inequality constraints on the FEV represented by (2.6).[8]

2.1.2 Existence and Uniqueness of a solution

Several papers have pointed out that there is a trade-off between sharp identification and computation, and this is especially true when using inequality constraints.[9] In fact restrictions that are too tight can lead to unfeasible or empty regions, i.e. the constraints in (2.6) are so demanding that they are rejected in the data. On the other hand, loosening the requirements in (2.6) eventually leads to identified sets so large to be barely informative. In this section we provide sufficient conditions for the existence of a solution to the constrained optimization problem. Doing so solves the trade-off by ensuring that an identification scheme can be found which is both informative and not rejected by data.

Recall that $\boldsymbol{q}_j^*$ for $j - 1, \ldots, k$ denotes the j-th column of the identified matrix $\boldsymbol{Q}_{1:k}^*$. For $j = 1$, given the constraints in (2.6)-(2.7), we define the following functions:

$$
\begin{aligned}
f_1 &= \frac{1}{2}\boldsymbol{q}_1' \left[\boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\mathbf{2}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\mathbf{1}}(\boldsymbol{\phi})\right] \boldsymbol{q}_1, \\
f_2 &= \frac{1}{2}\boldsymbol{q}_1' \left[\boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\mathbf{3}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\mathbf{1}}(\boldsymbol{\phi})\right] \boldsymbol{q}_1, \\
f_3 &= \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{q}_1 + \frac{1}{2}, \\
f_4 &= \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{q}_1 - \frac{1}{2}.
\end{aligned}
$$

Similar functions can be trivially defined for $j = 2, \ldots, k$.

We start with establishing a Gordan type alternative theorem, which will be instrumental to obtain the existence result.

Proposition 2.1 *Assume $j = 1$. If $\nexists \boldsymbol{\lambda} \in \mathcal{R}_+^4 \backslash \{0\}$ such that $(\forall \boldsymbol{q}_1 \in \mathcal{R}^n) \sum_{i=1}^4 \lambda_i f_i \geq 0$, $\boldsymbol{q}_1^*$ exists.*

[7] The orthogonality restriction matters only if we restrict multiple shocks simultaneously. For individual identification, we can always construct vectors in the Nullspace of the restricted shocks.

[8] In a different setup, i.e., identification of monetary policy via sign restrictions, Wolf (2020) stresses that in principle inequality constraints are necessary, but not sufficient, to successfully separate shocks because linear combinations of structural shocks can still satisfy the constraints. We find this is not the case in our simulation experiment and empirical application.

[9] See Amir-Ahmadi and Drautzburg (2021); Giacomini and Kitagawa (2020); Giacomini et al. (2020); Gafarov et al. (2018); Granziera et al. (2018); Volpicella (2021); Uhlig (2017).

The proof is provided in Appendix A. This proposition rules out that - for a given shock - the restrictions contradict each other and more generally it rules out that linear combinations of inequality constraints on the FEV violate the restrictions. Note that this proposition alone establishes existence of a solution to (2.5)-(2.6), but ignoring the orthogonality conditions (2.7). The satisfaction of orthogonality condition is essential for identifying simultaneously all of the shocks, avoiding the well known issue that shocks identified one-at-a-time can be correlated to each other.

Next we establish the conditions for the existence of a solution to the constrained optimization problem (2.5)-(2.7). Let $\boldsymbol{\sigma}$ denote a permutation of $1, \ldots, k$ among the $k!$ possible permutations and $\boldsymbol{\sigma}(z)$ for $z = 1, \ldots, k$ denote the z-th element of the permutation $\boldsymbol{\sigma}$. The following proposition holds:

Proposition 2.2 *(Existence) If there exists a permutation $\boldsymbol{\sigma}$ such that*

i) for $j = \boldsymbol{\sigma}(1)$ Proposition 2.1 is satisfied,

ii) conditions in Proposition 2.1 are met for all $j = \boldsymbol{\sigma}(2), \ldots, \boldsymbol{\sigma}(k)$ in the Nullspace of the previous $j - 1$ shocks,

then $\boldsymbol{Q}^*_{1:k}$ exists.

Appendix A provides a proof and a technical discussion. The permutation $\boldsymbol{\sigma}$ is instrumental to find at least one matrix $\boldsymbol{Q}^*_{1:k}$ such that its first k columns $\boldsymbol{q}^*_1, \ldots, \boldsymbol{q}^*_k$ satisfy Proposition 2.1 and are orthogonal to each other.

Proposition 2.2 offers sufficient conditions, which may or may not be satisfied depending on the application. The conditions were verified in the empirical application presented in this paper, but of course it may happen that they are not verified in other instances. Should this happen, one needs to abandon the convenience of simply checking the sufficient conditions and must implement a numerical procedure which we will illustrate in Section 2.2. Such procedure allows to investigate the feasibility of the optimization region.

In closing, note that Proposition 2.1 and Proposition 2.2 can be easily tweaked to include also sign restrictions to the set of requirements. These would be in the form of linear inequality constraints on $\boldsymbol{Q}_{1:k}$. Such an extension would allow users to combine the scheme proposed here with standard sign restrictions schemes.

The constrained optimization problem (2.5)-(2.7) is non-convex and in general allows a multiplicity of solutions which require time consuming numerical optimization, without guarantee of finding a global optimum. The proposition below establishes a sufficient condition for $\boldsymbol{Q}^*_{1:k}$ to be unique, which in turn implies that the numerical problem becomes easily tractable.

Proposition 2.3 *(Uniqueness) Assume that $\boldsymbol{Q}^*_{1:k}$ exists and is orthogonal. If $\boldsymbol{c}'_{ih}(\boldsymbol{\phi})\boldsymbol{q}_j \geq 0$ for $i, j = 1, \ldots, k$, $h = 0, \ldots, \tilde{h}$, then $\boldsymbol{Q}^*_{1:k}$ is unique.*

The formal proof is provided in Appendix A. Proposition 2.3 provides a sufficient condition that is both easy to verify and allows for an economic interpretation. Specifically, if there is a positive feedback - in either direction - between targeted variables in the system, then $\boldsymbol{Q}^*_{1:k}$ is selected over a closed convex feasibility region, and uniqueness follows. In the empirical application presented in this paper we have never found a case in which the sufficient condition of Proposition 2.3 was violated, but of course it may be not satisfied in some instances. In such cases researchers can still implement our identification strategy, but they would need to check for the possibility of multiple optima (this would need to be done in Step 3 of algorithm 2.1 below).

2.2 Implementation

The following Algorithm delivers the posterior distribution of the impulse response functions (or any other structural object) of interest.

Algorithm 2.1

1: Draw $\boldsymbol{\phi}$ from the posterior distribution of the reduced-form VAR.

2: Check existence and uniqueness of a solution using Proposition 2.2 and 2.3.

3: Obtain $\boldsymbol{Q}^*_{1:k}$ by solving the optimization problem (2.5)-(2.7) and compute the impulse response functions via (2.3).

4: Repeat Step 1-3, L times, e.g. $L = 1000$.

Algorithm 2.1 consists in a step of conventional sampling from the posterior of reduced-form parameters (Step 1), a step for investigation of feasibility (Step 2), and a step of numerical optimization (Step 3). The optimization involves a quadratic objective function with quadratic constraints, but can be reduced to a much more tractable problem using Proposition 2.3. Note that Step 1 uses a posterior distribution, which means it is based on a Bayesian estimation of the underlying reduced form VAR. This choice is simply based on the observation that Bayesian VARs are widely used in empirical macroeconomics. Still, Step 1 can be easily adapted to a frequentist framework, for example using maximum likelihood estimates and invoking large sample results or using a bootstrap approach to produce draws from the VAR coefficients. In either case the entire procedure would still remain valid, since the remaining steps condition on the reduced-form parameters ($\boldsymbol{\phi}$) and do not depend on a prior over $\boldsymbol{Q}$.

When conditions in Proposition 2.2 do not hold, researchers can consider the optimization region unfeasible if Step 3 in Algorithm 2.1 cannot detect an interior solution for a variety of starting points. Alternatively, one can stick to the conventional approach and think of the feasibility region as empty whether, for many draws from the orthonormal space, an admissible rotation matrix $\boldsymbol{Q}$ cannot be discovered.

Finally, in those cases in which Proposition 2.3 cannot be verified researchers need to ensure that Step 3 delivers a global - as opposed to a local - optimum.

2.3 Relation to alternative identification methods

The identification approach outlined above allows to avoid strong identification assumptions such as recursive orderings, and therefore it lends itself naturally to investigate questions in which one wants to remain agnostic about the direction of the various causal effects. The study of the effects of macroeconomic uncertainty shocks is just one example of such a situation, as both the theoretical literature and the empirical evidence so far are inconclusive on whether uncertainty is an exogenous impulse or an endogenous response.

Importantly, our strategy identifies all of the shocks simultaneously, thereby sidestepping the well known issue that shocks identified one-at-a-time can be correlated to each other, a problem which is particularly relevant in uncertainty literature. For instance, Cascaldi-Garcia and Galvão (2020) showed that news and uncertainty shocks tend to be correlated if identified separately; as such they are not truly structural. Caldara et al. (2016) separated uncertainty and financial shocks by imposing different ordering restrictions, finding that the order hugely affects the results. Of course, solving this problem comes at a cost. The cost of allowing simultaneous identification of a multiplicity of shocks is that the optimization problem can become non-convex. In Section 2.1.1 we establish mild conditions under which the problem is tractable and computationally fast.

Furthermore, the approach can resolve situations in which set-identification schemes are not sufficient to satisfactorily pin down the desired shock. For example Kilian and Murphy (2012) showed that qualitative information beyond sign restrictions is necessary to distinguish demand and supply shocks in the oil market. Similarly, separation between news and surprise shocks requires to rank the relative effect of those disturbances over target variables, see Amir-Ahmadi and Drautzburg (2021) for an example of such a situation. In order to separate credit and housing shocks Furlanetto et al. (2017) assumed that the former explain variation of total credits to households and firms more than the contributions to the fluctuations in the real estate value, and the other way around. The approach proposed in this paper achieves point-identification, avoiding the drawbacks of set-identification that affect most of the aforementioned studies (Baumeister & Hamilton, 2015; Giacomini & Kitagawa, 2020).

Finally, it is worth clarifying the differences between the approach pursued in this paper and that of Volpicella (2021). The two approaches both use the FEV decomposition, but are different conceptually and methodologically. In particular, Volpicella (2021) uses bounds on the FEV decomposition, in combination with traditional sign restrictions, to set-identify a single shock. The approach in this paper instead achieves point- (as opposed to set-) identification of a variety of shocks (as opposed to one) which are guaranteed to be mutually orthogonal. Furthermore, the approach of Volpicella (2021) requires the use of sign restrictions, which are essential to economically label the shocks, while the approach presented here can identify shocks without the need of imposing any sign restrictions. Methodologically, Volpicella (2021) requires specifying exact ad-hoc bounds on the FEV decomposition, while the approach proposed here only imposes milder inequality constraints that require the FEV of each shock to be larger relative to that of all the remaining shocks, but otherwise leave the FEV decomposition unbounded. This leads to major differences in both estimation and inference.

2.4 Relation to macro models

In this section we briefly discuss our identification assumptions in relation to the existing theoretical work on uncertainty. As discussed above (see Section 1 and the references therein), a large macroeconomic literature has developed models in which uncertainty is an exogenous source of fluctuations. In most of these models our identification assumptions are immediately satisfied, since they consider a single source of uncertainty and assume that the macroeconomic uncertainty shock explains 100% of the within period variation of macroeconomic uncertainty.

