Discussion Papers in Economics

GENERALIZING THE *MAX SHARE* IDENTIFICATION TO MULTIPLE SHOCKS IDENTIFICATION: AN APPLICATION TO UNCERTAINTY

By

Andrea Carriero
(Queen Mary University of London and University of Bologna)

&

Alessio Volpicella
(University of Surrey).

DP 03/22

School of Economics
University of Surrey
Guildford
Surrey GU2 7XH, UK
Telephone +44 (0)1483 689380
Facsimile +44 (0)1483 689548
Web https://www.surrey.ac.uk/school-economics
ISSN: 1749-5075

Generalizing the *Max Share Identification* to multiple shocks identification: an Application to Uncertainty*

Andrea Carriero[†] and Alessio Volpicella[‡]

July 18, 2022

Abstract

We generalize the *Max Share Identification* approach to allow for simultaneous identification of a multiplicity of shocks in a Structural Vector Autoregression. Our machinery therefore overcomes the well-known drawbacks that individually identified shocks (i) tend to be correlated to each other or (ii) can be separated under orthogonalizations with weak economic ground. We show that identification corresponds to solving a non-trivial optimization problem on the columns transforming reduced-form shocks into structural shocks. We provide conditions for existence and uniqueness of a solution, and Bayesian algorithms for estimation and inference. We use the approach to study the effects of uncertainty shocks, allowing for the possibility that uncertainty is an endogenous variable, and distinguishing macroeconomic from financial uncertainty. Using US data we find that macroeconomic uncertainty is mostly endogenous, and that overlooking this fact can lead to distortions on the estimates of its effects. We show that the distinction between macroeconomic and financial uncertainty is empirically relevant. Finally, we study the relation between uncertainty shocks and pure financial shocks, showing that the latter can have attenuated effects if one does not take into account the endogeneity of uncertainty.

Keywords: Causality, Forecast Error Variance, Identification, Structural Vector Autoregression, Uncertainty.

JEL: C11, C32, E32, E37, E44.

*This paper circulated with the title "Identification of Uncertainty Shocks", "The Identifying Information in the Forecast Error Variance: an Application to Endogenous and Heterogeneous Uncertainty and its Relationship with Financial Shocks" and "Identification through the Forecast Error Variance Decomposition: an Application to Uncertainty". We thank Ana Galvão, Valentina Corradi, Luca Fanelli, Stepana Lazarova and Matthew Read for insightful suggestions. We are also grateful to participants to 2021 RES conference, 2021 AMES conference, 2021 IAAE conference, 7^{th} RCEA Time Series workshop, 2021 EEA-ESEM Conference, Workshop in Macroeconometrics, 2021 EWMES, 2021 CEF for beneficial discussion, 2022 NASMES Conference, 2022 AMES-China Conference, and 2022 ESAM Conference.

[†]Queen Mary University of London and University of Bologna. Email: a.carriero@qmul.ac.uk

[‡]University of Surrey, School of Economics. Email: a.volpicella@surrey.ac.uk

1 Introduction and Related Literature

Since the influential paper of Bloom (2009), the business cycle relationship between uncertainty and macroeconomic variables and the underlying transmission mechanism have received extensive consideration.[1]

Three challenges come to the fore. First, most works usually employ structural vector autoregressions (SVARs) with some recursive identification scheme. The common assumption is that uncertainty is exogenous, i.e. it does not respond contemporaneously to economic variables, whereas economic variables react contemporaneously to uncertainty.[2] Recursive schemes are widespread due to the simplicity of implementation and interpretation, but for uncertainty it is extremely challenging to defend them as convincing identification strategies.

In fact, the current evidence makes researchers unable to take up a position on the direction of the causality between uncertainty and economic variables. On the contrary, both directions of causality are conceivable and macroeconomic theory is also ambiguous about the possible sign of the effects of uncertainty on the economy.

Uncertainty can affect the economy through firms' behavior, which is influenced by uncertainty because of (i) the real option argument (Bernanke, 1983; McDonald & Siegel, 1986); (ii) the delay of hiring and investment decisions (Bloom, 2009; Bloom et al., 2018; Leduc & Liu, 2016); (iii) the interaction with financial frictions that impact on firms' decisions (Arellano et al., 2018; Gilchrist et al., 2014; Alfaro et al., 2018). The uncertainty can influence the economy also through precautionary savings (Basu & Bundick, 2017; Fernández-Villaverde et al., 2011). On the other hand, some scholars point out that bad economic and/or credit conditions are likely to cause a rise in uncertainty (Van Nieuwerburgh & Veldkamp, 2006; Bachmann & Moscarini, 2011; Fajgelbaum et al., 2017; Brunnermeier & Sannikov, 2014; Atkinson et al., 2021; Plante et al., 2018). Empirical contributions that allow for both directions of causality include Carriero et al. (2021), Ludvigson et al. (2021), and Angelini et al. (2019). All these contributions show

[1]Bloom (2014) provides an excellent survey. A partial list of works consists of Bloom (2009), Bachmann et al. (2013), Caggiano et al. (2014), Jurado et al. (2015), Rossi and Sekhposyan (2015), Caldara et al. (2016), Baker et al. (2016), Basu and Bundick (2017), Cesa-Bianchi et al. (2018), Shin and Zhong (2020), Carriero et al. (2018b), Bloom et al. (2018), Angelini et al. (2019), Ludvigson et al. (2021), and Carriero et al. (2021).

[2]We use the terms exogenous (endogenous) as shorthand for predetermined (not predetermined) within the period.

that the direction of causality might depend on the uncertainty typology and measure of choice. Additional literature points out that uncertainty can stimulate economic activity (growth options theory): a mean-preserving spread in risk originated from an unbounded upside combined with a limited downside can lead firms to invest and hire, since the rise in mean preserving risk raises expected profits.[3]

A separate challenge is about the origins of uncertainty. Standard theories claim that uncertainty originates from macroeconomic fundamentals, e.g. productivity, and that such real economic uncertainty, when interacted with market frictions, decreases real activity. However, it has been argued that uncertainty depresses the economy via its impact on financial markets (Gilchrist et al., 2014), or through sources of uncertainty specific to financial markets (Bollerslev et al., 2009). Furthermore, Ng and Wright (2013) discuss that financial uncertainty –as distinct from macroeconomic uncertainty– could have a pivotal role in recessions after 1982, both as a cause and as a propagation channel. The challenge also arises because the theoretical literature has focused on volatility coming from fundamentals, while empirical efforts have usually tested those frameworks employing uncertainty proxies that are strongly correlated with financial market variables. This naturally leads to wonder whether it is macroeconomic uncertainty or financial uncertainty (or both) to drive business cycle fluctuations. The current literature does not disentangle the contributions of macroeconomic versus financial uncertainty to business cycle fluctuations, nor it allows feedback between macroeconomic and financial uncertainty. Exceptions are the small-scale models in Ludvigson et al. (2021) and Angelini et al. (2019) and the contribution in Shin and Zhong (2020).

The final challenge is that there is high degree of comovement between indicators of financial distress such as credit spreads and uncertainty proxies as both variables are "fast moving", as pointed out in several studies including Caldara et al. (2016), Brianti (2021), Caggiano et al. (2021). It is therefore difficult to impose plausible zero contemporaneous restrictions to identify these two disturbances. It is also difficult to impose sign restrictions as uncertainty and financial shocks could have theoretically the same qualitative effects on both prices and quantities.

This paper proposes a new approach to identification which allows to deal with the

[3] For instance, see Oi (1961), Hartman (1972), Abel (1983), Bar-Ilan and Strange (1996), Pástor and Veronesi (2006), Kraft et al. (2018), Segal et al. (2015), and Fernández-Villaverde and Guerrón-Quintana (2020).

three issues above. The approach allows for endogeneity of uncertainty, i.e. for a causal transmission channel going from uncertainty to the economic variables as well as the opposite. It also allows to separately identify different sources of uncertainty, and to disentangle uncertainty shocks from pure financial shocks. To our knowledge, this is the first paper to tackle these issues in a unified framework.

Within a SVAR, the proposed identification scheme generalizes the *Max Share Identification*, i.e. optimizing the Forecast Error Variance (FEV) decomposition for a single shock identification, to simultaneous identification of a multiplicity of shocks. While traditional *Max Share Identification*[4] is usually performed in combination with sign restrictions, we also introduce further constraints. Instead of constraining the FEV to a single shock, we simultaneously restrict (a function of) the FEV of target variables to more shocks. For example, consider the task of identifying a macroeconomic uncertainty shock, a financial uncertainty shock, and a credit supply shock:[5] the identifying assumption is that (i) the three shocks must maximize a function of the total variation of the three variables, (ii) subject to constraints that each shock of interest -say, macro uncertainty shock- needs to explain the variation of the corresponding target variable -say, some macro uncertainty proxy- more than the other target variables -say, financial uncertainty proxy and credit spread-. While we focus on uncertainty and financial disturbances, our identification and estimation toolkit is general, and can be applied in any SVAR where scholars want to identify more shocks (as discussed in Section 2.4). Inequality constraints on the FEV in (ii), which are an additional novelty to the literature, are tailored to our empirical exercise. Depending on the application, restrictions in (ii) can be replaced by, or combined with, sign restrictions.

The *Max Share Identification* for a single shock is a popular device. In our approach instead the optimization constrains the FEV decomposition of different variables and needs to be verified for all the shocks simultaneously. This is computationally more challenging, but has the advantage of (i) identifying simultaneously a multiplicity of shocks, and (ii) being well-suited when we want to distinguish competing shocks: uncertainty dis-

[4]Scholars have been applying *Max Share Identification*, or some variations of it, to numerous applications: a very incomplete list includes Uhlig (2004) for technology and wage-push shocks, Francis et al. (2014) for technology shocks, Barsky and Sims (2011) for news shocks, Mumtaz et al. (2018) for credit shocks, Mumtaz and Theodoridis (2018) for inflation target shocks, Caldara et al. (2016) for uncertainty and credit shocks, and Angeletos et al. (2020) for a variety of supply and demand shocks.

[5]A credit supply shock is defined as a shock to credit supply and measured through the credit spreads.

turbances are a natural example. Researchers have been increasingly identifying shocks simultaneously (Ludvigson et al., 2021; Brianti, 2021; Giacomini, Kitagawa, & Read, 2022; Cascaldi-Garcia & Galvão, 2020; Piffer & Podstawski, 2018; Furlanetto et al., 2017; Mertens & Ravn, 2013). Recently, literature started stressing that individually identified shocks are often correlated with other disturbances, and as such are not truly structural. For instance, Cascaldi-Garcia and Galvão (2020) find that uncertainty and news shocks, if singularly identified, are strongly correlated. Sequential, rather than simultaneous, identification comes with similar problems: sequentially identified shocks tend to be correlated, unless some orthogonality condition is imposed. However, the latter assumes some ordering restriction which usually have weak economic ground. For example, Uhlig (2004) explicitly argues that the *Max Share Identification* of more shocks implies further arbitrary orthogonalizations, making any economic interpretation hard. Caldara et al. (2016) apply the *Max Share Identification* to sequentially identify uncertainty and financial shocks, finding that changing the order of identification dramatically affects the results, e.g. uncertainty can be both expansionary and recessionary. While some identification strategies, such as sign restrictions and proxy SVARs, have toolkit for simultaneous identification, the methodological contribution of this paper is to make the *Max Share Identification* suitable for multiple shocks simultaneous identification.

Our identification strategy involves the solution of a constrained maximization problem, where the objective function is an equally weighted linear combination of the FEV of the (target) variables of interest and the constraints are the inequality restrictions on the FEV. We show that this corresponds to a non-convex quadratic optimization problem on the columns of the rotation matrix transforming reduced-form residuals into structural shocks. However, we provide a flexible toolkit and establish mild conditions under which the solution of the optimization problem exists and is unique. We develop simple algorithms to perform Bayesian estimation and inference, even though of course the identification result and properties do hold also in a frequentist setting. While the main text illustrates our machinery in the time domain, Appendix D shows that its implementation in the frequency domain is fully feasible.

The approach also differs from Amir-Ahmadi and Drautzburg (2021) and Volpicella (2022). Amir-Ahmadi and Drautzburg (2021), who employ set-identification through ranking restrictions on the impulse response functions, combined with standard sign restrictions. Volpicella (2022) puts sign restrictions and bounds on the FEV to set-identify a single shock; on the contrary in this paper we point-identify shocks, do not place bounds, and allow identification of a multiplicity of shocks.

Turning to the empirical application, we start with a simulation exercise showing that our identification successfully captures the effect of uncertainty on the economy regardless the exogeneity extent of the uncertainty disturbances in the Data Generating Process (DGP).

We apply the proposed identification scheme to a SVAR model estimated with US data. We find that both macroeconomic and financial uncertainty shocks act as negative demand shocks, i.e. decrease the real activity and trigger a deflationary pressure. The responses to the two shocks are quantitatively substantially different: macroeconomic uncertainty has a stronger and more persistent effect on the real activity variables. We also find evidence that separating macroeconomic from financial uncertainty is important, and not doing so can dramatically distort the impulse responses.

We find evidence that uncertainty is endogenous to some extent. In particular, dismissing the feedback effect from the macroeconomy to macroeconomic uncertainty changes the estimated responses in non trivial ways. This suggests that naive schemes such as sign restrictions and recursive ordering are too restrictive. These results are in line with Ludvigson et al. (2021) and Carriero et al. (2021). Angelini et al. (2019) instead find that both macro and financial uncertainty are exogenous.

In closing we turn our interest on the relation between financial conditions and uncertainty.[6] We find that this channel is crucial for the transmission of financial uncertainty shocks but negligible for the transmission of macroeconomic uncertainty shocks. We also find that financial shocks are recessionary and ignoring the endogenous role of uncertainty leads to under-estimating their effects on the economy.

The paper is organized as follows. Section 2 introduces the identification strategy; Section 3 illustrates the effectiveness of our approach via a simulation; Section 4 presents the empirical application; Section 5 concludes. Appendix A, B, C and D provide proofs, robustness checks, algorithms, and representation in the frequency domain, respectively.

[6]Several recent theoretical contributions emphasize the pivotal role that financial conditions might have in amplifying and propagating the effects of uncertainty to real economy (Arellano et al., 2018; Christiano et al., 2014; Gilchrist et al., 2014; Brunnermeier & Sannikov, 2014; Alfaro et al., 2018).

2 Theoretical framework

Consider a SVAR(p) model

$$\boldsymbol{A}_0 \boldsymbol{y}_t = \boldsymbol{a} + \sum_{j=1}^{p} \boldsymbol{A}_j \boldsymbol{y}_{t-j} + \boldsymbol{\epsilon}_t \tag{2.1}$$

for $t = 1, \ldots, T$, where $\boldsymbol{y}_t$ is an $n \times 1$ vector of endogenous variables, $\boldsymbol{\epsilon}_t$ an $n \times 1$ vector white noise process, normally distributed with mean zero and variance-covariance matrix $\boldsymbol{I}_n$, $\boldsymbol{A}_j$ is an $n \times n$ matrix of structural coefficient for $j = 0, \ldots, p$. The disturbances $\boldsymbol{\epsilon}_t$ are mutually uncorrelated, and are therefore interpretable as structural shocks. The initial conditions $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_p$ are given. Let $\boldsymbol{\theta} = (\boldsymbol{A}_0, \boldsymbol{A}_+)$ collect the structural parameters, where $\boldsymbol{A}_+ = (\boldsymbol{a}, \boldsymbol{A}_j)$ for $j = 1, \ldots, p$.

The reduced-form representation is a Vector Autoregression (VAR):

$$\boldsymbol{y}_t = \boldsymbol{b} + \sum_{j=1}^{p} \boldsymbol{B}_j \boldsymbol{y}_{t-j} + \boldsymbol{u}_t, \tag{2.2}$$

where $\boldsymbol{b} = \boldsymbol{A}_0^{-1}\boldsymbol{a}$ is an $n \times 1$ vector of constants, $\boldsymbol{B}_j = \boldsymbol{A}_0^{-1}\boldsymbol{A}_j$, $\boldsymbol{u}_t = \boldsymbol{A}_0^{-1}\boldsymbol{\epsilon}_t$ denotes the $n \times 1$ vector of reduced-form errors. $var(\boldsymbol{u}_t) = E(\boldsymbol{u}_t\boldsymbol{u}_t') = \boldsymbol{\Sigma} = \boldsymbol{A}_0^{-1}(\boldsymbol{A}_0^{-1})'$ is the $n \times n$ variance-covariance matrix of reduced-form errors. Let $\boldsymbol{\phi} = (\boldsymbol{B}, \boldsymbol{\Sigma}) \in \boldsymbol{\Phi}$ collect the reduced-form parameters, where $\boldsymbol{B} \equiv [\boldsymbol{b}, \boldsymbol{B}_1, \ldots, \boldsymbol{B}_p]$, $\boldsymbol{\Phi} \subset \mathcal{R}^{n+n^2p} \times \boldsymbol{\Xi}$, and $\boldsymbol{\Xi}$ is the space of symmetric positive semidefinite matrices.

We define the $n \times n$ matrix

$$\boldsymbol{IR}^h = \boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{A}_0^{-1} \tag{2.3}$$

as the impulse response at h-th horizon for $h = 0, 1, \ldots,$ where $\boldsymbol{C}_h(\boldsymbol{B})$ is the h-th coefficient matrix of $(\boldsymbol{I}_n - \sum_{h=1}^{p} \boldsymbol{B}_h L^h)^{-1}$. Its (i, j)-element denotes the effect on the i-th variable in $\boldsymbol{y}_{t+h}$ of a unit shock to the j-th element of $\boldsymbol{\epsilon}_t$. As is well-known there are several observationally equivalent $\boldsymbol{A}_0$ matrices, and expression (2.3) actually involves a set of impulse responses.

To formalize this fact we follow Uhlig (2005) and define the set of all IRFs through an $n \times n$ orthonormal matrix $\boldsymbol{Q} \in \boldsymbol{\Theta}(n)$, where $\boldsymbol{\Theta}(n)$ characterizes the set of all orthonormal

$n \times n$ matrices. Uhlig (2005) show that $\{\boldsymbol{A}_0 = \boldsymbol{Q}'\boldsymbol{\Sigma}_{tr}^{-1} : \boldsymbol{Q} \in \boldsymbol{\Theta}(n)\}$ is the set of observationally equivalent $\boldsymbol{A}_0$'s consistent with reduced-form parameters, where $\boldsymbol{\Sigma}$ relates to $\boldsymbol{A}_0$ by $\boldsymbol{\Sigma} = \boldsymbol{A}_0^{-1}(\boldsymbol{A}_0^{-1})'$, $\boldsymbol{\Sigma}_{tr}$ denotes the lower triangular Cholesky matrix with non-negative diagonal coefficients of $\boldsymbol{\Sigma}$. The likelihood function depends on $\boldsymbol{\phi}$ and does not contain any information about $\boldsymbol{Q}$, leading to ambiguity in decomposing $\boldsymbol{\Sigma}$. The identification problem arises because there is a multiplicity of $\boldsymbol{Q}$'s which deliver $\boldsymbol{A}_0$ given $\boldsymbol{\phi}$. Specifically, the impulse response of variable i to shock j at horizon h, i.e. (i, j)-element of $\boldsymbol{IR}^h$, can be expressed as $\boldsymbol{e}_i'\boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}\boldsymbol{Q}\boldsymbol{e}_j \equiv \boldsymbol{c}_{ih}'(\boldsymbol{\phi})\boldsymbol{q}_j$, where $\boldsymbol{e}_i$ is the i-th column vector of $\boldsymbol{I}_n$, $\boldsymbol{q}_j$ is the j-th column of $\boldsymbol{Q}$ and $\boldsymbol{c}_{ih}'(\boldsymbol{\phi})$ represents the i-th row vector of $\boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}$. Alternative identification schemes can be achieved by placing a set of restrictions on $\boldsymbol{Q}$. For example, imposing $\boldsymbol{Q} = \boldsymbol{I}_n$ implies a recursive ordering identification, i.e. the Cholesky decomposition, whereas sign restrictions specify a set of admissible $\boldsymbol{Q}$'s.

2.1 Sign restrictions

Assume that the researcher is interested in imposing some sign restrictions on the impulse response vector to the j-th structural shock, and let s_{hj} denote the number of sign restrictions on impulse responses at horizon h. In this case, the impulse response is given by the j-th column vector of $\boldsymbol{IR}^h = \boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}\boldsymbol{Q}$, and the sign restrictions are

$$\boldsymbol{S}_{hj}(\boldsymbol{\phi})\boldsymbol{q}_j \geq \boldsymbol{0},$$

where $\boldsymbol{S}_{hj}(\boldsymbol{\phi}) \equiv \boldsymbol{D}_{hj}\boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}$ is a $s_{hj} \times n$ matrix and $\boldsymbol{D}_{hj}$ is the $s_{hj} \times n$ selection matrix that selects the sign-restricted responses from the $n \times 1$ response vector $\boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}\boldsymbol{q}_j$. The nonzero elements of $\boldsymbol{D}_{hj}$ can be equal to 1 or to -1 depending on the sign of the restriction on the impulse response of interest. By considering multiple horizons, the whole set of sign restrictions placed on the $j-$th shock is

$$\boldsymbol{S}_j(\boldsymbol{\phi})\boldsymbol{q}_j \geq \boldsymbol{0}. \tag{2.4}$$

Specifically, $\boldsymbol{S}_j$ is a $\left(\sum_{h=0}^{\bar{h}_j} s_{hj}\right) \times n$ matrix defined by $\boldsymbol{S}_j(\boldsymbol{\phi}) = \left[\boldsymbol{S}_{0j}'(\boldsymbol{\phi}), \ldots, \boldsymbol{S}_{\bar{h}_j j}'(\boldsymbol{\phi})\right]'$. Let $\mathcal{I}_{\mathcal{S}} \subset \{1, 2, \ldots, n\}$ be the set of indices such that $j \in \mathcal{I}_{\mathcal{S}}$ if some of the impulse responses to the j-th structural shock are sign-constrained. Thus, the set of all sign restrictions is[7]

$$\boldsymbol{S}_j(\boldsymbol{\phi})\boldsymbol{q}_j \geq \boldsymbol{0}, \text{ for } j \in \mathcal{I}_{\mathcal{S}}. \tag{2.5}$$

2.2 Identification strategy

Our identification scheme identifies $k \leq n$ shocks $j \in 1,\ldots,k$, denoted by $\boldsymbol{q}_j = \boldsymbol{Q}\boldsymbol{e}_j$, where $\boldsymbol{q}_j'\boldsymbol{q}_{\tilde{j}} = 0$ for $j \neq \tilde{j}$ is the standard orthogonality condition.

In our empirical application we will set $k = 3$ and shocks of interest are those to macroeconomic uncertainty, financial uncertainty, and credit supply. The identifying assumption is that in the short-run (i) the three shocks must maximize a function of the total variation of the three variables (a function of the FEV of the three variables) (ii) subject to constraints that each shock of interest -say, macro uncertainty shock- needs to explain the variation of the corresponding target variable -say, some macro uncertainty proxy- more than the other target variables -say, financial uncertainty proxy and credit spread-.

Importantly, this approach does not require the researcher to take a stance in regards to the possible exogeneity or endogeneity of uncertainty: uncertainty can impact on macro variables, and vice-versa. In fact, Section 3 shows that our identification assumptions are consistent with DGPs regardless whether those frameworks consider endogenous or exogenous uncertainty, and successfully recover the impulse response functions of different DGPs.

2.2.1 Formal setup

In what follows we provide a formalization of the strategy described above. Let $CFEV_j^i(\tilde{h})$ denote the FEV at horizon $\tilde{h}$ of variable i explained by the j-th structural shock:

$$CFEV_j^i(\tilde{h}) = \boldsymbol{q}_j'\boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{i}}(\boldsymbol{\phi})\boldsymbol{q}_j, \tag{2.6}$$

where $\boldsymbol{\Upsilon}_{\tilde{\boldsymbol{h}}}^{\boldsymbol{i}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}(\boldsymbol{\phi})\boldsymbol{c}_{ih}'(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}'(\boldsymbol{\phi})\boldsymbol{c}_{ih}(\boldsymbol{\phi})}$ is a $n \times n$ positive semidefinite matrix. Expression (2.6) describes the percent contribution - expressed with a number in the interval $[0, 1]$ - of the shock j to the unexpected fluctuations of variable i at horizon $\tilde{h}$.

[7] Given the j-th shock, sign restrictions on $\boldsymbol{A}_0$ and $\boldsymbol{A}_+$ can be appended to equation (2.5), since they can be expressed as linear inequalities on $\boldsymbol{q}_j$.

Without loss of generality, suppose that (i) $j = 1$ is the first shock, $j = 2$ is the second shock, $j = 3$ is the third shock and so on; (ii) the n endogenous variables are ordered such that $i = 1$ is the macroeconomic uncertainty variable, $i = 2$ is the financial uncertainty variable, and $i = 3$ is the credit spreads. Define the following $I_{-j} = \{1, \ldots, k\}/\{j\}$ as a subset of the shocks of interest. The identification of $\boldsymbol{Q}_{1:k} = [\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_k]$, with $k = 3$ and $j \in 1, 2, 3$, requires to solve the following constrained optimization problem:[8]

$$\boldsymbol{Q}^*_{1:k} = \arg\max_{\boldsymbol{Q}_{1:k}} \sum_{i=1}^{k} \boldsymbol{q}_i' \boldsymbol{\Upsilon}^{\boldsymbol{i}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) \boldsymbol{q}_i \tag{2.7}$$

subject to

$$\boldsymbol{q}_j' \boldsymbol{\Upsilon}^{\boldsymbol{j}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) \boldsymbol{q}_j \geq \boldsymbol{q}_j' \boldsymbol{\Upsilon}^{\boldsymbol{i}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) \boldsymbol{q}_j \ for \ j = 1, \ldots, k, \quad \forall i \in I_{-j}, \tag{2.8}$$

$$\boldsymbol{S}_j(\boldsymbol{\phi}) \boldsymbol{q}_j \geq \boldsymbol{0}, \text{ for } j \in \mathcal{I}_{\mathcal{S}}, \tag{2.9}$$

and

$$\boldsymbol{Q}_{1:k}' \boldsymbol{Q}_{1:k} = \boldsymbol{I}_n. \tag{2.10}$$

In equation (2.7) and (2.8) $\tilde{h}$ is common across target variables $i = 1, \ldots, k$, but of course we can maximize and constrain the FEV of target variables at different horizons, depending on the application.[9]

Note that in our application $\mathcal{I}_{\mathcal{S}} = \emptyset$, i.e. no sign restrictions, but results for existence and uniqueness of solution accommodates them. Thus, the optimization process involves k variables only in the empirical exercise. For $j = 1$, we will identify the macroeconomic uncertainty shock as the innovation that maximizes its contribution to the sum of the FEV of the three target variables subject to the following constraints. Restrictions (2.8) establish that (for $j = 1$) the contribution of the macroeconomic uncertainty shock to the

[8] Once columns 1 to k are identified, we can always construct orthogonal columns $k+1$ to n.

[9] The literature has not reached a consensus on the nature and degree of persistence of uncertainty shocks, and there is no uncontroversial theory and/or empirical evidence supporting specific claims. For example, see opposite findings in Cerra and Saxena (2008), showing that financial shocks are more persistent than uncertainty disturbances, and Berger et al. (2020) and Bonciani and Oh (2022), who find that uncertainty shocks are very persistent. Carrière-Swallow and Céspedes (2013) find mixed results across countries. Brianti (2021) argues that uncertainty is more or equally persistent than credit supply shocks, while Caldara et al. (2016) find similar persistence. Thus, in order to restrict the system as less as possible, identifying assumptions in the application are imposed on $\tilde{h} = 0$. However, we re-estimate the model up to $\tilde{h} = 6$, that is a period of heightened economic uncertainty and credit deterioration, rather than just a one-off spike, finding no significant differences.

FEV of the macroeconomic uncertainty variable must be higher than the contribution to the FEV of financial uncertainty variables and credit spreads (upon impact). Those restrictions are instrumental to separate macroeconomic uncertainty shocks from financial uncertainty and credit supply shocks.

It is worth stressing that the constraints in (2.8) are not automatically satisfied by maximization in equation (2.7): the latter requires to maximize a sum, while the constraints are imposed on the components of the sum. In practice, the degree of relevance of restrictions in (2.8) depends on the empirical exercise. In our application, we find that inequality constraints on the FEV are pivotal to distinguish competing shocks, while their omission biases the results (see the fourth row in Figure 1 and 2 and the discussion in the simulation exercise). This is less likely the case with the single shock *Max Share Identification*.[10] Restrictions (2.10) ensure that the identified shocks are mutually orthogonal.[11] Similarly for $j = 2, 3$, the problem (2.7)-(2.10) identifies the financial uncertainty and the credit supply shock, respectively.

Our machinery, and below properties, are developed in the time domain. However, an increasing number of scholars has recently been focusing on frequency domain. For instance, Angeletos et al. (2020) argue that, unlike what is generally believed, targeting $6 - 32$ quarters recovers the business cycle in the frequency domain, but not in the time domain. Thus, Appendix D illustrates the full feasibility of our toolkit in the frequency domain.

2.2.2 Existence and Uniqueness of a solution

Several papers point out that there is a trade-off between sharp identification and computation, and this is especially true when using inequality constraints.[12] In fact restrictions that are too tight can lead to unfeasible or empty regions, i.e. the constraints are so demanding that they are rejected in the data. In this section we provide sufficient conditions for the existence of a solution to the constrained optimization problem. Doing so solves the trade-off by ensuring that an identification scheme can be found which is both informative and not rejected by data.

[10] For instance, see Table 2 in Angeletos et al. (2020).

[11] The orthogonality restriction matters only if we restrict multiple shocks simultaneously. For individual identification, we can always construct vectors in the Nullspace of the restricted shocks.

[12] See Amir-Ahmadi and Drautzburg (2021); Giacomini and Kitagawa (2020); Giacomini, Kitagawa, and Volpicella (2022); Gafarov et al. (2018); Granziera et al. (2018); Volpicella (2022); Uhlig (2017).

Recall that $\boldsymbol{q}_j^*$ for $j-1,\ldots,k$ denotes the j-th column of the identified matrix $\boldsymbol{Q}_{1:k}^*$. For $j=1$, given the constraints in (2.8)-(2.10), we define the following functions:

$$\begin{aligned}
f_1 &= \frac{1}{2}\boldsymbol{q}_1' \left[\boldsymbol{\Upsilon}^{\mathbf{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\mathbf{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\right] \boldsymbol{q}_1,\\
f_2 &= \frac{1}{2}\boldsymbol{q}_1' \left[\boldsymbol{\Upsilon}^{\mathbf{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\mathbf{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\right] \boldsymbol{q}_1,\\
f_3 &= \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{q}_1 + \frac{1}{2},\\
f_4 &= \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{q}_1 - \frac{1}{2}.
\end{aligned}$$

Similar functions can be trivially defined for $j=2,\ldots,k$ and/or when sign restrictions are imposed. In the case there were sign restrictions only, one could rely on the standard results in the literature to establish existence (Giacomini & Kitagawa, 2020; Amir-Ahmadi & Drautzburg, 2021; Granziera et al., 2018).

We start with establishing a Gordan type alternative theorem,[13] which will be instrumental to obtain the existence result.

Proposition 2.1 *Assume $j=1$. If $\nexists \boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ such that $(\forall \boldsymbol{q}_1 \in \mathcal{R}^n) \sum_{i=1}^4 \lambda_i f_i \geq 0$, $\boldsymbol{q}_1^*$ exists.*

The proof is provided in Appendix A. This proposition rules out that - for a given shock - the restrictions contradict each other and more generally it rules out that linear combinations of inequality constraints on the FEV violate the restrictions. Note that this proposition alone establishes existence of a solution to (2.7)-(2.9), but ignoring the orthogonality conditions (2.10). The satisfaction of orthogonality condition is essential for identifying simultaneously all of the shocks, avoiding the well-known issue that shocks identified one-at-a-time can be correlated to each other.

Next we establish the conditions for the existence of a solution to the constrained optimization problem (2.7)-(2.10). Let $\boldsymbol{\sigma}$ denote a permutation of $1,\ldots,k$ among the $k!$ possible permutations and $\boldsymbol{\sigma}(z)$ for $z=1,\ldots,k$ denote the z-th element of the permutation $\boldsymbol{\sigma}$. The following proposition holds:

Proposition 2.2 *(Existence) If there exists a permutation $\boldsymbol{\sigma}$ such that*

[13] Alternative theorems refer to stating that given two conditions, one of the two conditions is true. Gordan type alternative theorems specifically relate to alternative theorems deriving key results in optimization theory.

i) for $j = \boldsymbol{\sigma}(1)$ Proposition 2.1 is satisfied,

ii) conditions in Proposition 2.1 are met for all $j = \boldsymbol{\sigma}(2), \ldots, \boldsymbol{\sigma}(k)$ in the Nullspace of the previous $j-1$ shocks,

then $\boldsymbol{Q}^*_{1:k}$ exists.