Our identification assumptions are also satisfied in those models that consider more than one source of uncertainty. For example, Shin and Zhong (2020) built upon Basu and Bundick (2017) and Gertler and Karadi (2011) to construct a DSGE model with financial frictions (and credit supply shocks), exogenous macroeconomic uncertainty (as TFP volatility), and exogenous financial uncertainty (as capital quality volatility). Our identifying restrictions are confirmed by employing both the baseline parameterization in Shin and Zhong (2020)[10] and their battery of alternative calibrations[11]

Finally, there is some more recent literature modeling uncertainty as an endogenous response. In particular, Atkinson et al. (2021) departed from a Cobb-Douglas production function and suggested that complementarity between capital and labor inputs can generate endogenous uncertainty because the concavity in the production influences how output responds to productivity shocks.[12] However, even in that case credit shocks are not able to explain short run fluctuations of the uncertainty proxy more than uncertainty disturbances, which is in line with our identification approach.

3 Simulation exercise

This section presents a simulation showing that the proposed approach can recover the impulse response functions regardless on whether uncertainty is modeled as exogenous or endogenous.

We first employ a SVAR with endogenous uncertainty as Data Generating Process

[10]See Table A-7 of their paper.

[11]See Section D.2.1 of the their paper.

[12]When matching labor share and uncertainty moments, they found 16% of the volatility of uncertainty is endogenous in the short run.

(DGP) and generate artificial data for industrial production (IP), financial uncertainty (uF*), credit spread (CS), price index (PCEPI), monetary policy rate (FFR), and macroeconomic uncertainty (uM*). In order to generate endogeneity in uncertainty, macro and financial uncertainty are ordered after the other covariates. In the baseline scenario of Figure 1, financial uncertainty is ordered before macroeconomic uncertainty, but the results still hold if we reverse the order between uncertainty disturbances. The DGP is parameterized at the maximum likelihood estimates based on monthly US data for the period 1962 to 2016. Once the artificial data have been generated, we use them to estimate the impulse response functions. For brevity, here we provide simulated results mostly for financial uncertainty shocks, but the insights below apply to macroeconomic uncertainty and credit supply shocks as well.

Figure 1 shows that our identification strategy can successfully identify the uncertainty shocks in presence of endogeneity, while methods based on exclusion restrictions can not. In the figure the blue line denotes the true responses based on the DGP. Note the different scale across different rows of the figure.

In the panels on the first row of Figure 1, we employ our identification scheme to estimate the impulse responses (black lines). According to panels (a), (b) and (c), our strategy works well. The remaining rows in this figure we will depart from this ideal situation, showing that imposing too strong identification assumptions will distort the estimated responses.

In the panels on the second row of Figure 1 (panels (a′), (b′) and (c′)) we have shut down the short run[13] response of macroeconomic uncertainty in the estimated model; this corresponds to considering only one of the sources of uncertainty present in the DGP. As a consequence of this omission, the estimated responses of both real activity and credit spreads are biased. Similarly, in the panels on the third row of Figure 1 (panels (a″), (b″) and (c″)) we estimated a model that assumes that financial uncertainty is exogenous, i.e., no shock can contemporaneously affect it. Also in this case this leads to biased impulse responses.

The panels on the last row of Figure 1 (panels (a‴), (b‴) and (c‴)), illustrate a model in which impulse responses are identified using ordering restrictions between financial and uncertainty disturbances echoing the spirit of Caldara et al. (2016). Also in this case

[13]Short run is defined as up to $h = 4$ months, but the results remain unchanged for $h = 0, 1, \dots, 6$.

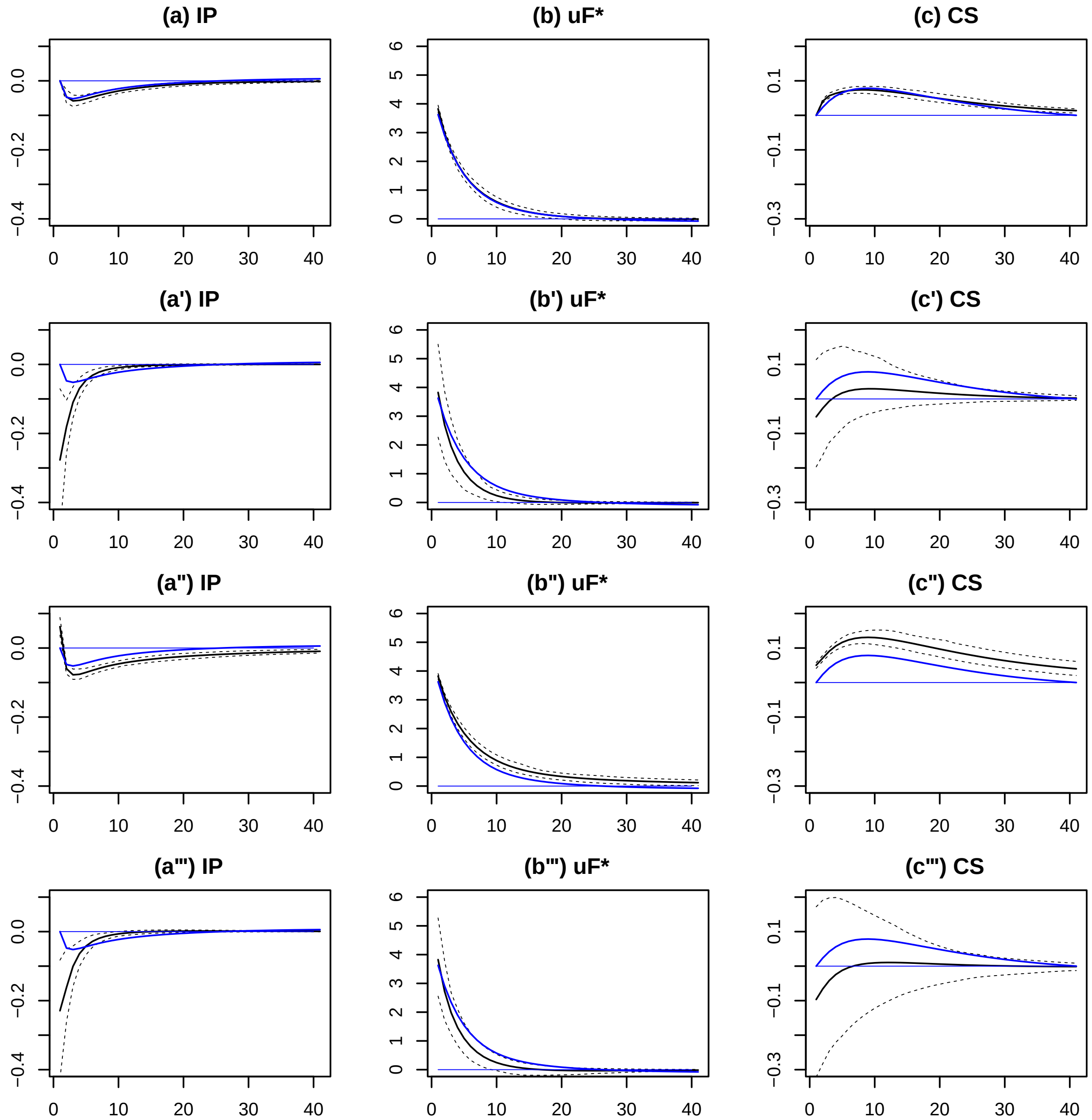

Figure 1: DGP with endogenous uncertainty: estimated responses to financial uncertainty shock. The blue line denotes the impulse responses to uncertainty shocks in the DGP. The black solid line represents the posterior mean of the estimated impulse responses, where in the first row ((a), (b), (c)) responses are identified through constraints on the FEV; in the second row ((a′), (b′), (c′)) the macroeconomic uncertainty response is shut down; in the third row ((a″), (b″), (c″)) the uncertainty is estimated as exogenous; in the fourth row (panels (a‴), (b‴) and (c‴)), impulse responses are estimated by replacing inequality constraints on the FEV with ordering restrictions between financial and uncertainty disturbances. The dashed black lines display the 68% Bayesian credibility region across replications. Shock size is set to 1 standard deviation.

impulse responses are biased.

We now turn on the effectiveness of our scheme when uncertainty is exogenous. Accordingly, we consider a DGP where uncertainty is ordered before the other variables.

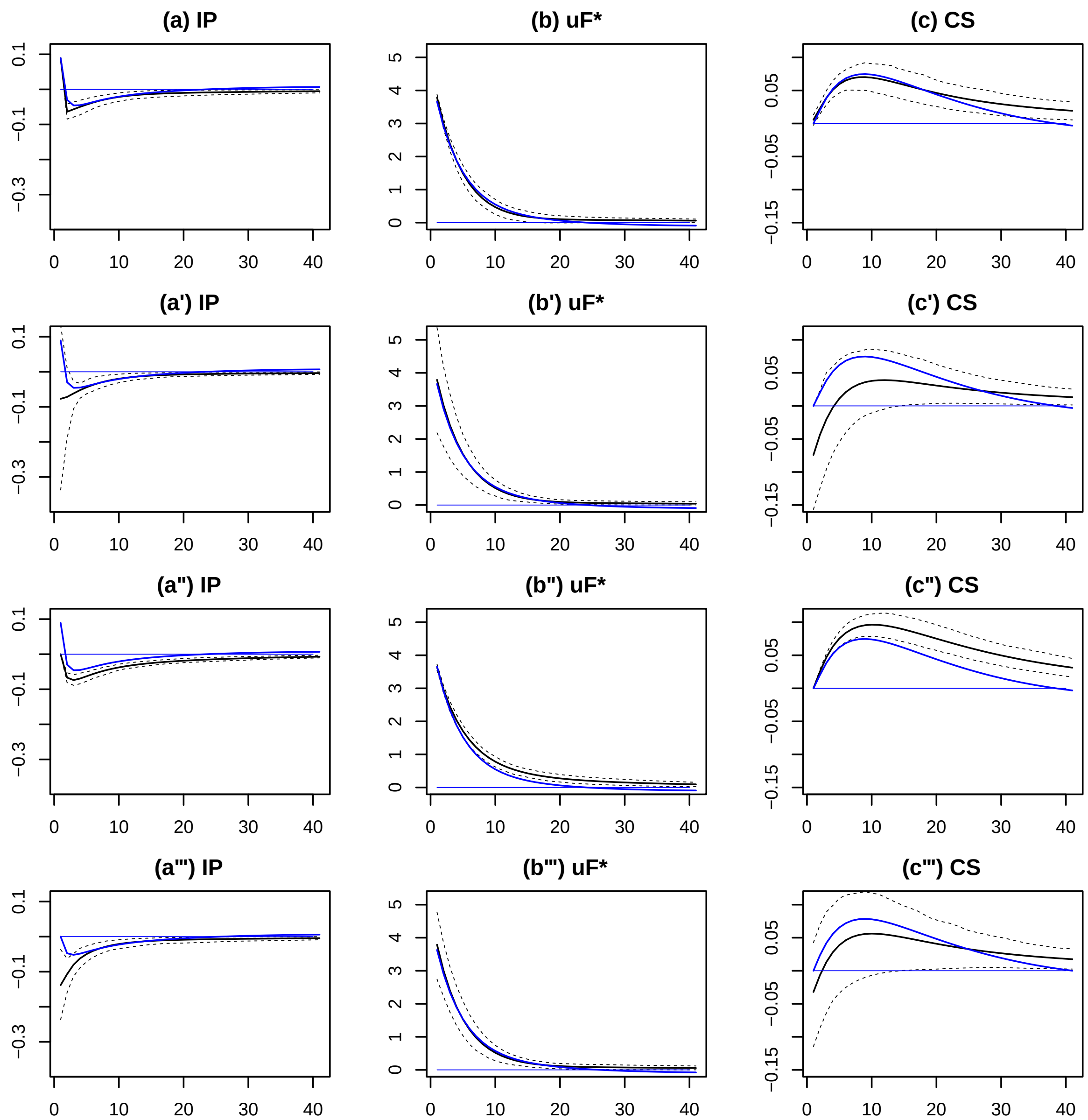

Figure 2: DGP with exogenous uncertainty: estimated responses to financial uncertainty shock. The blue line denotes the impulse responses to uncertainty shocks in the DGP. The black solid line represents the posterior mean of the estimated impulse responses, where in the first row ((a), (b), (c)) responses are identified through constraints on the FEV; in the second row ((a′), (b′), (c′)) the macroeconomic uncertainty response is shut down; in the third row ((a″), (b″), (c″)) the uncertainty is estimated as endogenous; in the fourth row (panels (a‴), (b‴) and (c‴)), impulse responses are estimated by replacing inequality constraints on the FEV with ordering restrictions between financial and uncertainty disturbances. The dashed black lines display the 68% Bayesian credibility region across replications. Shock size is set to 1 standard deviation.