Appendix A provides a proof and a technical discussion. The permutation $\boldsymbol{\sigma}$ is instrumental to find at least one matrix $\boldsymbol{Q}^*_{1:k}$ such that its first k columns $\boldsymbol{q}^*_1, \ldots, \boldsymbol{q}^*_k$ satisfy Proposition 2.1 and are orthogonal to each other.

In Appendix C, we provide two algorithms for the implementation of this proposition: an accept-reject sampler and an analytical detection of emptiness. Those algorithms are interesting per se as extend some contributions in the literature to multiple shocks identification. In the simulation exercise and empirical application the feasibility region is always non-empty.

The constrained optimization problem (2.7)-(2.10) is non-convex as we are optimizing over orthogonal vectors. Thus, in general we have a multiplicity of solutions which require time consuming numerical optimization, without guarantee of finding a global optimum. The proposition below establishes a sufficient condition for $\boldsymbol{Q}^*_{1:k}$ to be unique, which in turn implies that the numerical problem becomes easily tractable.

Proposition 2.3 *(Uniqueness) Assume that $\boldsymbol{Q}^*_{1:k}$ exists and is orthogonal. If $\mathbf{c}'_{ih}(\boldsymbol{\phi})\boldsymbol{q}_j \geq 0$ for $i = 1, \ldots, k$, $j = 1, \ldots, k$ and $h = 0, \ldots, \tilde{h}$, then $\boldsymbol{Q}^*_{1:k}$ is unique.*[14]

Note that the above proposition would not change with the presence of sign restrictions in (2.9) as the latter are linear inequality constraints in $\boldsymbol{Q}$. The formal proof is provided in Appendix A. Proposition 2.3 provides a sufficient condition that is both easy to verify and allows for an economic interpretation. Specifically, if targeted variables (in our application, macro uncertainty, financial uncertainty and credit spreads) react positively to macro uncertainty, financial uncertainty and credit supply shocks, then $\boldsymbol{Q}^*_{1:k}$ is selected over a closed convex feasibility region, and uniqueness follows. In the empirical application presented in this paper we never find a case in which the sufficient condition of Proposition 2.3 was violated, but of course it may be not satisfied in some instances.

[14] We present this proposition in terms of positive inequalities. However, uniqueness would still hold if negative linear inequalities were satisfied.

In such cases researchers can still implement our identification strategy, but they would need to check for the possibility of multiple optima (this would need to be done in Step 3 of algorithm 2.1 below).

2.3 Implementation

The following Algorithm delivers the posterior distribution of the impulse response functions (or any other structural object) of interest.

Algorithm 2.1

1: Draw $\boldsymbol{\phi}$ from the posterior distribution of the reduced-form VAR.

2: Check existence of a solution.

3: Obtain $\boldsymbol{Q}^*_{1:k}$ by solving the optimization problem (2.7)-(2.10) and compute the impulse response functions via (2.3).

4: Repeat Step 1-3, L times, e.g. $L = 1000$.

Algorithm 2.1 consists in a step of conventional sampling from the posterior of reduced-form parameters (Step 1), a step for investigation of feasibility (Step 2, see algorithms in Appendix C for its implementation), and a step of numerical optimization (Step 3). The optimization involves a quadratic objective function, but can be reduced to a much more tractable problem using Proposition 2.3. Note that Step 1 uses a posterior distribution, which means it is based on a Bayesian estimation of the underlying reduced form VAR. This choice is simply based on the observation that Bayesian VARs are widely used in empirical macroeconomics. Still, Step 1 can be easily adapted to a frequentist framework, for example using maximum likelihood estimates and invoking large sample results or using a bootstrap approach to produce draws from the VAR coefficients. In either case the entire procedure would still remain valid, since the remaining steps condition on the reduced-form parameters ($\boldsymbol{\phi}$) and do not depend on a prior over $\boldsymbol{Q}$.

Finally, in those cases in which Proposition 2.3 cannot be verified researchers need to ensure that Step 3 delivers a global - as opposed to a local - optimum, e.g. solving Step 3 from different starting points.

2.4 Relation to alternative identification methods

The identification approach outlined above allows to avoid strong identification assumptions such as recursive orderings, and therefore it lends itself naturally to investigate questions in which one wants to remain agnostic about the direction of the various causal effects. The study of the effects of uncertainty shocks is just one example of such a situation, as both the theoretical literature and the empirical evidence so far are inconclusive on whether uncertainty is an exogenous impulse or an endogenous response.

Importantly, our strategy identifies all of the shocks simultaneously, thereby sidestepping the well-known issue that shocks identified one-at-a-time can be correlated to each other, a problem which is particularly relevant in, but not limited to, uncertainty literature. For instance, Cascaldi-Garcia and Galvão (2020) show that news and uncertainty shocks tend to be correlated if identified separately; as such they are not truly structural. Caldara et al. (2016) separate uncertainty and financial shocks by imposing different ordering restrictions, finding that the order hugely affects the results. Of course, solving this problem comes at a cost. While standard *Max Share Identification* can be analytically solved (Uhlig, 2004), i.e. the identified shock corresponds to the eigenvector with the maximal eigenvalue associated to $\boldsymbol{\Upsilon}^i_{\underline{h}}(\boldsymbol{\phi})$, the cost of allowing simultaneous identification of a multiplicity of shocks is that the optimization problem can become non-convex. This is a consequence of optimizing a function over a multiplicity of orthonormal vectors. In Section 2.2.1 we establish mild conditions under which the problem is tractable and computationally fast.

A common use of the *Max Share Identification* is to reduce the identification uncertainty implied by sign restrictions, i.e. employing sign restrictions as constraints in the optimization problem. The same applies to our approach. For example, we can disentangle demand (say, government spending) from supply (say, productivity) shocks by using sign restrictions as constraints in the maximization problem. A positive demand shock is expected to increase both quantities and prices, a positive supply shock requires quantities and prices not to co-move. In the maximization, we would use target variables, e.g. long-run labour productivity and short-run government spending, for supply and demand shocks, respectively. Our strategy can resolve further situations in which set-identification schemes are not sufficient to satisfactorily pin down the desired shock.

For example Kilian and Murphy (2012) show that qualitative information beyond sign restrictions is necessary to distinguish demand and supply shocks in the oil market. Similarly, separation between news and surprise shocks requires to rank the relative effect of those disturbances over target variables, see Amir-Ahmadi and Drautzburg (2021) for an example of such a situation: since rank restrictions introduced by Amir-Ahmadi and Drautzburg (2021) are linear inequalities in $\boldsymbol{Q}$, they can be harmlessly nested in (2.9). In order to separate credit and housing shocks Furlanetto et al. (2017) assume that the former explain variation of total credits to households and firms more than the contributions to the fluctuations in the real estate value, and the other way around. The approach proposed in this paper achieves point-identification, avoiding the drawbacks of set-identification that affect most of the aforementioned studies (Baumeister & Hamilton, 2015; Giacomini & Kitagawa, 2020). Our machinery also naturally provides a new toolkit for researchers concentrating on the idea that a number of shocks can explain most of the movements in a possibly large set of macroeconomics aggregates. See the principal component analysis literature and, for a recent contribution, Angeletos et al. (2020).

Finally, it is worth clarifying the differences between the approach pursued in this paper and that of Volpicella (2022). The two approaches both use the FEV decomposition, but are different conceptually and methodologically. In particular, Volpicella (2022) uses bounds on the FEV decomposition, in combination with traditional sign restrictions, to set-identify a single shock. The approach in this paper instead achieves point- (as opposed to set-) identification of a variety of shocks (as opposed to one) which are guaranteed to be mutually orthogonal. Furthermore, the approach of Volpicella (2022) requires the use of sign restrictions, which are essential to economically label the shocks, while the approach presented here can identify shocks without necessarily imposing any sign restrictions. Methodologically, Volpicella (2022) requires specifying exact ad-hoc bounds on the FEV decomposition, while the empirical application here only imposes milder inequality constraints that require the FEV of each shock to be larger relative to that of all the remaining shocks, but otherwise leave the FEV decomposition unbounded (and we have also discussed that imposition of inequality restrictions on the FEV depends on the empirical application). Note that we can use the set-identifying restrictions in Volpicella (2022) as constraints in our optimization process: they are quadratic inequalities on $\boldsymbol{Q}$, mathematically they can be nested in restrictions (2.8).

2.5 Relation to macro models

In this section we briefly discuss our identification assumptions in relation to the existing theoretical work on uncertainty. As discussed above (see Section 1 and the references therein), a large macroeconomic literature has developed models in which uncertainty is an exogenous source of fluctuations. In most of these models our identification assumptions are immediately satisfied, since they consider a single source of uncertainty and assume that the macroeconomic uncertainty shock explains 100% of the within period variation of macroeconomic uncertainty.

Our identification assumptions are also satisfied in those models that consider more than one source of uncertainty. For example, Shin and Zhong (2020) build upon Basu and Bundick (2017) and Gertler and Karadi (2011) to construct a DSGE model with financial frictions (and credit supply shocks), exogenous macroeconomic uncertainty (as TFP volatility), and exogenous financial uncertainty (as capital quality volatility). Our identifying restrictions are confirmed by employing both the baseline parameterization in Shin and Zhong (2020)[15] and their battery of alternative calibrations[16]

Finally, there is some more recent literature modeling uncertainty as an endogenous response. In particular, Atkinson et al. (2021) depart from a Cobb-Douglas production function and suggest that complementarity between capital and labor inputs can generate endogenous uncertainty because the concavity in the production influences how output responds to productivity shocks.[17] However, even in that case credit shocks are not able to explain short run fluctuations of the uncertainty proxy more than uncertainty disturbances, which is in line with our identification approach.

3 Simulation exercise

This section presents a simulation showing that the proposed approach can recover the impulse response functions regardless on whether uncertainty is modeled as exogenous or endogenous.

We first employ a SVAR with endogenous uncertainty as Data Generating Process

[15]See Table A-7 of their paper.

[16]See Section D.2.1 of the their paper.

[17]When matching labor share and uncertainty moments, they found 16% of the volatility of uncertainty is endogenous in the short run.

(DGP) and generate artificial data for industrial production (IP), financial uncertainty (uF*), credit spread (CS), price index (PCEPI), monetary policy rate (FFR), and macroeconomic uncertainty (uM*). In order to produce endogeneity in uncertainty, data are generated by a recursive scheme with 1 lag,[18] where macro and financial uncertainty are ordered after the other covariates. In the baseline scenario of Figure 1, financial uncertainty is ordered before macroeconomic uncertainty, but the results still hold if we reverse the order between uncertainty disturbances. Ordering of the other variables do not affect the findings. In order to parameterize the DGP, we first estimate the recursive model via maximum likelihood with monthly US data for the period 1962 to 2016; we then fix the DGP to those estimates. Once the artificial data have been generated, we use our identification scheme to estimate the impulse response functions. For brevity, here we provide simulated results mostly for financial uncertainty shocks, but the insights below apply to macroeconomic uncertainty and credit supply shocks as well.

Figure 1 shows that our identification strategy can successfully identify the uncertainty shocks in presence of endogeneity, while methods based on exclusion restrictions can not. In the figure the blue line denotes the true responses based on the DGP. Note the different scale across different rows of the figure.

In the panels on the first row of Figure 1, we employ our identification scheme to estimate the impulse responses (black lines). According to panels (a), (b) and (c), our strategy works well. The remaining rows in this figure we will depart from this ideal situation, showing that imposing too strong identification assumptions will distort the estimated responses.

In the panels on the second row of Figure 1 (panels (a′), (b′) and (c′)) we shut down the response of macroeconomic uncertainty in the estimated model;[19] this corresponds to considering only one of the sources of uncertainty present in the DGP. As a consequence of this omission, the estimated responses of both real activity and credit spreads are biased. Similarly, in the panels on the third row of Figure 1 (panels (a″), (b″) and (c″)) we estimate a model that assumes that financial uncertainty is exogenous, i.e. no shock can contemporaneously affect it. Also in this case this leads to biased impulse responses.

The panels on the last row of Figure 1 (panels (a‴), (b‴) and (c‴)) illustrate a model in which impulse responses are identified without inequality constraints on the

[18]Further lags do not change the results.

[19]We silence the response up to $h = 4$ months, but the results remain unchanged for $h = 0, 1, \dots, 6$.

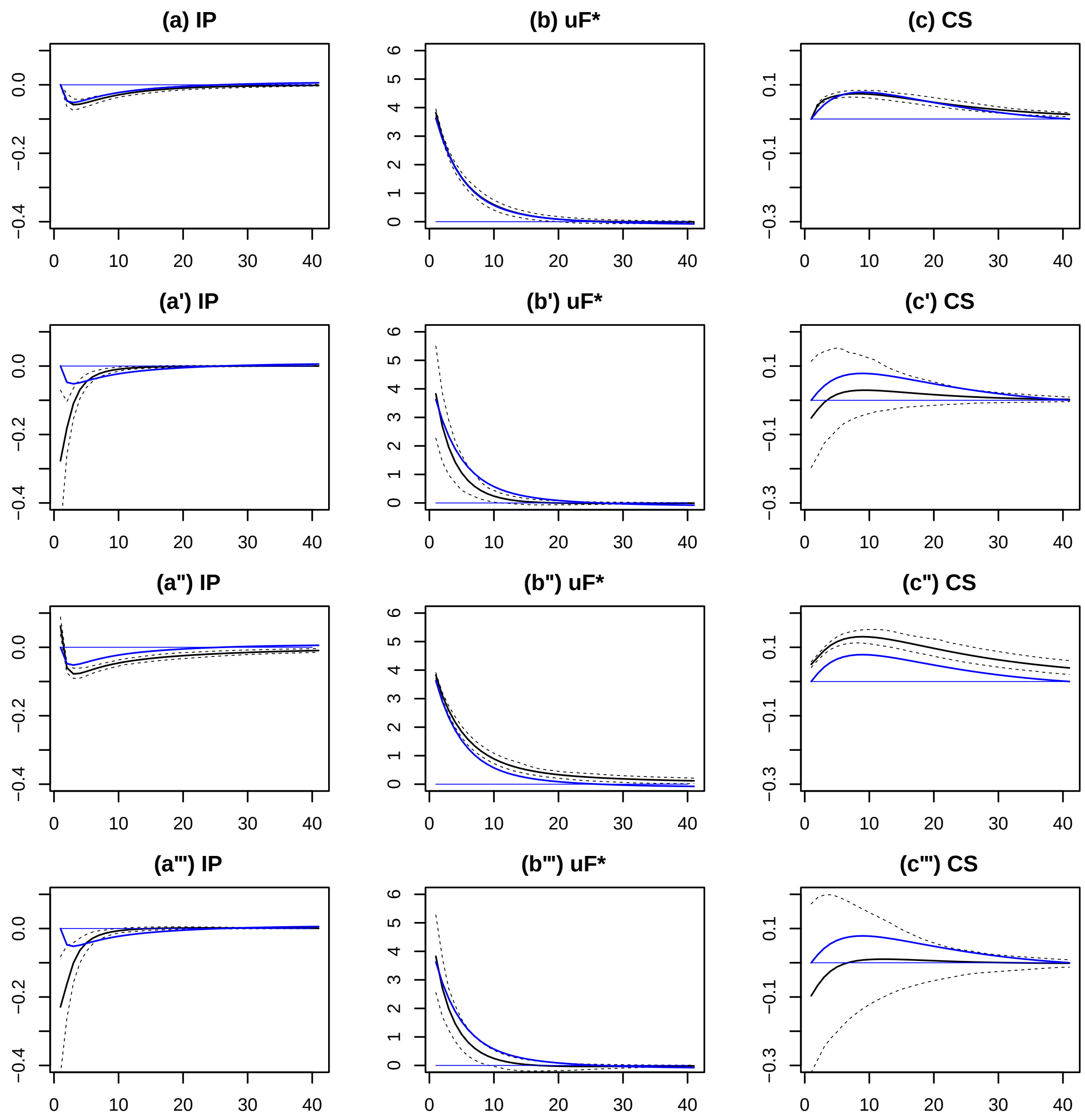

Figure 1: DGP with endogenous uncertainty: estimated responses to financial uncertainty shock. The blue line denotes the impulse responses to uncertainty shocks in the DGP. The black solid line represents the posterior mean of the estimated impulse responses, where in the first row ((a), (b), (c)) responses are identified through constraints on the FEV; in the second row ((a′), (b′), (c′)) the macroeconomic uncertainty response is shut down; in the third row ((a″), (b″), (c″)) the uncertainty is estimated as exogenous; in the fourth row (panels (a‴), (b‴) and (c‴)), impulse responses are estimated by removing inequality constraints on the FEV. The dashed black lines display the 68% Bayesian credibility region across replications. Shock size is set to 1 standard deviation.

FEV (inequalities in (2.8)). Also in this case impulse responses are biased. We also run sequential identification, i.e. ordering restrictions between financial and uncertainty disturbances, echoing the spirit of Caldara et al. (2016). Since the results are consistent

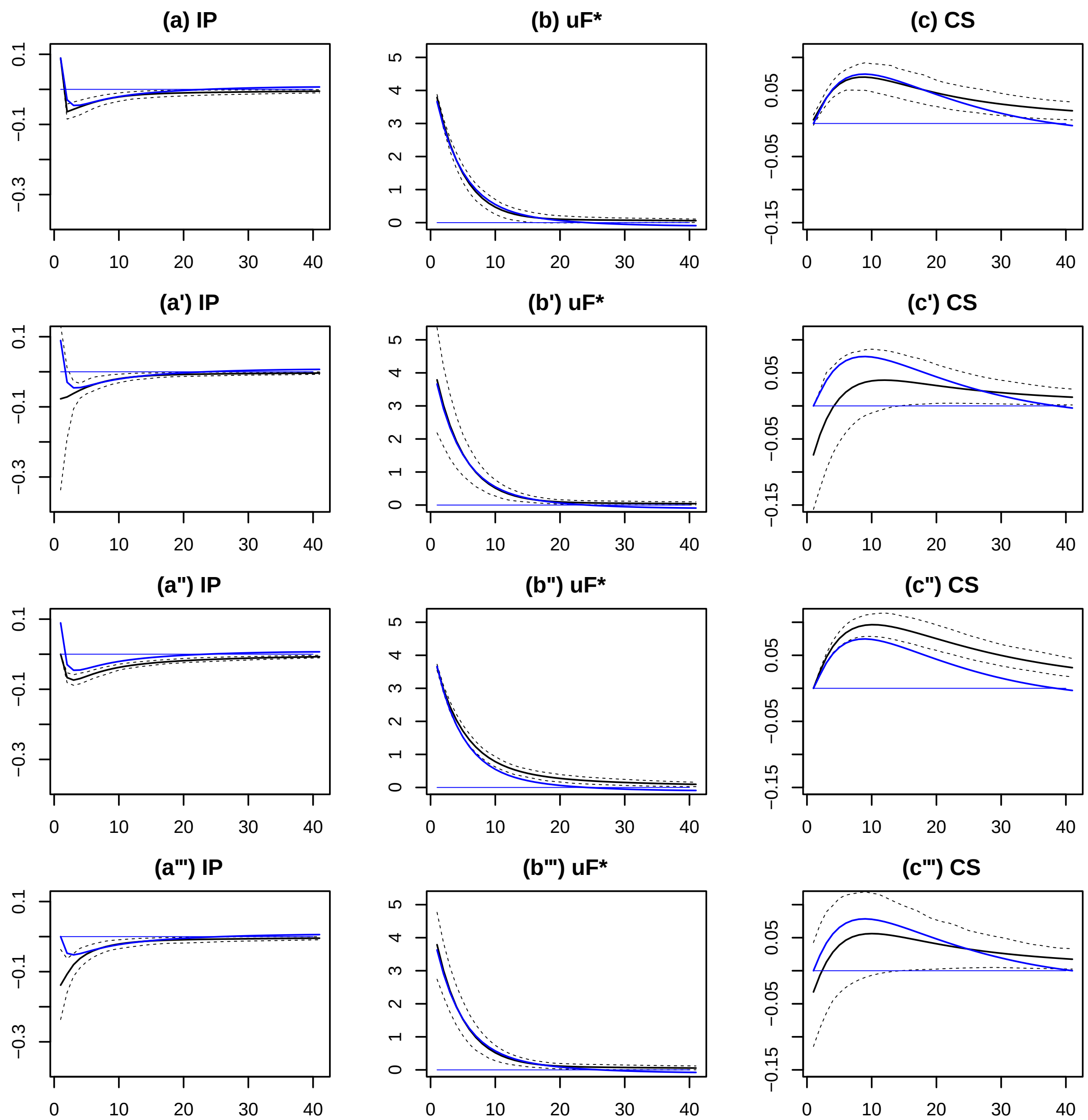

Figure 2: DGP with exogenous uncertainty: estimated responses to financial uncertainty shock. The blue line denotes the impulse responses to uncertainty shocks in the DGP. The black solid line represents the posterior mean of the estimated impulse responses, where in the first row ((a), (b), (c)) responses are identified through constraints on the FEV; in the second row ((a′), (b′), (c′)) the macroeconomic uncertainty response is shut down; in the third row ((a″), (b″), (c″)) the uncertainty is estimated as endogenous; in the fourth row (panels (a‴), (b‴) and (c‴)), impulse responses are estimated by removing inequality constraints on the FEV. The dashed black lines display the 68% Bayesian credibility region across replications. Shock size is set to 1 standard deviation.

with their findings, i.e. the effect on industrial production qualitatively depends on the sequence of identification, we omit it for brevity.

We now turn on the effectiveness of our scheme when uncertainty is exogenous. Accordingly, we consider a DGP where uncertainty is ordered before the other variables. This experiment is illustrated in Figure 2. Also in this case our proposed identification scheme recovers the correct responses; on the other hand, omitting distinctive sources of uncertainty (second row in the figure), imposing endogeneity when this is absent in the DGP (third row in the figure), and removing the inequality constraints on the FEV (fourth row in the figure) lead to biased impulse responses.

4 Empirical application

4.1 Specification and data

We now turn to our empirical application. Evaluating the relationship between economic variables and uncertainty needs selecting both a concept and metric of uncertainty. In the baseline model, we employ the Chicago Board Options Exchange S&P 100 Volatility Index as a measure of financial uncertainty and the the measure developed by Jurado et al. (2015) (JLN hereafter) as a measure of macroeconomic uncertainty. We check the robustness of our results to competing measures: for financial uncertainty, we also consider the measures of Carriero et al. (2018b) and Jurado et al. (2015); for macroeconomic uncertainty, we also use the measure of Carriero et al. (2018b).

Our baseline reduced form model is a VAR estimated with US monthly data ranging from from 1962m7 to 2016m12. We assume 7 lags[20] and a diffuse Normal Inverse Wishart prior.[21] The VAR includes 12 variables taken from the FRED database: macroeconomic uncertainty (JLN), financial uncertainty (VXO), credit spreads (CS), number of non-farm workers (PAYEM), industrial production (IP), weekly hours per worker (HOURS), real consumer spending (SPEND), real manufacturers' new orders (ORDER), real average earnings (EARNI), PCE price index (PCEPI), variation of federal funds rate (FFR), S&P 500 (S&P). The credit spread is measured as the difference between the BAA Corporate Bond Yield and the 10-year Treasury Constant Maturity rate; results are robust to employing the excess bond premium used in Caldara et al. (2016) and developed by Gilchrist and Zakrajšek (2012). All the variables enter the model growth rates, except for ORDER, PCEPI, FFR, CS, VXO, and JLN which enter in levels. All the variables

[20]This has been selected by maximizing the marginal likelihood.

[21]The prior is $\boldsymbol{\Sigma} \sim \mathcal{IW}(\boldsymbol{\Psi}, d)$ and $\boldsymbol{B}|\boldsymbol{\Sigma} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma} \otimes \boldsymbol{\Omega})$, where $\boldsymbol{\Psi} = \boldsymbol{I}_n$ is the location matrix, $d = n+1$ is a scalar degrees of freedom hyperparameter and $\boldsymbol{\Omega} = \boldsymbol{I}_{np+1}$ is the variance-covariance matrix of $\boldsymbol{B}$.

are demeaned prior to estimation. In order to facilitate comparisons with other studies, the impulse responses are expressed in percentage changes with respect to the levels. This implies that for those variables which were differenced the impulse responses are cumulated and the long run effects of transitory shocks do not vanish.

Appendix B provides further robustness checks, including re-estimation by explicitly controlling for a series of demand and supply shocks and a discussion of the historical narrative of macro vs financial uncertainty and financial uncertainty vs credit shocks. For example, from January 1990 – April 2022 using monthly data from FRED, the correlation between the VIX and the BAA 10-year spread is around 0.65, but we are still able to tease out time periods where credit shocks were small but financial uncertainty shocks were large (or vice versa).

4.2 The Effects of uncertainty shocks

Figure 3 and Figure 4 show the impulse responses to macro and financial uncertainty shocks, respectively. Uncertainty has a strong recessionary effect on employment, industrial production, hours worked, consumer spending, investment, and earnings; the financial conditions also deteriorate, as shown by the response of stock market and credit spreads. The shock leads to expansionary monetary policy trying to counteract the depressive effect of uncertainty. Notably, shocks to macroeconomic uncertainty increase financial uncertainty, and vice-versa.

To facilitate comparisons, Figure 5 overlays the impulse responses shown in Figure 3 and 4. The effects of macroeconomic and financial uncertainty are qualitatively comparable but there are some quantitative differences. For example, the recessionary effect on real activity variables seem more pronounced following macroeconomic uncertainty shocks, while credit spreads increase more with financial uncertainty shocks. In order to distinguish identified uncertainty shocks from demand type shocks, in Appendix B we (i) re-estimate the model by controlling for a series of demand and supply shocks, finding that the results are unchanged and (ii) show that the correlation between our identified uncertainty shocks and demand and supply shocks are not statistically significant.

We find a strong evidence in favor of a negative response of prices,[22] that is short-lived for macroeconomic uncertainty shocks but more persistent for financial uncertainty

[22]Using inflation rate provides similar results.

shocks, suggesting that uncertainty disturbances mimic demand shocks, namely they trigger a recession and a deflationary pressure on the economy. The slightly looser response of monetary policy for financial uncertainty might be driven by the more significant drop in prices relative to macroeconomic uncertainty.

This pronounced reduction in prices is in contrast with the existing empirical evidence on the impact of uncertainty on inflation, which is typically weak and rather mixed. Caggiano et al. (2014), Fernández-Villaverde et al. (2015), Leduc and Liu (2016), Basu and Bundick (2017) provide some empirical evidence that uncertainty is deflationary, while Mumtaz and Theodoridis (2015) find the opposite result. Carriero et al. (2018b) and Katayama and Kim (2018) argue that the effect of uncertainty on prices is not significant; the international evidence in Carriero et al. (2018a) suggests that the reaction of prices is country-specific and heterogeneous across the alternative measures of prices. While the effects of uncertainty on prices are different across these contributions, they are all based on simple recursive identification schemes: in most of these contributions uncertainty is modeled as exogenous.

4.2.1 Endogenous uncertainty?

Since our scheme allows for a contemporaneous feedback effect from economic and financial variables to uncertainty, it provides a natural ground to look into the issue of endogeneity of uncertainty. In order to tackle this question, we re-estimate the model adding a further restriction. Specifically we assume that each measure of uncertainty cannot be contemporaneously affected by structural shocks other than its own shock, i.e. uncertainty is exogenous. This is equivalent to order uncertainty first in a Cholesky decomposition scheme.

Panels (a)-(l) in Figure 6 display the responses to macroeconomic uncertainty shocks for the baseline identification (black line) and when macroeconomic uncertainty is assumed to be exogenous (red line): imposing exogeneity clearly changes several impulse response functions, which supports the view that macroeconomic uncertainty is endogenous to some extent. On the other hand, panels (a′)-(l′) display the responses to financial uncertainty shocks for the baseline identification (black line) and when financial uncertainty is assumed to be exogenous (red line): in this case the evidence in support of endogeneity is weaker.

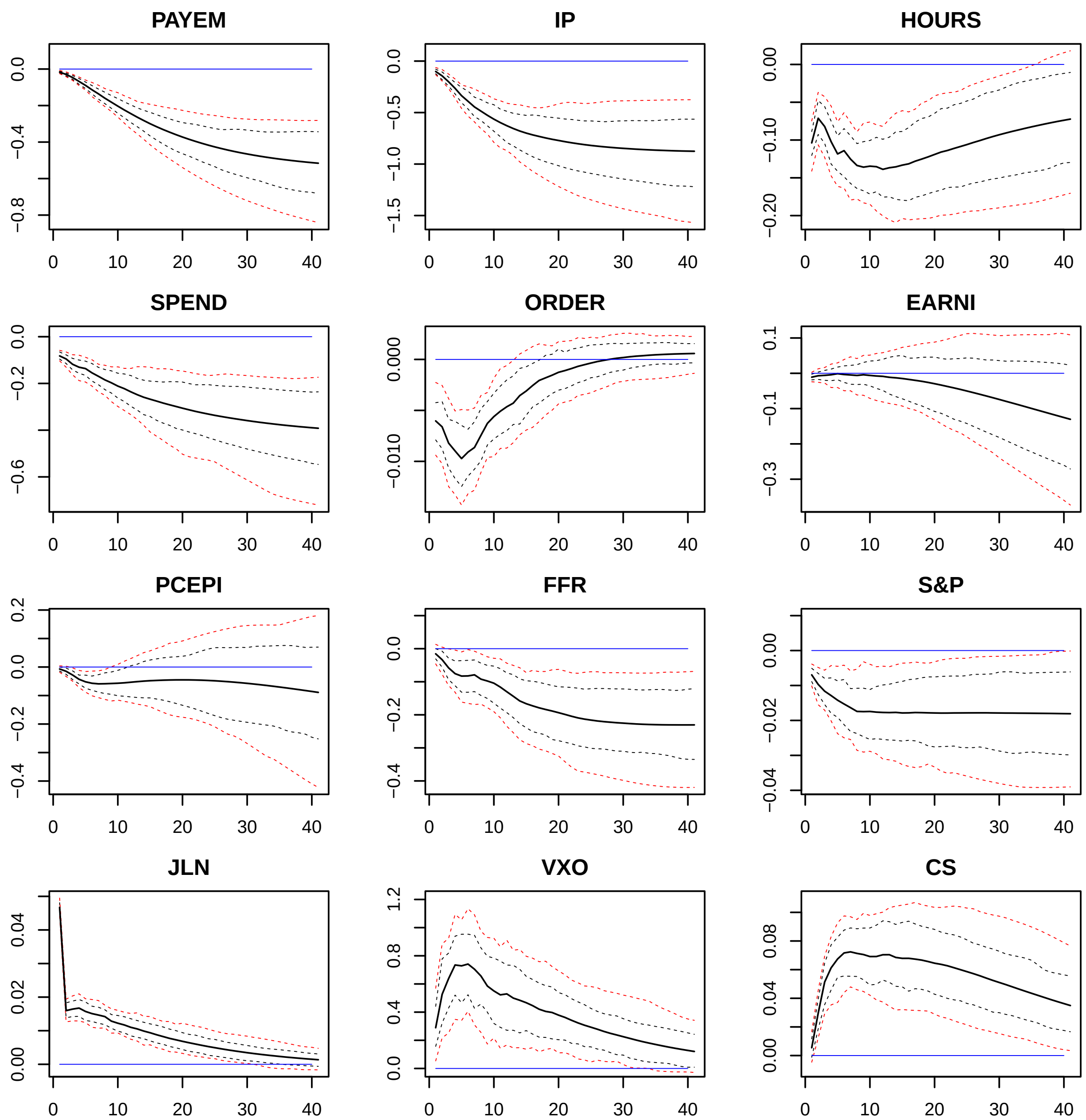

Figure 3: Responses to macroeconomic uncertainty shocks. Posterior medians are the black solid lines, the 68% Bayesian credibility region are the black dashed lines, and the 90% Bayesian credibility region are the red dashed lines. The blue solid line is the zero line. The shock size is set to one standard deviation.