This experiment is illustrated in Figure 2. Also in this case our proposed identification scheme recovers the correct responses; on the other hand, omitting distinctive sources of uncertainty (second row in the figure), imposing endogeneity when this is absent in the DGP (third row in the figure), and removing the inequality constraints on the FEV (fourth row in the figure) lead to biased impulse responses.[14]

4 Empirical application

4.1 Specification and data

We now turn to our empirical application. Evaluating the relationship between economic variables and uncertainty needs selecting both a concept and metric of uncertainty. In the baseline model, we employ the Chicago Board Options Exchange S&P 100 Volatility Index as a measure of financial uncertainty and the the measure developed by Jurado et al. (2015) (JLN hereafter) as a measure of macroeconomic uncertainty. We have checked the robustness of our results to competing measures: for financial uncertainty, we also considered the measures of Carriero et al. (2018b) and Jurado et al. (2015); for macroeconomic uncertainty, we also used the measure of Carriero et al. (2018b).

Our baseline reduced form model is a VAR estimated with US monthly data ranging from from 1962m7 to 2016m12. We assume 7 lags[15] and a diffuse Normal Inverse Wishart prior. The VAR includes 12 variables taken from the FRED database: macroeconomic uncertainty (JLN), financial uncertainty (VXO), credit spreads (CS), number of non-farm workers (PAYEM), industrial production (IP), weekly hours per worker (HOURS), real consumer spending (SPEND), real manufacturers' new orders (ORDER), real average earnings (EARNI), PCE price index (PCEPI), variation of federal funds rate (FFR), S&P 500 (S&P). The credit spread is measured as the difference between the BAA Corporate Bond Yield and the 10-year Treasury Constant Maturity rate; results are robust to employing the excess bond premium used in Caldara et al. (2016) and developed by Gilchrist and Zakrajšek (2012). All the variables enter the model growth rates, except for ORDER, PCEPI, FFR, CS, VXO, and JLN which enter in levels. All the variables are demeaned prior to estimation. In order to facilitate comparisons with other studies, the impulse responses are expressed in percentage changes with respect to the levels. This implies that for those variables which were differenced the impulse responses are

[14]Removing the orthogonality assumption yielded similar results, suggesting that the inequality restrictions are sufficient to disentangle the various disturbances, which is also the case in our empirical application

[15]This has been selected by maximizing the marginal likelihood.

cumulated and the long run effects of transitory shocks do not vanish.

4.2 The Effects of uncertainty shocks

Figure 3 and Figure 4 show the impulse responses to macro and financial uncertainty shocks, respectively. Uncertainty has a strong recessionary effect on employment, industrial production, hours worked, consumer spending, investment, and earnings; the financial conditions also deteriorate, as shown by the response of stock market and credit spreads. The shock leads to expansionary monetary policy trying to counteract the depressive effect of uncertainty. Notably, shocks to macroeconomic uncertainty increase financial uncertainty, and vice-versa.

To facilitate comparisons, Figure 5 overlays the impulse responses shown in Figure 3 and 4. The effects of macroeconomic and financial uncertainty are qualitatively comparable but there are some quantitative differences. For example, the recessionary effect on real activity variables seem more pronounced following macroeconomic uncertainty shocks, while credit spreads increase more with financial uncertainty shocks.

We find a strong evidence in favor of a negative response of prices, that is short-lived for macroeconomic uncertainty shocks but more persistent for financial uncertainty shocks, suggesting that uncertainty disturbances mimic demand shocks, namely they trigger a recession and a deflationary pressure on the economy. The slightly looser response of monetary policy for financial uncertainty might be driven by the more significant drop in prices relative to macroeconomic uncertainty.

This pronounced reduction in prices is in contrast with the existing empirical evidence on the impact of uncertainty on inflation, which is typically weak and rather mixed. Caggiano et al. (2014), Fernández-Villaverde et al. (2015), Leduc and Liu (2016), Basu and Bundick (2017) provided some empirical evidence that uncertainty is deflationary, while Mumtaz and Theodoridis (2015) found the opposite. Carriero et al. (2018b) and Katayama and Kim (2018) argued that the effect of uncertainty on prices is not significant; the international evidence in Carriero et al. (2018a) pointed out that the reaction of prices is country-specific and heterogeneous across the alternative measures of prices. While the effects of uncertainty on prices are different across these contributions, they are all based on simple recursive identification schemes: in most of these contributions uncertainty is modeled as exogenous.

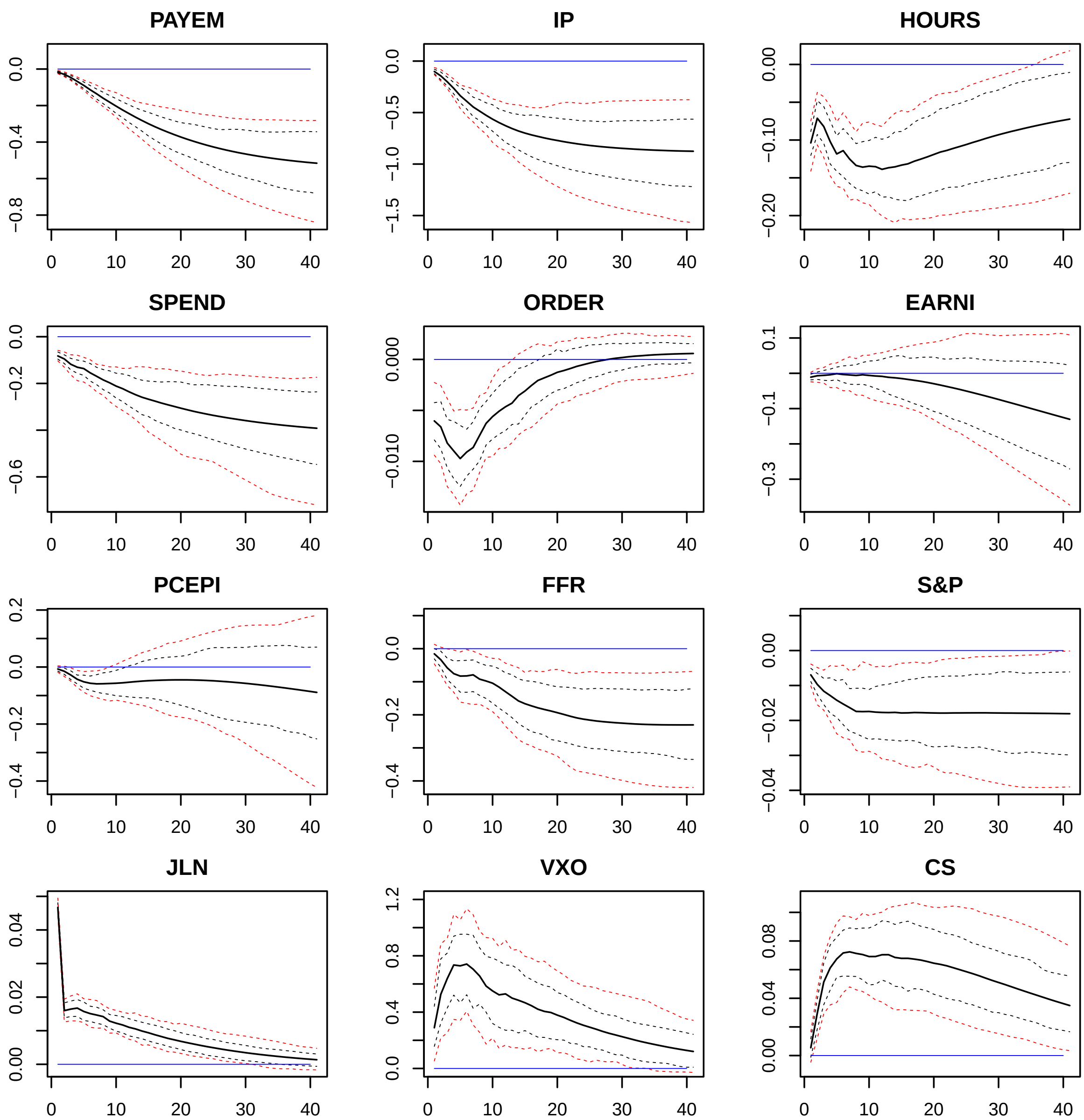

Figure 3: Responses to macroeconomic uncertainty shocks. Posterior medians are the black solid lines, the 68% Bayesian credibility region are the black dashed lines, and the 90% Bayesian credibility region are the red dashed lines. The blue solid line is the zero line. The shock size is set to one standard deviation.

4.2.1 Endogenous uncertainty?

Since our scheme allows for a contemporaneous feedback effect from economic and financial variables to uncertainty, it provides a natural ground to look into the issue of endogeneity of uncertainty. In order to tackle this question, we re-estimated the model adding a further restriction. Specifically we assumed that each measure of uncertainty cannot be contemporaneously affected by structural shocks other than its own shock,