The pattern shown in Figure 6 is in line with Ludvigson et al. (2021), who argue that while financial uncertainty is mainly exogenous, macroeconomic uncertainty presents some endogeneity. However, such a conclusion is not clear-cut in the literature. For example Angelini et al. (2019) find that both macroeconomic and financial uncertainty are mostly exogenous, and Carriero et al. (2021) point out that macroeconomic uncertainty displays some endogeneity, though more at quarterly than monthly frequency.[23]

[23]In unreported results news-based policy uncertainty as proxy for macro volatility turns out to be

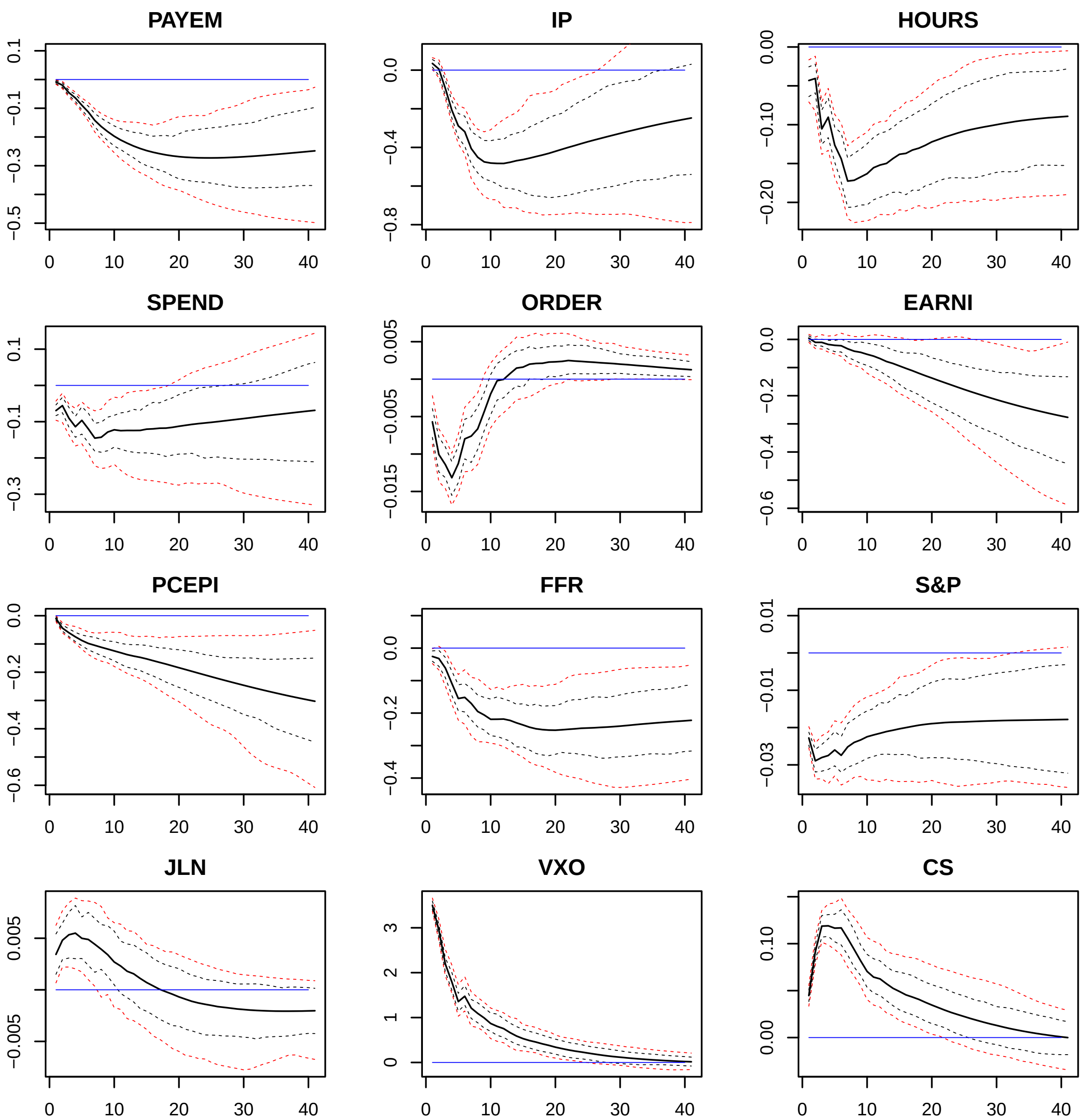

Figure 4: Responses to financial uncertainty shocks. Posterior medians are the black solid lines, the 68% Bayesian credibility region are the black dashed lines, and the 90% Bayesian credibility region are the red dashed lines. The blue solid line is the zero line. The shock size is set to one standard deviation.

Each study above adopts a different identification strategy. Ludvigson et al. (2021) use a small-scale model and a set-identification approach based on narrative restrictions requiring the shocks to be consistent with some historical episodes and correlated with some external instruments. Instead, we use a large-scale model, which reduces the problems of possible omitted variable bias, and a point-identification approach, which avoids

endogenous.

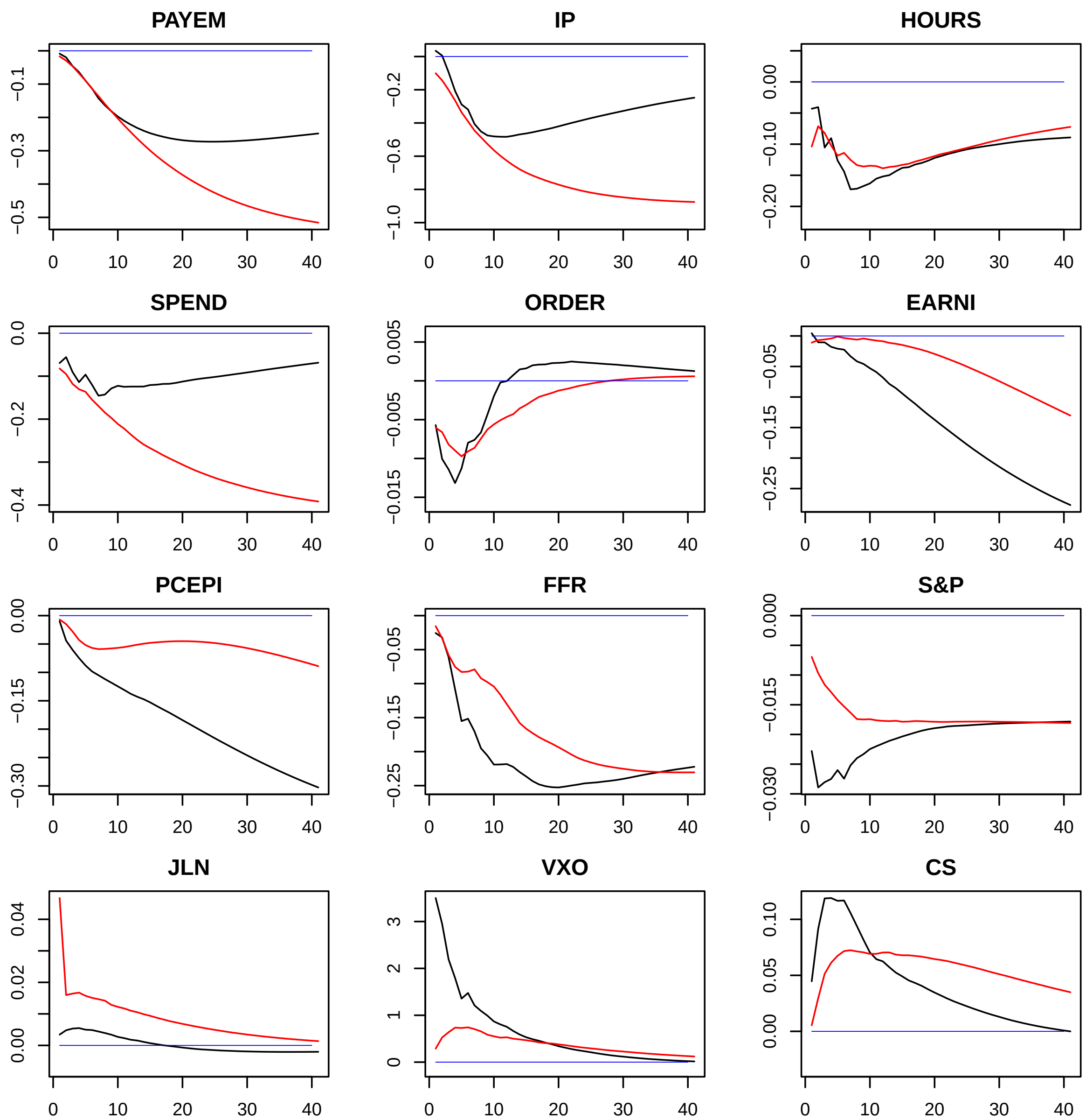

Figure 5: Comparison between macro and financial uncertainty shocks. The red and black line denote the posterior median of the impulse response functions to macroeconomic and financial uncertainty shock, respectively. The blue solid line is the zero line. The shock size is set to one standard deviation.

the problems inherent in set-identification discussed e.g. in Giacomini et al. (2021).[24] On the one hand, for our choice of observables, we implement their methodology, finding that both macro and financial uncertainty do not have real effects. This is expected given the

[24]Ludvigson et al. (2021) employed bootstrap to construct confidence intervals for the impulse response functions, but their frequentist validity is unknown. The fact that confidence intervals are presented for a specific point-estimate only (rather than for the identified sets as such) makes hard to evaluate the effect of sample bias and identification uncertainty in their setting. On the other hand, Bayesian inference naturally follows in our point-identified model.

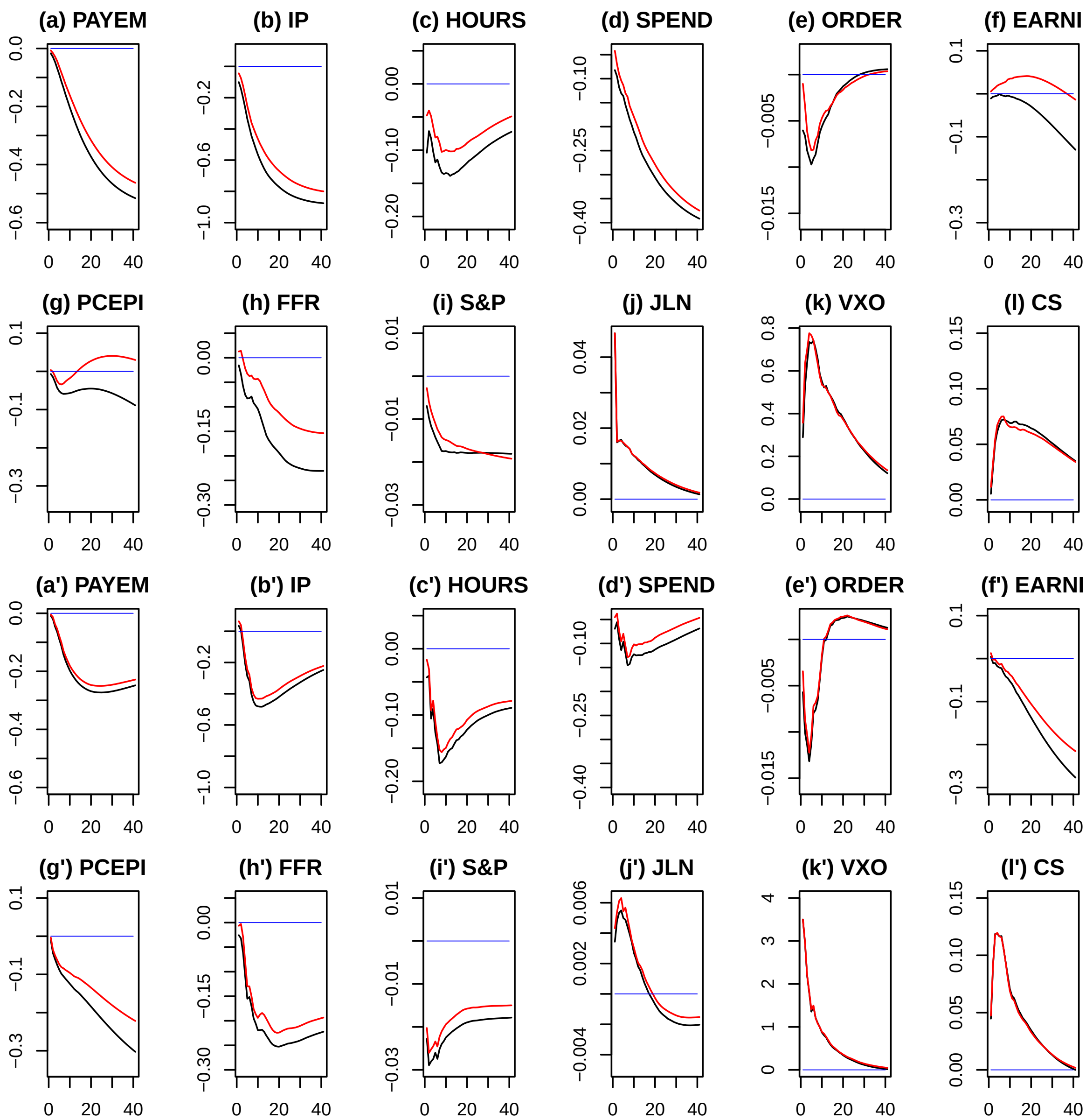

Figure 6: Baseline scenario vs exogenous uncertainty. The figure reports the posterior median of the impulse response functions to macro (panels (a)-(l)) and financial (panels (a′)-(l′)) uncertainty shocks for the baseline identification (black line) and when uncertainty is assumed to be exogenous (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

set-identification feature of their machinery. On the other hand, we adapt our strategy to their 3-variable model, i.e. macro and financial uncertainty, industrial production: we find that both typologies of uncertainty have recessionary effects, and macro uncertainty is endogenous to industrial production, unlike financial uncertainty.

Angelini et al. (2019) also use a small-scale model in which there are no proxies for financial conditions. They achieve identification by assuming that in the sample preceding January 2008 financial uncertainty shocks could neither contemporaneously impact on nor been impacted by macro variables directly.[25] However, an indirect channel on real variables through the impact from financial uncertainty to macro uncertainty is allowed since the Great Moderation. Differently from them, we never assume exogeneity of financial uncertainty, not even in some sub-sample, and we use a large-scale model which includes financial variables and a credit channel. The latter is relevant, as we shall see below in section 4.2.3, to disentangle uncertainty shocks from pure financial shocks.

Carriero et al. (2021) employ a large model and achieve point-identification exploiting heteroskedasticity in the error terms of the SVAR. However their approach has a major drawback insofar their model does not include macroeconomic and financial uncertainty in the same unified framework. As we shall see in section 4.2.2 such a choice does not guarantee that the macroeconomic and financial uncertainty shock are mutually orthogonal, which can lead to substantial distortions in the estimated responses. Furthermore, their approach requires an ordering restriction on the block of macroeconomic variables in which pure financial shocks are not explicitly identified. Instead, the approach of this paper allows to identify shocks to financial and macroeconomic uncertainty which are orthogonal by construction, and to disentangle them from pure financial shocks.

Another nice feature of our framework is that it allows for formal tests of exogeneity. We formally test the exogeneity restrictions, with the null being $(\boldsymbol{q}_1^*)'\boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{1}}(\boldsymbol{\phi})\boldsymbol{q}_1^* = 1$ for macroeconomic uncertainty and $(\boldsymbol{q}_2^*)'\boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{2}}(\boldsymbol{\phi})\boldsymbol{q}_2^* = 1$ for financial uncertainty, and we find that exogeneity is rejected at 1% significance level for macro uncertainty. For financial uncertainty, exogeneity in not rejected when we consider real variables only; on the other hand, exogeneity is rejected when we take financial variables (credit spreads and stock market) into account. We then conclude that financial uncertainty is likely to be exogenous to real economy, but endogenous to financial conditions.

4.2.2 Uncertainty and its sources

In the next experiment we evaluate the importance of having both a measure of macroeconomic and a measure of financial uncertainty in the model.[26]

[25]This is an intriguing assumption because financial markets are usually expected to react fast to news, while macroeconomic variables are relatively slower (Gertler & Karadi, 2015; Lettau et al., 2002).

[26]Ludvigson et al. (2021) and Shin and Zhong (2020) use set-identification schemes to separate macro and financial uncertainty shocks. Both papers find differences in the responses of the economy to these two types of shocks.

Panels (a)-(l) in Figure 7 display the responses to macroeconomic uncertainty shock for the baseline identification (black line) and when the response of financial uncertainty is muted (zero response) for 6 months (red line).[27] Similarly, panels (a′)-(l′) show the responses to financial uncertainty shock for the baseline identification (black line) and when the response of macroeconomic uncertainty is muted (red line). Our results show that omitting either one of the two uncertainty measures can lead to distortions in the estimated responses. In particular, neglecting this channel seems to attenuate the estimated impact of uncertainty. More formally, in our framework we always reject the hypothesis of zero impact response of macroeconomic (financial) uncertainty to financial (macroeconomic) disturbance.