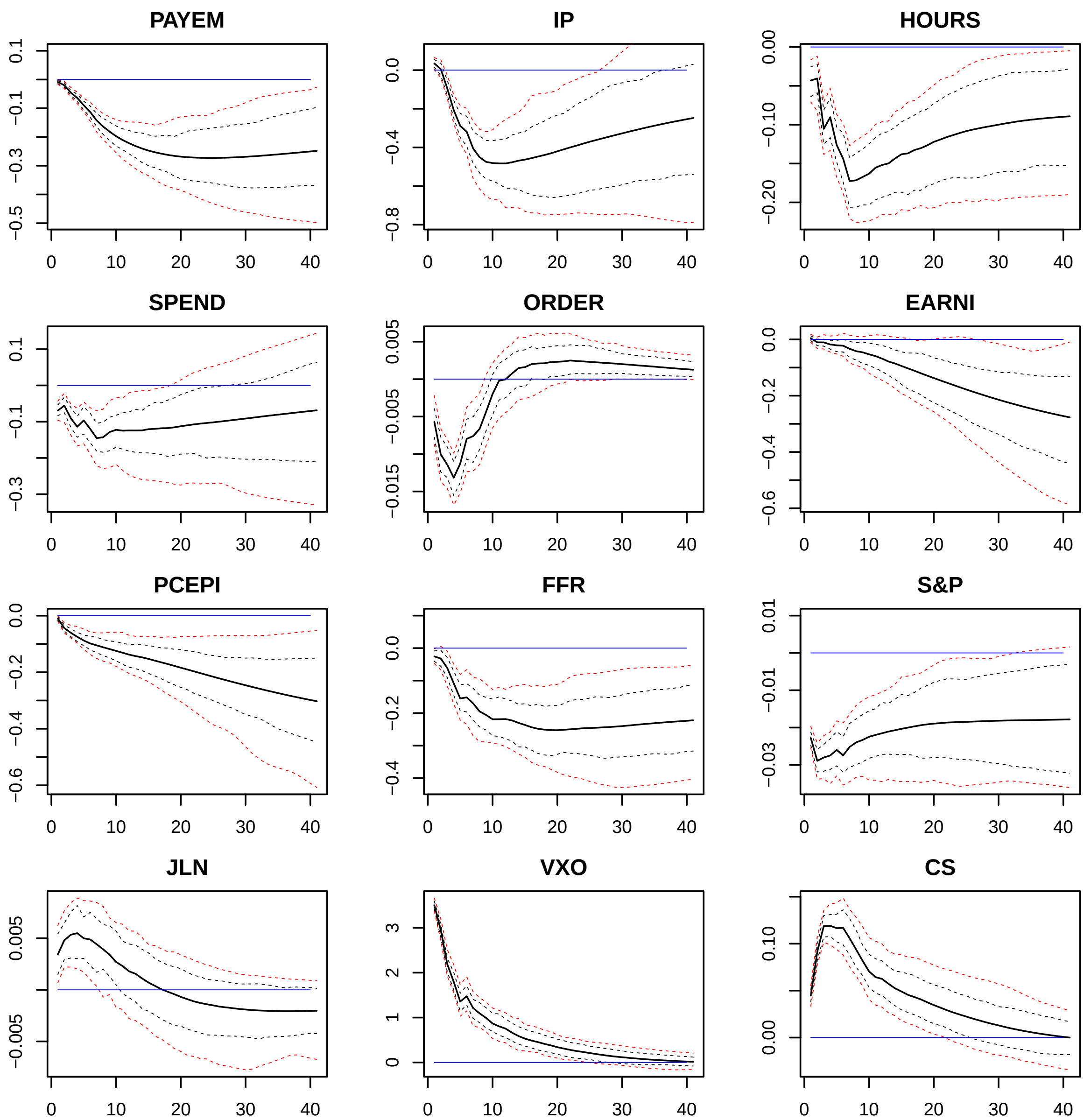

Figure 4: Responses to financial uncertainty shocks. Posterior medians are the black solid lines, the 68% Bayesian credibility region are the black dashed lines, and the 90% Bayesian credibility region are the red dashed lines. The blue solid line is the zero line. The shock size is set to one standard deviation.

i.e., uncertainty is exogenous. This is equivalent to order uncertainty first in a Cholesky decomposition scheme.

Panels (a)-(l) in Figure 6 display the responses to macroeconomic uncertainty shocks for the baseline identification (black line) and when macroeconomic uncertainty is assumed to be exogenous (red line): imposing exogeneity clearly changes several impulse response functions, which supports the view that macroeconomic uncertainty is endogenous to some extent. On the other hand, panels (a′)-(l′) display the responses to financial

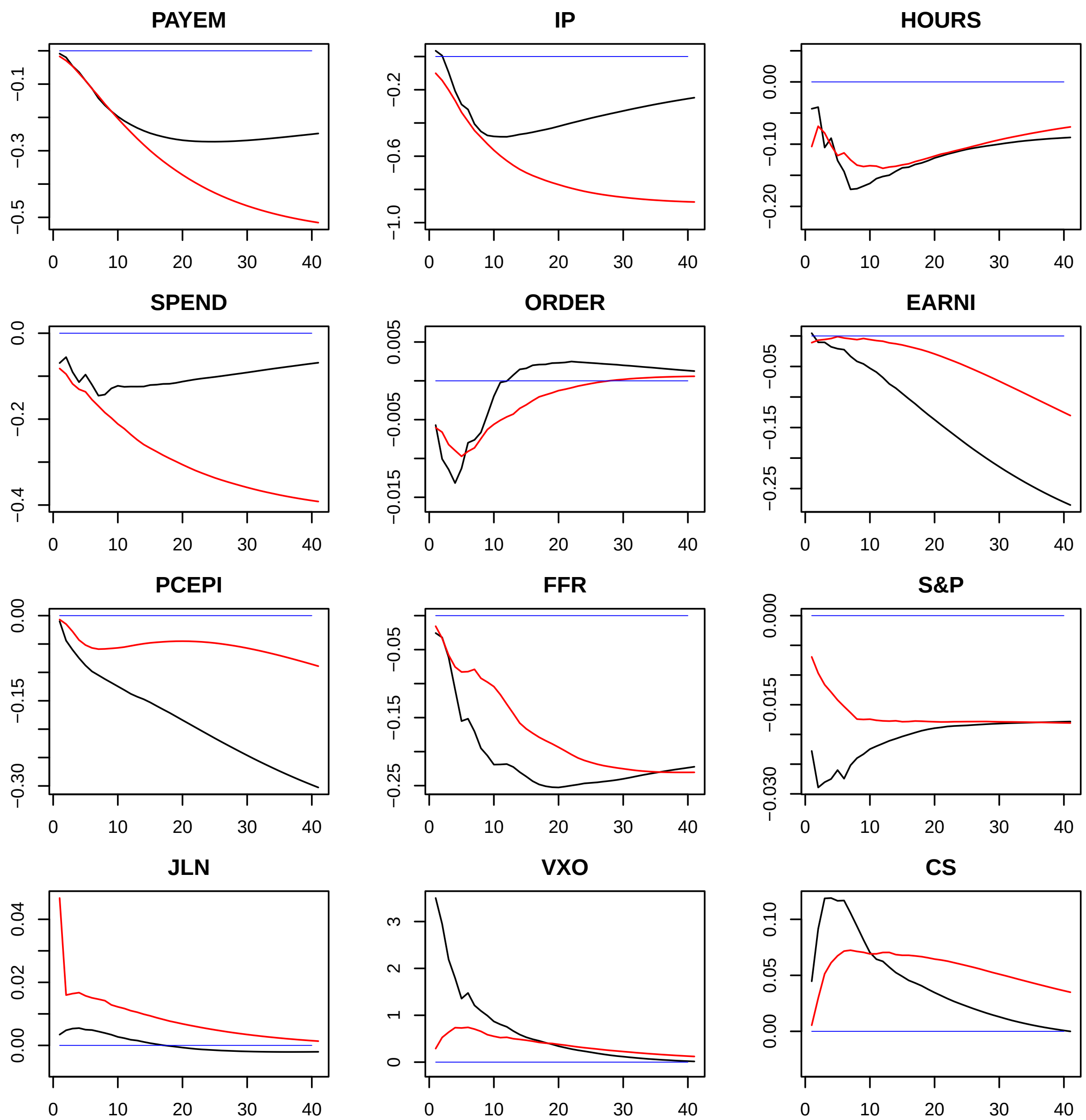

Figure 5: Comparison between macro and financial uncertainty shocks. The red and blue line denote the posterior median of the impulse response functions to macroeconomic and financial uncertainty shock, respectively. The blue solid line is the zero line. The shock size is set to one standard deviation.

uncertainty shocks for the baseline identification (black line) and when financial uncertainty is assumed to be exogenous (red line): in this case the evidence in support of endogeneity is milder.

The pattern shown in Figure 6 is in line with Ludvigson et al. (2021), who argued that while financial uncertainty is mainly exogenous, macroeconomic uncertainty presents some endogeneity. However, such a conclusion is not clear-cut in the literature. For example Angelini et al. (2019) found that both macroeconomic and financial uncertainty are

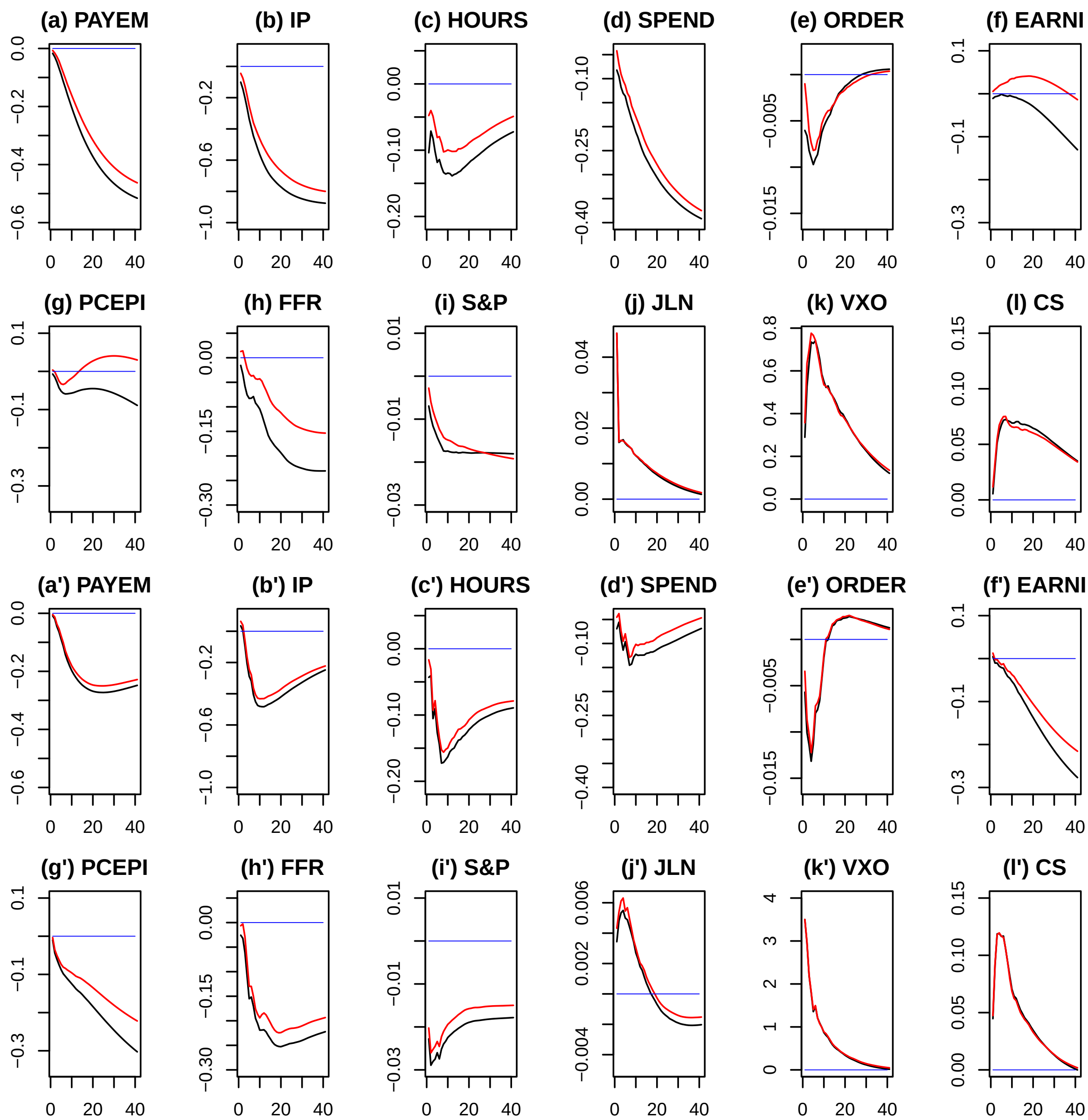

Figure 6: Baseline scenario vs exogenous uncertainty. The figure reports the posterior median of the impulse response functions to macro (panels (a)-(l)) and financial (panels (a′)-(l′)) uncertainty shocks for the baseline identification (black line) and when uncertainty is assumed to be exogenous (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

mostly exogenous, and Carriero et al. (2021) pointed out that macroeconomic uncertainty displays some endogeneity, though more at quarterly than monthly frequency.[16]

The studies cited above are the only three - to the best of our knowledge - which allow for endogeneity in uncertainty. They each adopt a different identification strat-

[16] In unreported results news-based policy uncertainty as proxy for macro volatility turns out to be endogenous.

egy. Ludvigson et al. (2021) uses a small-scale model and a set-identification approach based on narrative restrictions requiring the shocks to be consistent with some historical episodes and correlated with some external instruments. Instead, we use a large-scale model, which reduces the problems of possible omitted variable bias, and a point-identification approach, which avoids the problems inherent in set-identification discussed e.g. in Giacomini et al. (2021).[17]

Angelini et al. (2019) also use a small-scale model in which there are no proxies for financial conditions. They achieve identification by assuming that in the sample preceding January 2008 financial uncertainty shocks could neither contemporaneously impact on nor been impacted by macro variables directly.[18] However, an indirect channel on real variables through the impact from financial uncertainty to macro uncertainty is allowed since the Great Moderation. Differently from them, we never assume exogeneity of financial uncertainty, not even in some sub-sample, and we use a large-scale model which includes financial variables and a credit channel. The latter is relevant, as we shall see below in section 4.2.3, to disentangle uncertainty shocks from pure financial shocks.