4.2.3 The financial channel

There are some contributions arguing that financial conditions play a key role in amplifying and transmitting uncertainty shocks. For example, Arellano et al. (2018), Christiano et al. (2014), and Gilchrist et al. (2014) develop models featuring a financial channel in which the cost of external finance goes up in reaction to an increase in uncertainty; Alfaro et al. (2018) find that financial frictions can double the recessionary effect of uncertainty. On the other hand, Brunnermeier and Sannikov (2014) emphasize that a worsening of borrowers' financial position leads to higher uncertainty. Caldara et al. (2016), Brianti (2021), and Caggiano et al. (2021) find evidence that deterioration of financial conditions magnify the impact of uncertainty shocks on real activity.

In light of these contributions we investigate the role of financial channel within our identification scheme. Figure 8 compares the responses to macro (panels (a)-(l)) and financial (panels (a′)-(l′)) uncertainty shock for the baseline identification (black line) and for an alternative model in which the financial channel is shut down (red line) by imposing that there is no contemporaneous feedback between financial variables (credit spreads and stock market) and uncertainty. The picture emerging is one in which the financial channel seems relevant in the transmission mechanism of both financial and macroeconomic uncertainty shocks, with larger effects on the former.

[27] We try horizons other than 6, and the results are qualitatively unchanged.

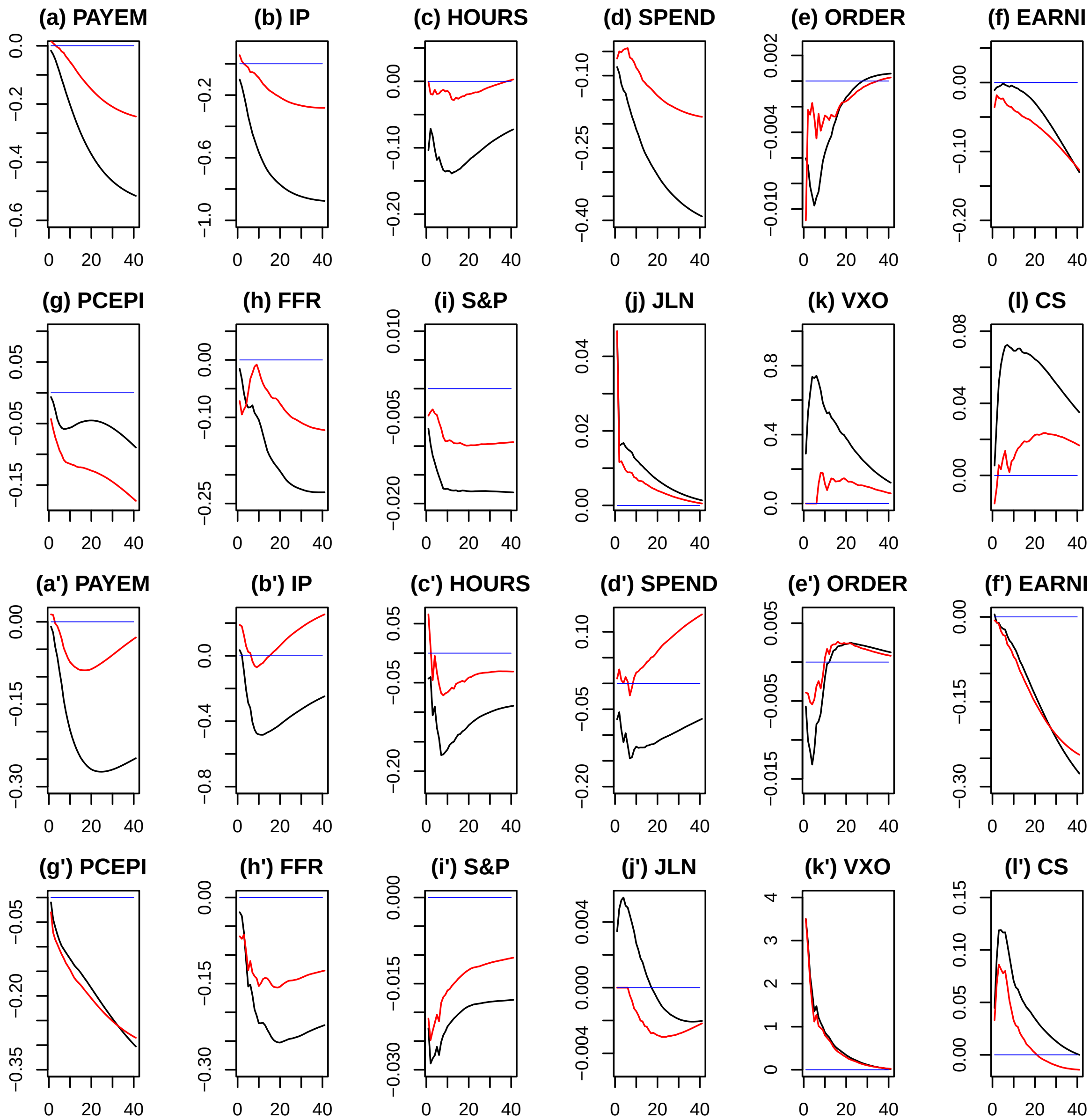

Figure 7: Baseline scenario vs shutting down the channel between macro and financial uncertainty. Panels (a)-(l) display the responses to macroeconomic uncertainty shock for the baseline identification (black line) and when the response of financial uncertainty is muted (red line). Panels (a′)-(l′) show the responses to financial uncertainty shock for the baseline identification (black line) and when the response of macroeconomic uncertainty is muted (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

.

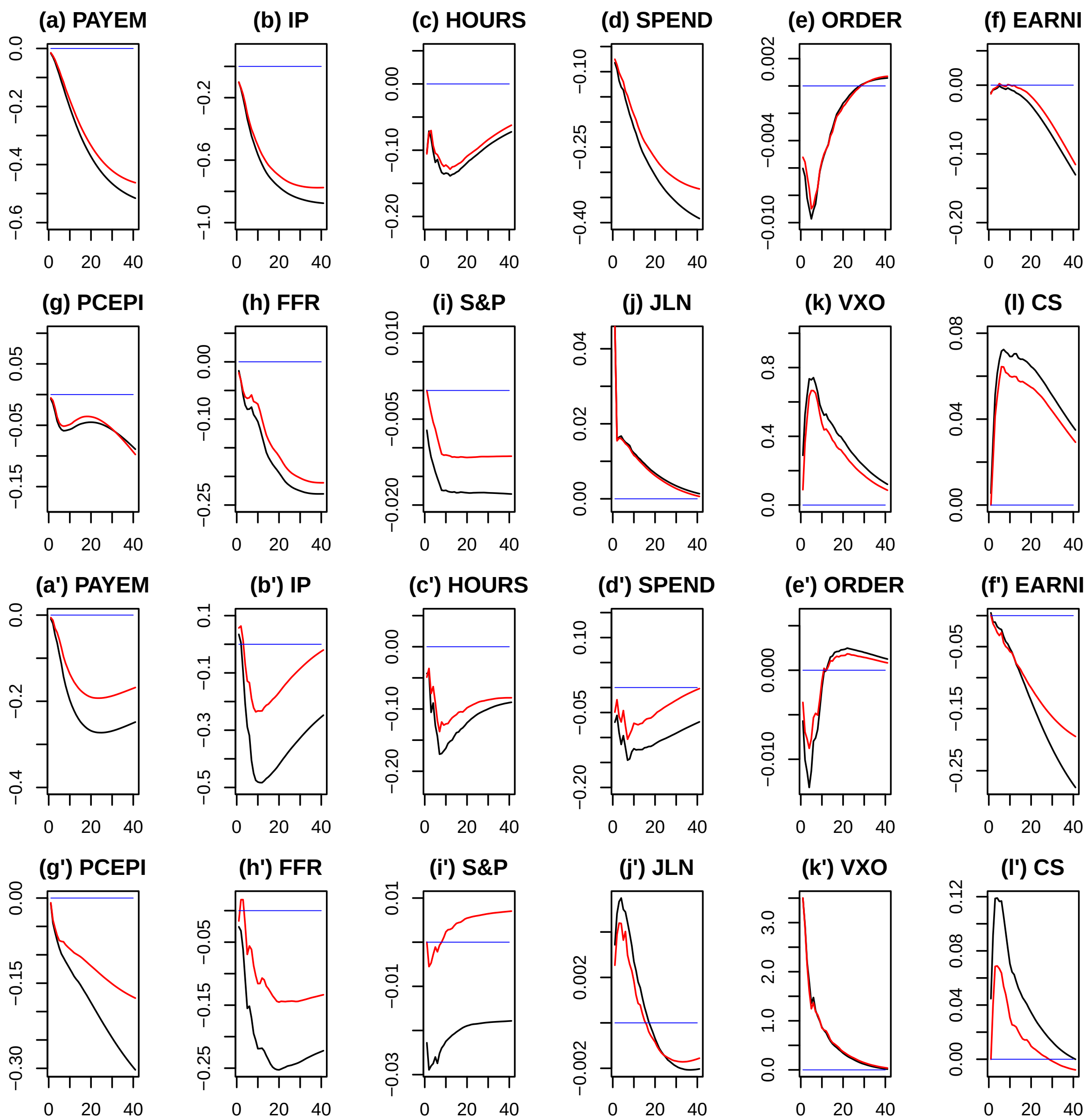

Figure 8: Financial channel. The figure reports the posterior median of the impulse response functions to macro (panels (a)-(l)) and financial (panels (a′)-(l′)) uncertainty shocks for the baseline identification (black line) and when the financial channel is shut down (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

4.3 The Effects of financial shocks

Prompted by the Great Recession and the debt crisis in the Euro-Area, a number of works attempt to identify and estimate the effects of credit supply shocks.[28] The overall picture

[28] Peersman (2011), Bijsterbosch and Falagiarda (2014), Eickmeier and Ng (2015), and Gambetti and Musso (2017) employ sign restrictions; Gilchrist and Zakrajšek (2012) develop a measure of credit spreads based on firm level data, finding that a component of this index is an indicator for credit supply; alternative proxies of credit supply have been put forward by Kashyap and Wilcox (1993), Gertler and

emerging from these studies is one in which the estimated effects of financial shocks are sensitive to identification schemes and some identification strategies are likely to provide misleading results, see e.g. the discussion in Mumtaz et al. (2018).

While identification of financial shocks and measurement of credit spreads significantly differ across the contributions in this literature, a common feature is the absence of interaction between financial factors and uncertainty, in the sense that most specifications exclude measures of uncertainty or overlook its role in the transmission mechanism of financial shocks. Prominent exceptions are Caldara et al. (2016), Furlanetto et al. (2017), Caggiano et al. (2021) and Brianti (2021), which are discussed below.

4.3.1 Uncertainty and the transmission mechanism

We use our model to shed light on the contribution of uncertainty to the transmission mechanism of financial shocks. Figure 9 shows that an increase in credit spreads has a depressive and deflationary effect on macroeconomic variables and leads to higher uncertainty, especially financial uncertainty. We then consider an experiment in which for the initial six months we shut down the response of macroeconomic uncertainty to credit spreads shock (Figure 10, panels (a)-(l)).[29] The changes in the responses is relevant and is consistent with the endogenous features of macroeconomic uncertainty. Also, shutting down the response of financial uncertainty dramatically mitigates the responses to financial shocks and leads to substantial distortions, as seen in Figure 10, panels (a′)-(l′). This confirms that although there is evidence for exogeneity in financial uncertainty with respect to macroeconomic variables, the feedback effect from financial conditions is substantial. Formal tests reject the null of no response on impact of uncertainty to credit spread shocks.[30] Overall, omitting the role of either form of uncertainty results in under-estimating the effects of credit shocks. Table 1 presents the FEV of macroeconomic uncertainty proxy, financial uncertainty proxy, and credit spreads due to the three shocks of interest. It seems confirmed the intuition that macro uncertainty is contemporaneously affected by more shocks, while financial uncertainty tend to be contemporaneously exogenous to macro variables.

Gilchrist (1994), and Lown and Morgan (2006).

[29] We also constrain horizons other than 6, and the findings do not change.

[30] The null being $\boldsymbol{c}_{10}'(\boldsymbol{\phi})\boldsymbol{q}_3^* = \boldsymbol{c}_{20}'(\boldsymbol{\phi})\boldsymbol{q}_3^* = 0$.

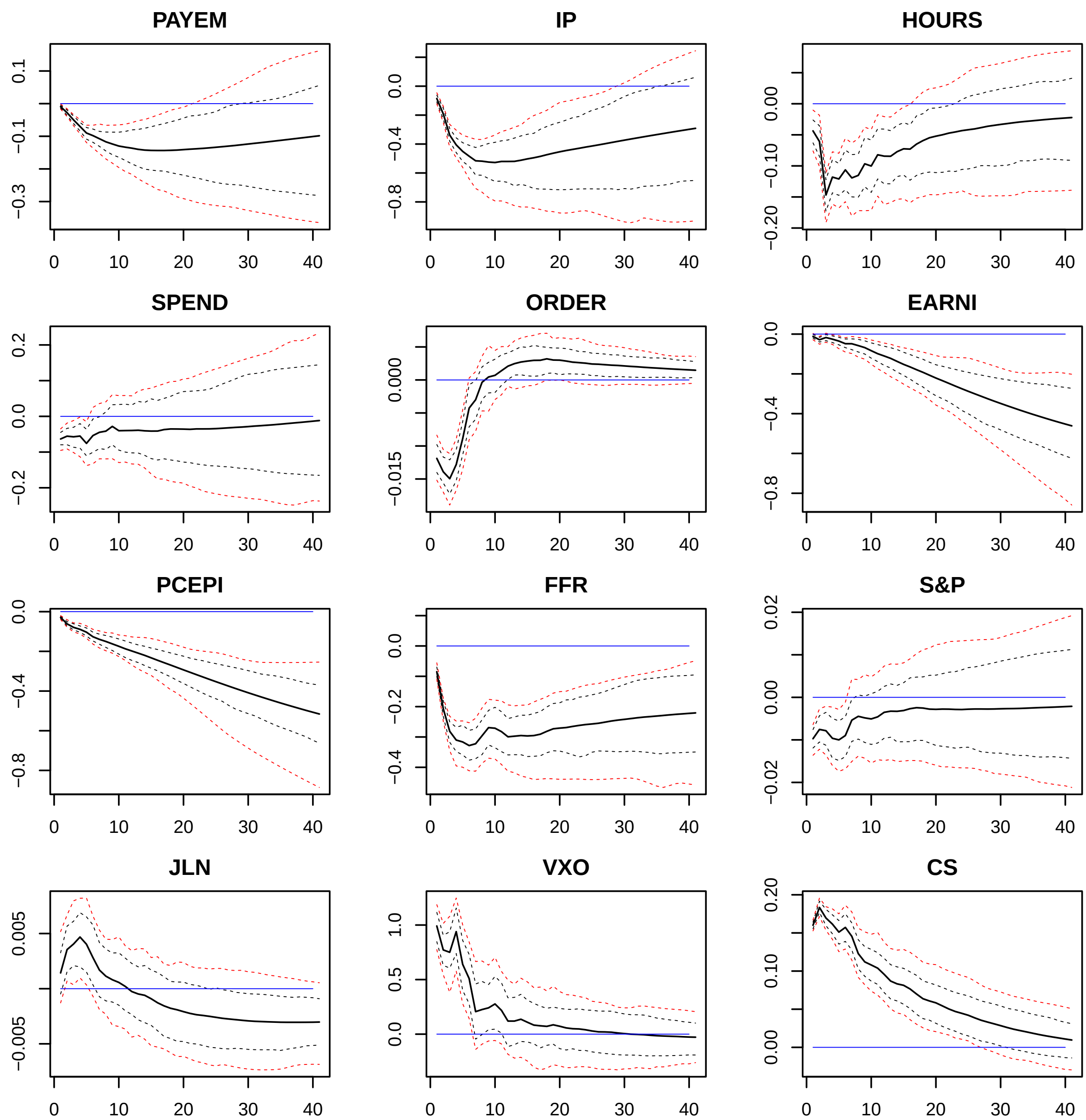

Figure 9: Responses to financial shocks. The figure reports the posterior median (black solid lines), the 68% Bayesian credibility region (black dashed lines), and the 90% Bayesian credibility region (red dashed lines) of the impulse response functions to financial shocks. The blue solid line is the zero line. The shock size is set to one standard deviation.