Carriero et al. (2021) employ a large model and achieve point-identification exploiting heteroskedasticity in the error terms of the SVAR. However their approach has a major drawback insofar their model does not include macroeconomic and financial uncertainty in the same unified framework. As we shall see in section 4.2.2 such a choice does not guarantee that the macroeconomic and financial uncertainty shock are mutually orthogonal, which can lead to substantial distortions in the estimated responses. Furthermore, their approach requires an ordering restriction on the block of macroeconomic variables in which pure financial shocks are not explicitly identified. Instead, the approach of this paper allows to identify shocks to financial and macroeconomic uncertainty which are orthogonal by construction, and to disentangle them from pure financial shocks.

Another nice feature of our framework is that it allows for formal tests of exogeneity. We formally tested the exogeneity restrictions, with the null being $(\boldsymbol{q}_1^*)'\boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{1}}(\boldsymbol{\phi})\boldsymbol{q}_1^* = 1$ for macroeconomic uncertainty and $(\boldsymbol{q}_2^*)'\boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{2}}(\boldsymbol{\phi})\boldsymbol{q}_2^* = 1$ for financial uncertainty, and we

[17]Ludvigson et al. (2021) employed bootstrap to construct confidence intervals for the impulse response functions, but their frequentist validity is unknown. The fact that confidence intervals are presented for a specific point-estimate only (rather than for the identified sets as such) makes hard to evaluate the effect of sample bias and identification uncertainty in their setting. On the other hand, Bayesian inference naturally follows in our point-identified model.

[18]This is an intriguing assumption because financial markets are usually expected to react fast to news, while macroeconomic variables are relatively slower (Gertler & Karadi, 2015; Lettau et al., 2002).

found that exogeneity is rejected at 1% significance level for both macro and financial uncertainty. The evidence for endogeneity of financial uncertainty is milder and it disappears if one removes the stock index and credit spreads from the model.

4.2.2 Uncertainty and its sources

In the next experiment we evaluate the importance of having both a measure of macroeconomic and a measure of financial uncertainty in the model.[19]

Panels (a)-(l) in Figure 7 display the responses to macroeconomic uncertainty shock for the baseline identification (black line) and when the response of financial uncertainty is muted for 6 months (red line).[20] Similarly, panels (a′)-(l′) show the responses to financial uncertainty shock for the baseline identification (black line) and when the response of macroeconomic uncertainty is muted (red line). Our results show that omitting either one of the two uncertainty measures can lead to distortions in the estimated responses. In particular, neglecting this channel seems to attenuate the estimated impact of uncertainty. More formally, in our framework we always reject the hypothesis of zero impact response of macroeconomic (financial) uncertainty to financial (macroeconomic) disturbance.

4.2.3 The financial channel

There are some contributions that have argued that financial conditions play a key role in amplifying and transmitting uncertainty shocks. For example, Arellano et al. (2018), Christiano et al. (2014), and Gilchrist et al. (2014) developed models featuring a financial channel in which the cost of external finance goes up in reaction to an increase in uncertainty; Alfaro et al. (2018) found that financial frictions can double the recessionary effect of uncertainty. On the other hand, Brunnermeier and Sannikov (2014) emphasized that a worsening of borrowers' financial position leads to higher uncertainty. Caldara et al. (2016), Brianti (2021), and Caggiano et al. (2021) found evidence that deterioration of financial conditions magnify the impact of uncertainty shocks on the real activity.

In light of these contributions we investigated the role of financial channel within our identification scheme. Figure 8 compares the responses to macro (panels (a)-(l)) and

[19] Ludvigson et al. (2021) and Shin and Zhong (2020) used set-identification schemes to separate macro and financial uncertainty shocks. Both papers found differences in the responses of the economy to these two types of shocks.

[20] We tried horizons other than 6, and the results are qualitatively unchanged.

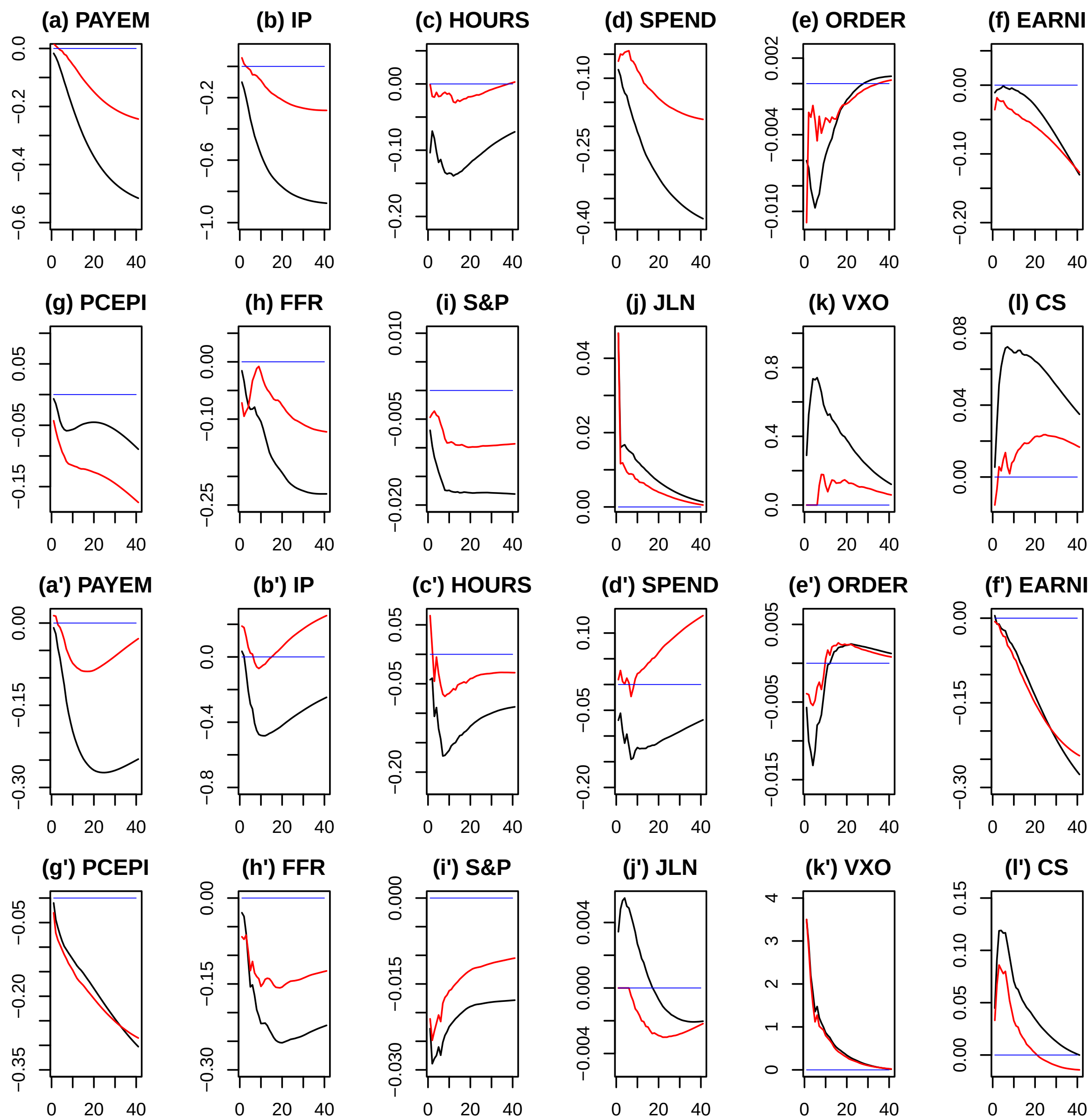

Figure 7: Baseline scenario vs shutting down the channel between macro and financial uncertainty. Panels (a)-(l) display the responses to macroeconomic uncertainty shock for the baseline identification (black line) and when the response of financial uncertainty is muted (red line). Panels (a′)-(l′) show the responses to financial uncertainty shock for the baseline identification (black line) and when the response of macroeconomic uncertainty is muted (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

financial (panels (a′)-(l′)) uncertainty shock for the baseline identification (black line) and for an alternative model in which the financial channel is shut down (red line) by imposing that there is no contemporaneous feedback between financial variables (credit spreads and stock market) and uncertainty. The picture emerging is one in which the financial channel seems relevant in the transmission mechanism of both financial and

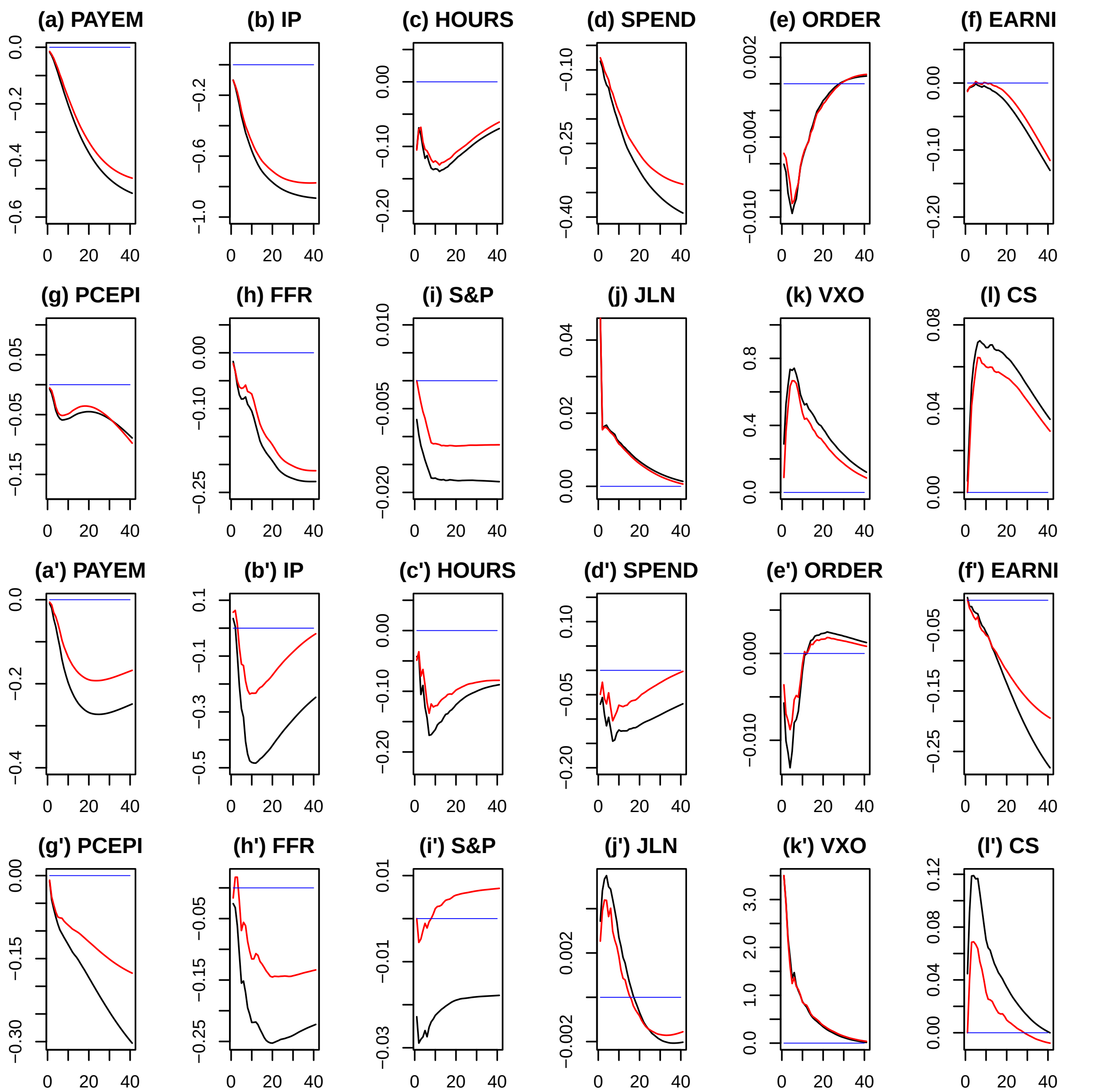

Figure 8: Financial channel. The figure reports the posterior median of the impulse response functions to macro (panels (a)-(l)) and financial (panels (a′)-(l′)) uncertainty shocks for the baseline identification (black line) and when the financial channel is shut down (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

macroeconomic uncertainty shocks, with larger effects on the former.

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4.3 The Effects of financial shocks

Prompted by the Great Recession and the debt crisis in the Euro-Area, a number of works have attempted to identify and estimate the effects of credit supply shocks.[21] The overall picture emerging from these studies is one in which the estimated effects of financial shocks are sensitive to identification schemes and some identification strategies are likely to provide misleading results, see e.g. the discussion in Mumtaz et al. (2018).

While identification of financial shocks and measurement of credit spreads significantly differ across the contributions in this literature, a common feature is the absence of interaction between financial factors and uncertainty, in the sense that most specifications exclude measures of uncertainty or overlook its role in the transmission mechanism of financial shocks. Prominent exceptions are Caldara et al. (2016), Furlanetto et al. (2017), Caggiano et al. (2021) and Brianti (2021), which are discussed below.

4.3.1 Uncertainty and the transmission mechanism

We use our model to shed light on the contribution of uncertainty to the transmission mechanism of financial shocks. Figure 9 shows that an increase in credit spreads has a depressive and deflationary effect on macroeconomic variables and leads to higher uncertainty, especially financial uncertainty. We then consider an experiment in which for the initial six months we shut down the response of macroeconomic uncertainty to credit spreads shock (Figure 10, panels (a)-(l)).[22] The changes in the responses is relevant and is consistent with the endogenous features of macroeconomic uncertainty. Also, shutting down the response of financial uncertainty dramatically mitigates the responses to financial shocks and leads to substantial distortions, as seen in Figure 10, panels (a′)-(l′). This confirms that although the evidence for endogeneity in financial uncertainty is mild with respect to macroeconomic variables, the feedback effect from financial conditions is substantial. Formal tests reject the null of no response on impact of uncertainty to

[21]Peersman (2011), Bijsterbosch and Falagiarda (2014), Eickmeier and Ng (2015), and Gambetti and Musso (2017) employed sign restrictions; Gilchrist and Zakrajšek (2012) developed a measure of credit spreads based on firm level data, finding that a component of this index is an indicator for credit supply; alternative proxies of credit supply have been put forward by Kashyap and Wilcox (1993), Gertler and Gilchrist (1994), and Lown and Morgan (2006).

[22]We also constrained horizons other than 6, and the findings do not change.

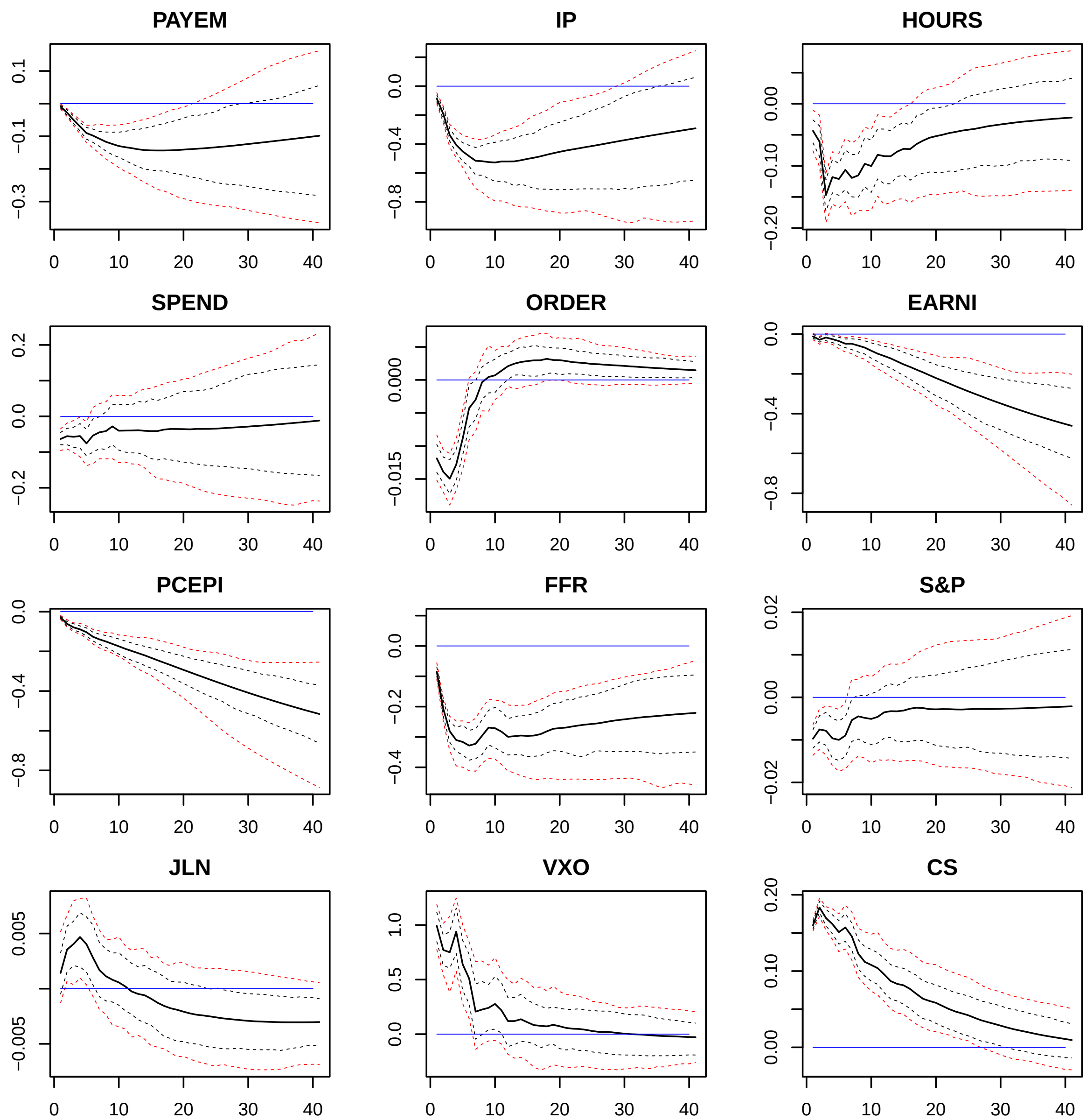

Figure 9: Responses to financial shocks. The figure reports the posterior median (black solid lines), the 68% Bayesian credibility region (black dashed lines), and the 90% Bayesian credibility region (red dashed lines) of the impulse response functions to financial shocks. The blue solid line is the zero line. The shock size is set to one standard deviation.

credit spread shocks.[23] Overall, omitting the role of either form of uncertainty results in under-estimating the effects of credit shocks.

[23]The null being $\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{q}_3^* = \boldsymbol{c}'_{20}(\boldsymbol{\phi})\boldsymbol{q}_3^* = 0$.

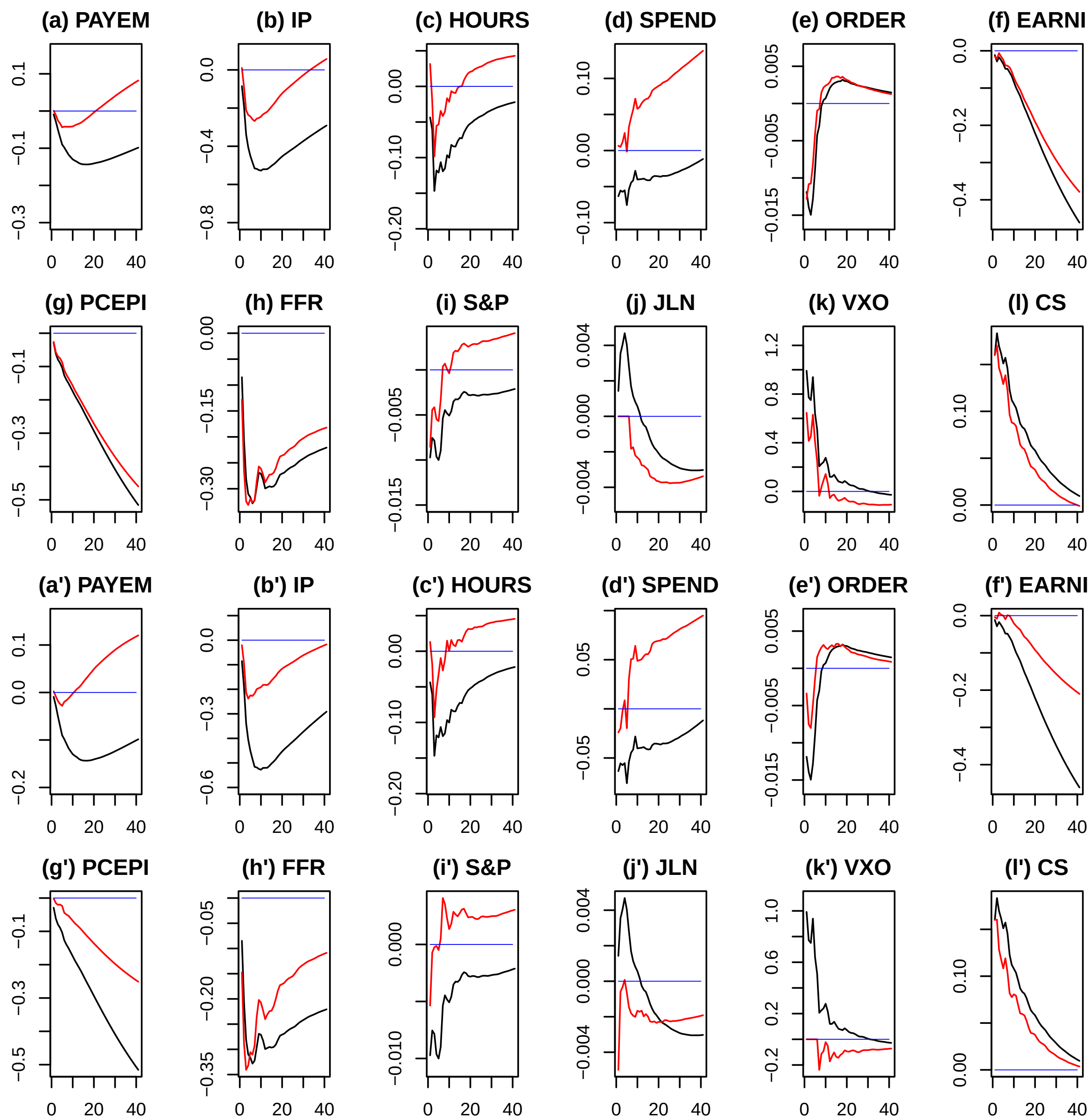

Figure 10: Financial shocks: shutting down uncertainty. Panels (a)-(l) report the posterior median of the impulse response functions to financial shocks for the baseline identification (black line) and when the response of macroeconomic uncertainty is shut down (red line). Panels (a′)-(l′) report the posterior median of the impulse response functions to financial shocks for the baseline identification (black line) and when the response of financial uncertainty is shut down (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