4.3.2 Relation with other studies

There are other papers which analyze the issue of the role of uncertainty in the transmission of financial shocks. In what follows we describe the main differences in the identification approach with respect to these studies.

Caldara et al. (2016) identify level (financial) and second moment (uncertainty) shocks

Table 1: FEV (%)

uF^* shock	$h = 0$	$h = 10$	$h = 20$	$h \to \infty$
uF^*	91.30	80.60	80.90	73.06
uM^*	10.48	20.43	13.97	13.63
CS	20.00	20.65	17.8	14.97
uM^* shock				
uF^*	2.00	7.07	10.47	12.58
uM^*	30.45	35.51	37.71	37.33
CS	5.00	7.20	8.84	7.43
CS shock				
uF^*	6.07	6.71	6.59	11.10
uM^*	11.87	12.83	10.02	26.86
CS	44.50	42.44	41.53	39.79

by employing a penalty function approach which relies on the ordering of the two first and second moment proxies and find that results are very sensitive to ordering. They assume a single source of uncertainty within the economy. The advantage of the identification scheme we propose here is that we do not require any ordering and do not exclude multiple forms of uncertainty. Furlanetto et al. (2017) and Caggiano et al. (2021) identify financial uncertainty and pure financial shocks by employing a mix of sign, ratio, and narrative restrictions. They do not separately identify macroeconomic uncertainty shocks. Compared to these contributions, our approach achieves point-identification thereby avoiding the problem inherent with is set-identified models, i.e. it is not clear how much the posterior estimation is driven by the prior distributions (Baumeister & Hamilton, 2015; Giacomini & Kitagawa, 2020). Brianti (2021) identify credit supply and macroeconomic uncertainty shocks relying on the qualitatively different responses of corporate cash holdings to a macroeconomic uncertainty shock (that pushes firms to increase their cash holdings for precautionary reasons) and a first-moment financial shock (that leads firms to reduce cash reserves as they lose access to external finance). However, i) those restrictions come from a theoretical framework with exogenous uncertainty and ii) the financial shocks as estimated by Brianti (2021) are a mix between first- and second-moment shocks within the financial sector, and as such cannot separate financial uncertainty shocks from pure credit supply disturbances.

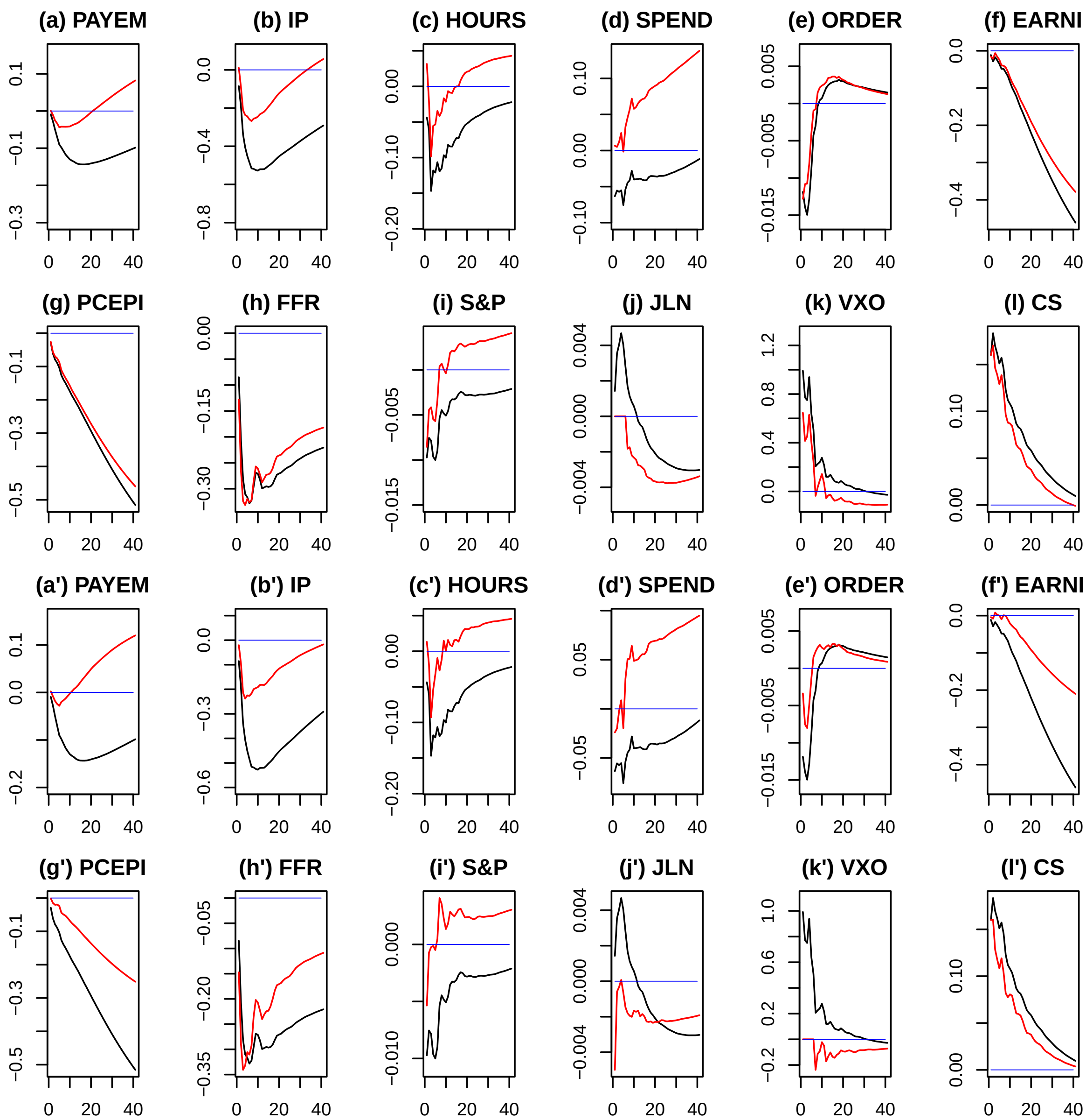

Figure 10: Financial shocks: shutting down uncertainty. Panels (a)-(l) report the posterior median of the impulse response functions to financial shocks for the baseline identification (black line) and when the response of macroeconomic uncertainty is shut down (red line). Panels (a′)-(l′) report the posterior median of the impulse response functions to financial shocks for the baseline identification (black line) and when the response of financial uncertainty is shut down (red line). The blue solid line is the zero line. The shock size is set to one standard deviation.

5 Conclusions

This paper developed a novel multiple shocks identification scheme for SVARs, based on generalizing the *Max Share Identification* to simultaneous identification of a multiplicity of shocks. Our approach overcomes some drawbacks induced by individually identified shocks, i.e. those shocks (i) tend to be correlated or (ii) can be separated under orthogonalizations with weak economic ground. The methodology involves the solution of a quadratic optimization problem. We characterized the properties of this approach, such as existence and uniqueness of a solution, and provided an algorithm for its implementation. The identification and estimation toolkit developed in this paper is general, and can be applied in any SVAR where standard ordering and sign restrictions are not desirable or sufficient to identify all of the shocks of interest. We used the approach to investigate the effects of uncertainty allowing for endogeneity. We also considered the interaction of uncertainty with financial shocks. Using US data, we found that some variables have a significant contemporaneous feedback effect on macroeconomic uncertainty, and overlooking this endogenous channel can lead to distortions. On the other hand, our empirical results suggest that financial uncertainty is likely to be an exogenous source of business cycle fluctuations for real variables. Finally, we found that omitting the role of uncertainty in the transmission mechanism can lead to underestimate the effects of financial shocks on the economy.

Appendix

A Omitted Proofs

For simplicity, in our proofs we assume $k = 3$ and $\mathcal{I}_S = \emptyset$, but results trivially hold for any finite discrete scalar $k > 0$ and for any non-empty $\mathcal{I}_S$.

Proof of Proposition 2.1.

Note that the feasibility region for $\boldsymbol{q}_1$ is characterized by $f_i \leq 0$ for $i = 1, \ldots, 4$ in Section 2.2.2 of the main text. Let us write f_i for $i = 1, \ldots, 4$ more compactly:

$$f_1 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_1(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.1}$$

$$f_2 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_2(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.2}$$

$$f_3 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_3(\boldsymbol{\phi})\boldsymbol{q}_1 + \frac{1}{2}, \tag{A.3}$$

$$f_4 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_4(\boldsymbol{\phi})\boldsymbol{q}_1 - \frac{1}{2}, \tag{A.4}$$

where $\boldsymbol{A}_1(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\mathbf{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\mathbf{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})$, $\boldsymbol{A}_2(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\mathbf{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\mathbf{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})$, and $\boldsymbol{A}_3(\boldsymbol{\phi}) = \boldsymbol{A}_4(\boldsymbol{\phi}) = \boldsymbol{I}_n$, with $\boldsymbol{I}_n$ being the identity matrix. Note that $\boldsymbol{A}_i(\boldsymbol{\phi})$ for $i = 1, \ldots, 4$ is a square $n \times n$ matrix and can be as such decomposed into symmetric and skew-symmetric (or anti-symmetric) components (Toeplitz decomposition): $\boldsymbol{A}_i(\boldsymbol{\phi}) \equiv \boldsymbol{A}_{iS}(\boldsymbol{\phi}) + \boldsymbol{A}_{iAS}(\boldsymbol{\phi})$, where $\boldsymbol{A}_{iS}(\boldsymbol{\phi}) = \frac{\boldsymbol{A}_i(\boldsymbol{\phi})+(\boldsymbol{A}_i(\boldsymbol{\phi}))'}{2}$ and $\boldsymbol{A}_{iAS}(\boldsymbol{\phi}) = \frac{\boldsymbol{A}_i(\boldsymbol{\phi})-(\boldsymbol{A}_i(\boldsymbol{\phi}))'}{2}$ are the symmetric and antisymmetric components of $\boldsymbol{A}_i(\boldsymbol{\phi})$, respectively. It is trivial to show that $\boldsymbol{q}_1'\boldsymbol{A}_i(\boldsymbol{\phi})\boldsymbol{q}_1 = \boldsymbol{q}_1'\boldsymbol{A}_{iS}(\boldsymbol{\phi})\boldsymbol{q}_1$. As a result, we obtain

$$f_1 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{1S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.5}$$

$$f_2 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{2S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.6}$$

$$f_3 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{3S}(\boldsymbol{\phi})\boldsymbol{q}_1 + \frac{1}{2}, \tag{A.7}$$

$$f_4 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{4S}(\boldsymbol{\phi})\boldsymbol{q}_1 - \frac{1}{2}. \tag{A.8}$$

We now define the following objects:

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{A}_{1S} & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{H}_2 = \begin{bmatrix} \boldsymbol{A}_{2S} & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{H}_3 = \begin{bmatrix} \boldsymbol{A}_{3S} & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{H}_4 = \begin{bmatrix} \boldsymbol{A}_{3S} & 0 \\ 0 & -1 \end{bmatrix}. \tag{A.9}$$

Note that $\boldsymbol{H}_i$ for $i = 1, \ldots, 4$ is a Z-matrix, i.e. the off-diagonal elements of a symmetric matrix are non-positive. Define a set $\boldsymbol{\Omega}_0$:

$$\boldsymbol{\Omega}_0 := \{(\frac{1}{2}\boldsymbol{a}'\boldsymbol{H}_1\boldsymbol{a}, \ldots, \frac{1}{2}\boldsymbol{a}'\boldsymbol{H}_4\boldsymbol{a}) : \boldsymbol{a} \in \mathcal{R}^{n+1}\} + int\mathcal{R}^4_+. \tag{A.10}$$

It suffices to prove that if $\boldsymbol{q}_1^*$ does not exist, then $\exists \boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ such that $(\forall \boldsymbol{q}_1 \in \mathcal{R}^n) \sum_{i=1}^4 \lambda_i f_i \geq 0$. In doing so, we follow the argument in Jeyakumar et al. (2009), Theorem 5.2. Assume that $\boldsymbol{q}_1^*$ does not exist. This is equivalent to state that the following system has no solution: $\boldsymbol{q}_1 \in \mathcal{R}^n$, $f_i < 0$, $i = 1, \ldots, 4$. Introduce 4 homogeneous functions

$\bar{f}_i : \mathcal{R}^{n+1} \to \mathcal{R}$, with $\bar{f}_i = \frac{1}{2}(\boldsymbol{q}, t)\boldsymbol{H}_i(\boldsymbol{q}, t)'$, where t is a scalar:

$$\bar{f}_1 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{1S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.11}$$

$$\bar{f}_2 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{2S}(\boldsymbol{\phi})\boldsymbol{q}_1, \tag{A.12}$$

$$\bar{f}_3 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{3S}(\boldsymbol{\phi})\boldsymbol{q}_1 + \frac{1}{2}t^2, \tag{A.13}$$

$$\bar{f}_4 = \frac{1}{2}\boldsymbol{q}_1'\boldsymbol{A}_{4S}(\boldsymbol{\phi})\boldsymbol{q}_1 - \frac{1}{2}t^2. \tag{A.14}$$

Note that $f_1 = \bar{f}_1$ and $f_2 = \bar{f}_2$. Then $\boldsymbol{0} \notin \boldsymbol{\Omega}_0$. Otherwise, there exists some $\boldsymbol{q} \in \mathcal{R}^n$ such that $f_i < 0$, $i = 1, \ldots, 4$, which is a contradiction (Jeyakumar et al. (2009) in Theorem 5.2 provide the technical proof of this). Since $\boldsymbol{H}_i$ for $i = 1, \ldots, 4$ are Z-matrices, $\boldsymbol{\Omega}_0$ is a convex set (see Theorem 5.1 in Jeyakumar et al. (2009)). By the convex separation theorem, there must exist $\boldsymbol{\lambda} \in \mathcal{R}^4 \backslash \{0\}$ such that for all $(y_1, \ldots, y_4) \in \boldsymbol{\Omega}_0$, $\sum_{i=1}^4 \lambda_i y_i \geq 0$. In turn, this means that there must exist $\boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ and for all $(\boldsymbol{q}, t) \in \mathcal{R}^{n+1}$, $\sum_{i=1}^4 \lambda_i \bar{f}_i \geq 0$. Setting $t = 1$ implies $f_i = \bar{f}_i$ for $i = 1, \ldots, 4$, namely $\exists \boldsymbol{\lambda} \in \mathcal{R}^4_+ \backslash \{0\}$ such that $(\forall \boldsymbol{q}_1 \in \mathcal{R}^n) \sum_{i=1}^4 \lambda_i f_i \geq 0$.

■

Proof of Proposition 2.2.

Without loss of generality, assume a permutation of the set $\{1, \ldots, k\}$: for instance, $\boldsymbol{\sigma} = (1, \ldots, k)$. According to condition i), we then obtain that for $j = \boldsymbol{\sigma}(1) = 1$ Proposition 2.1 is satisfied, namely $\boldsymbol{q}_1$ satisfies optimization constraints. Consider the following projector operator: $proj_{\boldsymbol{q}}(\boldsymbol{v}) = \frac{\langle \boldsymbol{q}, \boldsymbol{v} \rangle}{\langle \boldsymbol{q}, \boldsymbol{q} \rangle}\boldsymbol{q}$, where $\langle \boldsymbol{q}, \boldsymbol{v} \rangle$ denotes the inner product of vectors $\boldsymbol{q}$ and $\boldsymbol{v}$, with $\boldsymbol{q}, \boldsymbol{v} \in \mathcal{R}^n$. Put it another way, we are projecting $\boldsymbol{v}$ orthogonally into the line spanned by $\boldsymbol{q}$. Given $\boldsymbol{q}_1$, assume the following Gram–Schmidt process for $\boldsymbol{q}_j$ with $j = \boldsymbol{\sigma}(2), \ldots, \boldsymbol{\sigma}(k)$:

$$\boldsymbol{q}_2 = \boldsymbol{v}_2 - proj_{\boldsymbol{q}_1}(\boldsymbol{v}_2) \tag{A.15}$$

$$\vdots \tag{A.16}$$

$$\boldsymbol{q}_k = \boldsymbol{v}_k - \sum_{j=1}^{k-1} proj_{\boldsymbol{q}_j}(\boldsymbol{v}_k). \tag{A.17}$$

Given $\boldsymbol{q}_1$, this corresponds to generate vectors $\boldsymbol{q}_j$ which are in the Nullspace of $\boldsymbol{q}_{j-1}$ for $j = 2, \ldots, k$, i.e. generating a series of orthogonal vectors. If $\boldsymbol{q}_j$ for $j = 1, \ldots, k$ satisfies

Proposition 2.1, there must exist an orthogonal matrix $\boldsymbol{Q}_{1:k} = [\boldsymbol{q}_1, \ldots, \boldsymbol{q}_k]$ consistent with restrictions (2.8) and (2.10) in the main text. Existence follows.