4.3.2 Relation with other studies

There are other papers which have analyzed the issue of the role of uncertainty in the transmission of financial shocks. In what follows we describe the main differences in the identification approach with respect to these studies.

Caldara et al. (2016) identified level (financial) and second moment (uncertainty) shocks by employing a penalty function approach which relies on the ordering of the two first and second moment proxies and found that results are very sensitive to ordering. They assumed a single source of uncertainty within the economy. The advantage of the identification scheme we propose here is that we do not require any ordering and do not exclude multiple forms of uncertainty. Furlanetto et al. (2017) and Caggiano et al. (2021) identified financial uncertainty and pure financial shocks by employing a mix of sign, ratio, and narrative restrictions. They do not separately identify macroeconomic uncertainty shocks. Compared to these contributions, our approach achieves point-identification thereby avoiding the problem inherent with is set-identified models, i.e., it is not clear how much the posterior estimation is driven by the prior distributions (Baumeister & Hamilton, 2015; Giacomini & Kitagawa, 2020). Brianti (2021) identified credit supply and macroeconomic uncertainty shocks relying on the qualitatively different responses of corporate cash holdings to a macroeconomic uncertainty shock (that pushes firms to increase their cash holdings for precautionary reasons) and a first-moment financial shock (that leads firms to reduce cash reserves as they lose access to external finance). However, i) those restrictions come from a theoretical framework with exogenous uncertainty and ii) the financial shocks as estimated by Brianti (2021) are a mix between first- and second-moment shocks within the financial sector, and as such cannot separate financial uncertainty shocks from pure credit supply disturbances.

5 Conclusions

This paper developed a novel multiple shocks identification scheme for SVARs, based on constraining the FEV decomposition. The approach involves the solution of a quadratic optimization problem. We characterized the properties of this approach, such as existence and uniqueness of a solution, and provided an algorithm for its implementation. The identification and estimation toolkit developed in this paper is general, and can be applied in any SVAR where standard ordering and sign restrictions are not desirable or sufficient to identify all of the shocks of interest. We used the approach to investigate the effects of uncertainty allowing for endogeneity. We also considered the interaction of uncertainty with financial shocks. Using US data, we found that some variables have a significant contemporaneous feedback effect on macroeconomic uncertainty, and overlooking this endogenous channel can lead to distortions. On the other hand, our empirical results suggest that financial uncertainty is likely to be an exogenous source of business cycle fluctuations. Finally, we found that omitting the role of uncertainty in the transmission mechanism can lead to underestimate the effects of financial shocks on the economy.

Appendix A: Proofs

For simplicity, in our proofs we assume $k = 3$, but results trivially hold for any finite discrete scalar $k > 0$.

Proof of Proposition 2.1.

Note that the feasibility region for $\boldsymbol{q}_1$ is characterized by $f_i \leq 0$ for $i = 1, \ldots, 4$ in Section 2.1.2 of the main text. Let us write f_i for $i = 1, \ldots, 4$ more compactly:

$$f_1 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_1(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{5.1}$$

$$f_2 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_2(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{5.2}$$

$$f_3 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_3(\boldsymbol{\phi})\boldsymbol{q}_1 + \frac{1}{2}, \tag{5.3}$$

$$f_4 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_4(\boldsymbol{\phi})\boldsymbol{q}_1 - \frac{1}{2}, \tag{5.4}$$

where $\boldsymbol{A}_1(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\boldsymbol{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})$, $\boldsymbol{A}_2(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\boldsymbol{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})$, and $\boldsymbol{A}_3(\boldsymbol{\phi}) = \boldsymbol{A}_4(\boldsymbol{\phi}) = \boldsymbol{I}_n$, with $\boldsymbol{I}_n$ being the identity matrix. Note that $\boldsymbol{A}_i(\boldsymbol{\phi})$ for $i = 1, \ldots, 4$ is a square $n \times n$ matrix and can be as such decomposed into symmetric and skew-symmetric (or anti-symmetric) components (Toeplitz decomposition): $\boldsymbol{A}_i(\boldsymbol{\phi}) \equiv \boldsymbol{A}_{iS}(\boldsymbol{\phi}) + \boldsymbol{A}_{iAS}(\boldsymbol{\phi})$, where $\boldsymbol{A}_{iS}(\boldsymbol{\phi}) = \frac{\boldsymbol{A}_i(\boldsymbol{\phi}) + (\boldsymbol{A}_i(\boldsymbol{\phi}))'}{2}$ and $\boldsymbol{A}_{iAS}(\boldsymbol{\phi}) = \frac{\boldsymbol{A}_i(\boldsymbol{\phi}) - (\boldsymbol{A}_i(\boldsymbol{\phi}))'}{2}$ are the symmetric and antisymmetric components of $\boldsymbol{A}_i(\boldsymbol{\phi})$, respectively. It is trivial to show that $\boldsymbol{q}_1'\boldsymbol{A}_i(\boldsymbol{\phi})\boldsymbol{q}_1 = \boldsymbol{q}_1'\boldsymbol{A}_{iS}(\boldsymbol{\phi})\boldsymbol{q}_1$. As a result, we obtain

$$f_1 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{1S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{5.5}$$

$$f_2 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{2S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{5.6}$$

$$f_3 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{3S}(\boldsymbol{\phi})\boldsymbol{q}_1 + \frac{1}{2}, \tag{5.7}$$

$$f_4 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{4S}(\boldsymbol{\phi})\boldsymbol{q}_1 - \frac{1}{2}. \tag{5.8}$$

We now define the following objects:

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{A}_{1S} & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{H}_2 = \begin{bmatrix} \boldsymbol{A}_{2S} & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{H}_3 = \begin{bmatrix} \boldsymbol{A}_{3S} & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{H}_4 = \begin{bmatrix} \boldsymbol{A}_{3S} & 0 \\ 0 & -1 \end{bmatrix}. \tag{5.9}$$

Note that $\boldsymbol{H}_i$ for $i = 1, \ldots, 4$ is a Z-matrix, i.e., the off-diagonal elements of a symmetric matrix are non-positive. Define a set $\boldsymbol{\Omega}_0$:

$$\boldsymbol{\Omega}_0 := \{(\frac{1}{2}\boldsymbol{a'}\boldsymbol{H}_1\boldsymbol{a}, \ldots, \frac{1}{2}\boldsymbol{a'}\boldsymbol{H}_4\boldsymbol{a}) : \boldsymbol{a} \in \mathcal{R}^{n+1}\} + int\mathcal{R}^4_+. \tag{5.10}$$

It suffices to prove that if $\boldsymbol{q}_1^*$ does not exist, then $\exists \boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ such that $(\forall \boldsymbol{q}_1 \in \mathcal{R}^n) \sum_{i=1}^4 \lambda_i f_i \geq 0$. In doing so, we follow the argument in Jeyakumar et al. (2009), Theorem 5.2. Assume that $\boldsymbol{q}_1^*$ does not exist. This is equivalent to state that the following system has no solution: $\boldsymbol{q}_1 \in \mathcal{R}^n$, $f_i < 0$, $i = 1, \ldots, 4$. Introduce 4 homogeneous functions $\bar{f}_i : \mathcal{R}^{n+1} \to \mathcal{R}$, with $\bar{f}_i = \frac{1}{2}(\boldsymbol{q}, t)\boldsymbol{H}_i(\boldsymbol{q}, t)'$, where t is a scalar:

$$\bar{f}_1 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{1S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{5.11}$$
$$\bar{f}_2 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{2S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{5.12}$$
$$\bar{f}_3 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{3S}(\boldsymbol{\phi})\boldsymbol{q}_1 + \frac{1}{2}t^2, \tag{5.13}$$
$$\bar{f}_4 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{4S}(\boldsymbol{\phi})\boldsymbol{q}_1 - \frac{1}{2}t^2. \tag{5.14}$$

Note that $f_1 = \bar{f}_1$ and $f_2 = \bar{f}_2$. Then $\boldsymbol{0} \notin \boldsymbol{\Omega}_0$. Otherwise, there exists some $\boldsymbol{q} \in \mathcal{R}^n$ such that $f_i < 0$, $i = 1, \ldots, 4$, which is a contradiction (Jeyakumar et al. (2009) in Theorem 5.2 provide the technical proof of this). Since $\boldsymbol{H}_i$ for $i = 1, \ldots, 4$ are Z-matrices, $\boldsymbol{\Omega}_0$ is a convex set (see Theorem 5.1 in Jeyakumar et al. (2009)). By the convex separation theorem, there must exist $\boldsymbol{\lambda} \in \mathcal{R}^4 \backslash \{0\}$ such that for all $(y_1, \ldots, y_4) \in \boldsymbol{\Omega}_0$, $\sum_{i=1}^4 \lambda_i y_i \geq 0$. In turn, this means that there must exist $\boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ and for all $(\boldsymbol{q}, t) \in \mathcal{R}^{n+1}$, $\sum_{i=1}^4 \lambda_i \bar{f}_i \geq 0$. Setting $t = 1$ implies $f_i = \bar{f}_i$ for $i = 1, \ldots, 4$, namely $\exists \boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ such that $(\forall \boldsymbol{q}_1 \in \mathcal{R}^n) \sum_{i=1}^4 \lambda_i f_i \geq 0$.

■

Proof of Proposition 2.2.

Without loss of generality, assume a permutation of the set $\{1, \ldots, k\}$: for instance, $\boldsymbol{\sigma} = (1, \ldots, k)$. According to condition i), we then obtain that for $j = \boldsymbol{\sigma}(1) = 1$ Proposition 2.1 is satisfied, namely $\boldsymbol{q}_1$ satisfies optimization constraints. Consider the following projector operator: $proj_{\boldsymbol{q}}(\boldsymbol{v}) = \frac{\langle \boldsymbol{q},\boldsymbol{v}\rangle}{\langle \boldsymbol{q},\boldsymbol{q}\rangle}\boldsymbol{q}$, where $\langle \boldsymbol{q}, \boldsymbol{v}\rangle$ denotes the inner product of vectors $\boldsymbol{q}$ and $\boldsymbol{v}$, with $\boldsymbol{q}, \boldsymbol{v} \in \mathcal{R}^n$. Put it another way, we are projecting $\boldsymbol{v}$ orthogonally into the line spanned by $\boldsymbol{q}$. Given $\boldsymbol{q}_1$, assume the following Gram–Schmidt process for $\boldsymbol{q}_j$ with $j = \boldsymbol{\sigma}(2), \ldots, \boldsymbol{\sigma}(k)$:

$$\boldsymbol{q}_2 = \boldsymbol{v}_2 - proj_{\boldsymbol{q}_1}(\boldsymbol{v}_2) \tag{5.15}$$

$$\vdots \tag{5.16}$$

$$\boldsymbol{q}_k = \boldsymbol{v}_k - \sum_{j=1}^{k-1} proj_{\boldsymbol{q}_j}(\boldsymbol{v}_k). \tag{5.17}$$

Given $\boldsymbol{q}_1$, this corresponds to generate vectors $\boldsymbol{q}_j$ which are in the Nullspace of $\boldsymbol{q}_{j-1}$ for $j = 2, \ldots, k$, i.e., generating a series of orthogonal vectors. If $\boldsymbol{q}_j$ for $j = 1, \ldots, k$ satisfies Proposition 2.1, there must exist an orthogonal matrix $\boldsymbol{Q}_{1:k} = [\boldsymbol{q}_1, \ldots, \boldsymbol{q}_k]$ consistent with restrictions (2.6) and (2.7) in the main text. Existence follows.

Technical remark: the orthogonal vectors generated by the above Gram-Schmidt process can be easily adjusted to make them unit vectors, i.e. construct $\frac{\boldsymbol{q}}{||\boldsymbol{q}||}$. However, in our case this would be redundant as Proposition 2.1 imposes unit vectors condition.

■

Proof of Proposition 2.3.

Assume that $\boldsymbol{Q}^*_{1:k}$ exists and is orthogonal. This proof shows that under the conditions in Proposition 2.3, $\boldsymbol{q}^*_1, \ldots, \boldsymbol{q}^*_k$ are unique. Without loss of generality, for $j = 1$ restrictions (2.6) are reduced to:

$$\boldsymbol{q}_1'[\boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})]\boldsymbol{q}_1 \geq 0 \tag{5.18}$$

$$\boldsymbol{q}_1'[\boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})]\boldsymbol{q}_1 \geq 0, \tag{5.19}$$

where $\boldsymbol{\Upsilon}^{\boldsymbol{i}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}(\boldsymbol{\phi})\boldsymbol{c}'_{ih}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{ih}(\boldsymbol{\phi})\boldsymbol{c}_{ih}(\boldsymbol{\phi})}$. Let $\boldsymbol{\Upsilon}^{\boldsymbol{12}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{1h}(\boldsymbol{\phi})\boldsymbol{c}'_{1h}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{1h}(\boldsymbol{\phi})\boldsymbol{c}_{1h}(\boldsymbol{\phi})} - \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{2h}(\boldsymbol{\phi})\boldsymbol{c}'_{2h}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{2h}(\boldsymbol{\phi})\boldsymbol{c}_{2h}(\boldsymbol{\phi})}$ and $\boldsymbol{\Upsilon}^{\boldsymbol{13}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{1h}(\boldsymbol{\phi})\boldsymbol{c}'_{1h}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{1h}(\boldsymbol{\phi})\boldsymbol{c}_{1h}(\boldsymbol{\phi})} - \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{3h}(\boldsymbol{\phi})\boldsymbol{c}'_{3h}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{3h}(\boldsymbol{\phi})\boldsymbol{c}_{3h}(\boldsymbol{\phi})}$.

Thus, restrictions on $\boldsymbol{q}_1$ are

$$\boldsymbol{q}_1' \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\mathbf{12}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0 \tag{5.20}$$

$$\boldsymbol{q}_1' \boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\mathbf{13}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0. \tag{5.21}$$

For simplicity, and without loss of generality, assume $\tilde{h} = 0$:

$$\boldsymbol{q}_1' \boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{12}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0 \tag{5.22}$$

$$\boldsymbol{q}_1' \boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{13}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0, \tag{5.23}$$

where

$$\boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{12}}(\boldsymbol{\phi}) = \frac{\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}'(\boldsymbol{\phi})}{\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})} - \frac{\boldsymbol{c}_{20}(\boldsymbol{\phi})\boldsymbol{c}_{20}'(\boldsymbol{\phi})}{\boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{c}_{20}(\boldsymbol{\phi})} \tag{5.24}$$

$$\boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{13}}(\boldsymbol{\phi}) = \frac{\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}'(\boldsymbol{\phi})}{\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})} - \frac{\boldsymbol{c}_{30}(\boldsymbol{\phi})\boldsymbol{c}_{30}'(\boldsymbol{\phi})}{\boldsymbol{c}_{30}'(\boldsymbol{\phi})\boldsymbol{c}_{30}(\boldsymbol{\phi})}. \tag{5.25}$$

Thus, we obtain

$$\boldsymbol{q}_1' \boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{12}}(\boldsymbol{\phi})\boldsymbol{q}_1 = \boldsymbol{q}_1' \left[m_1(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}'(\boldsymbol{\phi}) - m_2(\boldsymbol{\phi})\boldsymbol{c}_{20}(\boldsymbol{\phi})\boldsymbol{c}_{20}'(\boldsymbol{\phi}) \right] \boldsymbol{q}_1 \tag{5.26}$$

$$= m_1(\boldsymbol{\phi})\boldsymbol{q}_1'\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{q}_1 - m_2(\boldsymbol{\phi})\boldsymbol{q}_1'\boldsymbol{c}_{20}(\boldsymbol{\phi})\boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{q}_1 \tag{5.27}$$

$$= m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{q}_1 \right)^2 - m_2(\boldsymbol{\phi}) \left(\boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{q}_1 \right)^2, \tag{5.28}$$

where $m_1(\boldsymbol{\phi}) = \frac{1}{\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})}$ and $m_2(\boldsymbol{\phi}) = \frac{1}{\boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{c}_{20}(\boldsymbol{\phi})}$ are positive scalar. Similarly, we get

$$\boldsymbol{q}_1' \boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{13}}(\boldsymbol{\phi})\boldsymbol{q}_1 = m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{q}_1 \right)^2 - m_3(\boldsymbol{\phi}) \left(\boldsymbol{c}_{30}'(\boldsymbol{\phi})\boldsymbol{q}_1 \right)^2, \tag{5.29}$$

where $m_3(\boldsymbol{\phi}) = \frac{1}{\boldsymbol{c}_{30}'(\boldsymbol{\phi})\boldsymbol{c}_{30}(\boldsymbol{\phi})}$. Thus, for $j = 1$ restrictions (2.6) are equivalent to

$$m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{q}_1 \right)^2 - m_2(\boldsymbol{\phi}) \left(\boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{q}_1 \right)^2 \geq 0 \tag{5.30}$$

$$m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{q}_1 \right)^2 - m_3(\boldsymbol{\phi}) \left(\boldsymbol{c}_{30}'(\boldsymbol{\phi})\boldsymbol{q}_1 \right)^2 \geq 0. \tag{5.31}$$

Recall that for $j = 1$, conditions in Proposition 2.3 are

$$\boldsymbol{c}_{i0}'(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0 \text{ for } i = 1, \ldots, 3. \tag{5.32}$$

Combining (5.30)-(5.32) delivers the following restrictions for $j = 1$:

$$\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq \sqrt{\frac{m_2(\boldsymbol{\phi})}{m_1(\boldsymbol{\phi})}}\boldsymbol{c}'_{20}(\boldsymbol{\phi})\boldsymbol{q}_1 \tag{5.33}$$

$$\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq \sqrt{\frac{m_3(\boldsymbol{\phi})}{m_1(\boldsymbol{\phi})}}\boldsymbol{c}'_{30}(\boldsymbol{\phi})\boldsymbol{q}_1. \tag{5.34}$$

Thus, conditional on the existence of $\boldsymbol{q}_1^*$, constraints of the optimization problem become linear, and as such, convex for $\boldsymbol{q}_1$. Also, it is easy to observe that conditions in (5.32) make the objective function convex in $\boldsymbol{q}_1$. Since the problem is now convex, $\boldsymbol{q}_1^*$ must be unique. Extension to $\tilde{h} > 0$ is trivial. The same proof applies to $j = 2, 3$, i.e., $\boldsymbol{q}_2^*$ and $\boldsymbol{q}_3^*$ are unique. As a result, conditional on $\boldsymbol{q}_1^*$, $\boldsymbol{q}_2^*$ and $\boldsymbol{q}_3^*$ to exist and be orthogonal to each other, matrix $\boldsymbol{Q}_{1:k}^*$ is unique (with $k = 3$ in our setting).

■

Appendix B: Shocks Series and Robustness Checks

Here we present some evidence that the three identified shocks are truly structural and exogenous to a set of structural shocks previously identified by the literature. We have re-estimated the impulse responses by explicitly controlling, i.e, imposing orthogonality condition, for military news (Ramey, 2016), expected tax (Leeper et al., 2013), unanticipated and anticipated tax (Mertens & Ravn, 2011), monetary policy (Romer & Romer, 1989), and technology surprise (Basu et al., 2006). All the results presented in the main text are robust to this additional control. Furthermore, we have computed the correlations between the three identified shocks and those disturbances, finding that correlations are never significant at 1% and 5% level.

Ludvigson et al. (2021) and Caggiano et al. (2021) stressed that credible identification regimes need to estimate shocks consistent with specific episodes in the history. In particular, we focus on three events: Black Monday (October 1987), Lehman collapse (September 2008), and Covid outbreak (March 2020). For the Black Monday, our estimated financial uncertainty shock is large,[24] while this is not the case for credit supply disturbances. This is in line with the narrative of Ludvigson et al. (2021) and Caggiano et al. (2021), where the Black Monday is featured by significant financial volatility but low

[24] Bigger than the median. This definition of large shocks is consistent with Ludvigson et al. (2021).

credit conditions disruption. In September 2008, we find that our three estimated shocks are all large, which is consistent with the consensus view of the Great Recession as a mix of financial and uncertainty shocks. Also, we have extended our sample up to 2020: both macro and financial uncertainty shocks are large and bigger than pure financial shock in March 2020, which is compatible with the belief that the pandemic prompted a spike in uncertainty but significant fiscal and monetary policy interventions prevented credit supply deterioration.

The findings we have obtained are also robust to the following battery of checks, which are available upon request: lag length from three to twelve; selecting the prior tightness by maximizing the marginal likelihood rather than employing a flat specification; using the measures of Carriero et al. (2018b) and Jurado et al. (2015) as alternative proxies of financial uncertainty and the proposal in Carriero et al. (2018b) as proxy of macroeconomic uncertainty; employing the excess bond premium in Caldara et al. (2016) and Gilchrist and Zakrajšek (2012) as credit spreads. Moreover, we have run a quarterly specification, finding that results are equivalent to what shown so far.

Finally, a number of papers have pointed out that the effect of uncertainty shocks is more intense when the Zero Lower Bound (ZLB) holds (Caldara et al., 2016; Caggiano et al., 2014; Basu & Bundick, 2017; Johannsen, 2014). Thus, we have estimated the model over the sample up to 2008m9, which removes the years of the Great Recession where the ZLB binds. The results are qualitatively equivalent to Figure 3 and Figure 4; quantitatively, the response of the variables is slightly less pronounced. Since this is fully consistent with the previous literature, we omit it for brevity.

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