Technical remark: the orthogonal vectors generated by the above Gram-Schmidt process can be easily adjusted to make them unit vectors, i.e. construct $\frac{\boldsymbol{q}}{||\boldsymbol{q}||}$. However, in our case this would be redundant as Proposition 2.1 imposes unit vectors condition.

■

Proof of Proposition 2.3.

Assume that $\boldsymbol{Q}^*_{1:k}$ exists and is orthogonal. This proof shows that under the conditions in Proposition 2.3, $\boldsymbol{q}^*_1, \ldots, \boldsymbol{q}^*_k$ are unique. Without loss of generality, for $j = 1$ restrictions (2.8) are reduced to:

$$\boldsymbol{q}_1'[\boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})]\boldsymbol{q}_1 \geq 0 \tag{A.18}$$

$$\boldsymbol{q}_1'[\boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})]\boldsymbol{q}_1 \geq 0, \tag{A.19}$$

where $\boldsymbol{\Upsilon}^{\boldsymbol{i}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{ih}(\boldsymbol{\phi})\boldsymbol{c}'_{ih}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{ih}(\boldsymbol{\phi})\boldsymbol{c}_{ih}(\boldsymbol{\phi})}$. Let $\boldsymbol{\Upsilon}^{\boldsymbol{12}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{2}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{1h}(\boldsymbol{\phi})\boldsymbol{c}'_{1h}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{1h}(\boldsymbol{\phi})\boldsymbol{c}_{1h}(\boldsymbol{\phi})} - \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{2h}(\boldsymbol{\phi})\boldsymbol{c}'_{2h}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{2h}(\boldsymbol{\phi})\boldsymbol{c}_{2h}(\boldsymbol{\phi})}$ and $\boldsymbol{\Upsilon}^{\boldsymbol{13}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \boldsymbol{\Upsilon}^{\boldsymbol{1}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) - \boldsymbol{\Upsilon}^{\boldsymbol{3}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi}) = \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{1h}(\boldsymbol{\phi})\boldsymbol{c}'_{1h}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{1h}(\boldsymbol{\phi})\boldsymbol{c}_{1h}(\boldsymbol{\phi})} - \frac{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}_{3h}(\boldsymbol{\phi})\boldsymbol{c}'_{3h}(\boldsymbol{\phi})}{\sum_{h=0}^{\tilde{h}} \boldsymbol{c}'_{3h}(\boldsymbol{\phi})\boldsymbol{c}_{3h}(\boldsymbol{\phi})}$. Thus, restrictions on $\boldsymbol{q}_1$ are

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\boldsymbol{12}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0 \tag{A.20}$$

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\boldsymbol{13}}_{\tilde{\boldsymbol{h}}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0. \tag{A.21}$$

For simplicity, and without loss of generality, assume $\tilde{h} = 0$:

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\boldsymbol{12}}_{\boldsymbol{0}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0 \tag{A.22}$$

$$\boldsymbol{q}_1'\boldsymbol{\Upsilon}^{\boldsymbol{13}}_{\boldsymbol{0}}(\boldsymbol{\phi})\boldsymbol{q}_1 \geq 0, \tag{A.23}$$

where

$$\boldsymbol{\Upsilon}^{\boldsymbol{12}}_{\boldsymbol{0}}(\boldsymbol{\phi}) = \frac{\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}'_{10}(\boldsymbol{\phi})}{\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})} - \frac{\boldsymbol{c}_{20}(\boldsymbol{\phi})\boldsymbol{c}'_{20}(\boldsymbol{\phi})}{\boldsymbol{c}'_{20}(\boldsymbol{\phi})\boldsymbol{c}_{20}(\boldsymbol{\phi})} \tag{A.24}$$

$$\boldsymbol{\Upsilon}^{\boldsymbol{13}}_{\boldsymbol{0}}(\boldsymbol{\phi}) = \frac{\boldsymbol{c}_{10}(\boldsymbol{\phi})\boldsymbol{c}'_{10}(\boldsymbol{\phi})}{\boldsymbol{c}'_{10}(\boldsymbol{\phi})\boldsymbol{c}_{10}(\boldsymbol{\phi})} - \frac{\boldsymbol{c}_{30}(\boldsymbol{\phi})\boldsymbol{c}'_{30}(\boldsymbol{\phi})}{\boldsymbol{c}'_{30}(\boldsymbol{\phi})\boldsymbol{c}_{30}(\boldsymbol{\phi})}. \tag{A.25}$$

Thus, we obtain

$$\boldsymbol{q}_1' \boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{12}}(\boldsymbol{\phi}) \boldsymbol{q}_1 = \boldsymbol{q}_1' \left[m_1(\boldsymbol{\phi}) \boldsymbol{c}_{10}(\boldsymbol{\phi}) \boldsymbol{c}_{10}'(\boldsymbol{\phi}) - m_2(\boldsymbol{\phi}) \boldsymbol{c}_{20}(\boldsymbol{\phi}) \boldsymbol{c}_{20}'(\boldsymbol{\phi}) \right] \boldsymbol{q}_1 \tag{A.26}$$

$$= m_1(\boldsymbol{\phi}) \boldsymbol{q}_1' \boldsymbol{c}_{10}(\boldsymbol{\phi}) \boldsymbol{c}_{10}'(\boldsymbol{\phi}) \boldsymbol{q}_1 - m_2(\boldsymbol{\phi}) \boldsymbol{q}_1' \boldsymbol{c}_{20}(\boldsymbol{\phi}) \boldsymbol{c}_{20}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \tag{A.27}$$

$$= m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}_{10}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \right)^2 - m_2(\boldsymbol{\phi}) \left(\boldsymbol{c}_{20}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \right)^2, \tag{A.28}$$

where $m_1(\boldsymbol{\phi}) = \frac{1}{\boldsymbol{c}_{10}'(\boldsymbol{\phi}) \boldsymbol{c}_{10}(\boldsymbol{\phi})}$ and $m_2(\boldsymbol{\phi}) = \frac{1}{\boldsymbol{c}_{20}'(\boldsymbol{\phi}) \boldsymbol{c}_{20}(\boldsymbol{\phi})}$ are positive scalar. Similarly, we get

$$\boldsymbol{q}_1' \boldsymbol{\Upsilon}_{\mathbf{0}}^{\mathbf{13}}(\boldsymbol{\phi}) \boldsymbol{q}_1 = m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}_{10}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \right)^2 - m_3(\boldsymbol{\phi}) \left(\boldsymbol{c}_{30}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \right)^2, \tag{A.29}$$

where $m_3(\boldsymbol{\phi}) = \frac{1}{\boldsymbol{c}_{30}'(\boldsymbol{\phi}) \boldsymbol{c}_{30}(\boldsymbol{\phi})}$. Thus, for $j = 1$ restrictions (2.8) are equivalent to

$$m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}_{10}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \right)^2 - m_2(\boldsymbol{\phi}) \left(\boldsymbol{c}_{20}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \right)^2 \geq 0 \tag{A.30}$$

$$m_1(\boldsymbol{\phi}) \left(\boldsymbol{c}_{10}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \right)^2 - m_3(\boldsymbol{\phi}) \left(\boldsymbol{c}_{30}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \right)^2 \geq 0. \tag{A.31}$$

Recall that for $j = 1$, conditions in Proposition 2.3 are

$$\boldsymbol{c}_{i0}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \geq 0 \text{ for } i = 1, \ldots, 3. \tag{A.32}$$

Combining (A.30)-(A.32) delivers the following restrictions for $j = 1$:

$$\boldsymbol{c}_{10}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \geq \sqrt{\frac{m_2(\boldsymbol{\phi})}{m_1(\boldsymbol{\phi})}} \boldsymbol{c}_{20}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \tag{A.33}$$

$$\boldsymbol{c}_{10}'(\boldsymbol{\phi}) \boldsymbol{q}_1 \geq \sqrt{\frac{m_3(\boldsymbol{\phi})}{m_1(\boldsymbol{\phi})}} \boldsymbol{c}_{30}'(\boldsymbol{\phi}) \boldsymbol{q}_1. \tag{A.34}$$

Thus, conditional on the existence of $\boldsymbol{q}_1^*$, constraints of the optimization problem become linear, and as such, convex for $\boldsymbol{q}_1$. Also, it is easy to observe that conditions in (A.32) make the objective function convex in $\boldsymbol{q}_1$. Since the problem is now convex, $\boldsymbol{q}_1^*$ must be unique. Extension to $\tilde{h} > 0$ is trivial. The same proof applies to $j = 2, 3$, i.e. $\boldsymbol{q}_2^*$ and $\boldsymbol{q}_3^*$ are unique. As a result, conditional on $\boldsymbol{q}_1^*$, $\boldsymbol{q}_2^*$ and $\boldsymbol{q}_3^*$ to exist and be orthogonal to each other, matrix $\boldsymbol{Q}_{1:k}^*$ is unique (with $k = 3$ in our setting).

■

B Shocks Series and Robustness Checks

Here we present some evidence that the three identified shocks are truly structural and exogenous to a set of structural shocks previously identified by the literature. We re-estimate the impulse responses by explicitly controlling, i.e, imposing orthogonality condition, for military news (Ramey, 2016), expected tax (Leeper et al., 2013), unanticipated and anticipated tax (Mertens & Ravn, 2011), monetary policy (Romer & Romer, 1989), and technology surprise (Basu et al., 2006). All the results presented in the main text are robust to this additional control. Furthermore, we compute the correlations between the three identified shocks and those disturbances, finding that correlations are never significant at 1% and 5% level.

Ludvigson et al. (2021) and Caggiano et al. (2021) stress that credible identification regimes need to estimate shocks consistent with specific historical episodes. First, we focus on three 'big (historical) shocks " in the language of Ludvigson et al. (2021): Black Monday (October 1987), break-up of Bretton Woods (December 1970) and Lehman collapse (September 2008). For the Black Monday, our estimated financial uncertainty shock is large,[31] while this is not the case for credit supply disturbances. This is in line with the narrative of Ludvigson et al. (2021) and Caggiano et al. (2021), where the Black Monday is featured by significant financial volatility but low credit conditions disruption. In December 1970, consistently with Ludvigson et al. (2021), we find a significant increase in the estimated macro uncertainty shock, and no significant changes in the series of financial uncertainty and credit supply shocks. In September 2008, we find that our three estimated shocks are all large, which is consistent with the consensus view of the Great Recession as a mix of financial and uncertainty shocks. Also, we extend our sample up to 2020: both macro and financial uncertainty shocks are large and bigger than pure financial shock in March 2020 (Covid outbreak), which is compatible with the belief that the pandemic prompted a spike in uncertainty but significant fiscal and monetary policy interventions prevented credit supply deterioration. Second, we consider two further events, where Ludvigson et al. (2021) argue that the sign of the shocks can be fairly deducted from historical reading of the times: both for the Volcker experiment (October 1979) and the debt-ceiling crisis (July and August 2011) the identified (macro

[31] Bigger than the median. This definition of large shocks is consistent with Ludvigson et al. (2021).

and financial) uncertainty shocks are positive, in line with the findings of Ludvigson et al. (2021). Third, the correlation between the estimated uncertainty shocks and stock market returns is negative, while the correlation with the price of gold is positive, consistent with the argument in Piffer and Podstawski (2018) and Ludvigson et al. (2021).

The findings we obtain are also robust to the following battery of checks, which are available upon request: lag length from three to twelve; selecting the prior tightness by maximizing the marginal likelihood rather than employing a flat specification; using the measures of Carriero et al. (2018b) and Jurado et al. (2015) as alternative proxies of financial uncertainty and the proposal in Carriero et al. (2018b) as proxy of macroeconomic uncertainty; employing the excess bond premium in Caldara et al. (2016) and Gilchrist and Zakrajšek (2012) as credit spreads. Moreover, we run a quarterly specification, finding that results are equivalent to what shown so far.

Finally, a number of papers points out that the effect of uncertainty shocks is more intense when the Zero Lower Bound (ZLB) holds (Caldara et al., 2016; Caggiano et al., 2014; Basu & Bundick, 2017; Johannsen, 2014). Thus, we estimate the model over the sample up to 2008m9, which removes the years of the Great Recession where the ZLB binds. The results are qualitatively equivalent to Figure 3 and Figure 4; quantitatively, the response of the variables is slightly less pronounced. Since this is fully consistent with the previous literature, we omit it for brevity.

C Algorithms for Checking Existence of Solutions

This appendix provides two algorithms to check if the optimization region is feasible. Those algorithms are interesting per se as extend some contributions in the literature to multiple shocks identification. The first proposal (Algorithm C.1) is an adjustment of Algorithm A.1 in Giacomini, Kitagawa, and Volpicella (2022) and Algorithm 1 in Giacomini and Kitagawa (2020) and employs random draws of $\boldsymbol{Q}$ to assess whether any of these satisfies the inequality constraints. The second proposal (Algorithm C.2) analytically checks for non-emptiness and extends Algorithm A.2 in Giacomini, Kitagawa, and Volpicella (2022) to multiple shocks identification. The first algorithm is simple to run but can give a misleading conclusion if the feasibility region is small. The second algorithm provides the right answer, but (i) can be implemented only if we can convexify the optimization problem and (ii) can become cumbersome if the number of restrictions gets larger.

Algorithm C.1

1: Draw $\boldsymbol{\phi}$ from the posterior distribution of the reduced-form VAR.

2: Generate $\boldsymbol{Q}$.[32]

3: Check if $\boldsymbol{Q}$ satisfies inequality restrictions. If it does, we conclude that the feasibility region is non-empty. Otherwise, repeat Step 2 K times, e.g. $K = 3000$, until $\boldsymbol{Q}$ satisfies restrictions. If none of the K draws satisfies the restrictions, approximate the feasibility region as empty and return to Step 1.

4: Repeat Step 1-3, Z times, e.g. $Z = 1000$.

The next algorithm relies on the linear structure of restrictions, and as such can be applied when Proposition 2.3 applies. First, we choose a permutation $\boldsymbol{\sigma}$ of $1, \ldots, k$ shocks among the $k!$ possible permutations. The algorithm is based on the observation that any non-empty region for the first element of $\boldsymbol{\sigma}$, i.e. $\boldsymbol{q}_{\boldsymbol{\sigma}(1)}$, contains a vertex on the unit sphere where (at least) $n-1$ constraints are binding. We select all the possible candidates for such vertex by choosing any combination of $n-1$ constraints and making them binding. If we could find a vertex that satisfies the constraints ruled out in the selection, we can claim this vertex is contained in the feasibility region for $\boldsymbol{q}_{\boldsymbol{\sigma}(1)}$, allowing us to conclude that the region for $\boldsymbol{q}_{\boldsymbol{\sigma}(1)}$ non-empty. If we cannot find any such vertex, we conclude that the feasibility for $\boldsymbol{Q}$ is empty. Second, given $\boldsymbol{q}_{\boldsymbol{\sigma}(1)}$, we generate the additional $k-1$ columns of $\boldsymbol{Q}$ following the order of the permutation. If restrictions on those columns are satisfied, we conclude that $\boldsymbol{Q}$ exists. Otherwise, we choose a different permutation. If none of the permutations leads to a feasible $\boldsymbol{Q}$, the region is empty.

Algorithm C.2

1: Draw $\boldsymbol{\phi}$ from the posterior distribution of the reduced-form VAR.

2: Select $\boldsymbol{\sigma}$.

[32] There are several ways to generate orthonormal matrices. Among others, see Uhlig (2005), Rubio-Ramirez et al. (2010), Giacomini and Kitagawa (2020).

3: Find unit length vectors $\boldsymbol{q}_{\boldsymbol{\sigma}(1)}$ and $-\boldsymbol{q}_{\boldsymbol{\sigma}(1)}$ satisfying active constraints. Check if $\boldsymbol{q}_{\boldsymbol{\sigma}(1)}$ or $-\boldsymbol{q}_{\boldsymbol{\sigma}(1)}$ satisfy the inactive constraints. If so, go to Step 4. Otherwise, keep constructing different combinations of active constraints and verify if the corresponding solution satisfies the inactive constraints. If none of the solutions satisfies the inactive restrictions, the feasibility region for $\boldsymbol{Q}$ is empty.

4: Construct the further orthogonal vectors, e.g. use the Gram–Schmidt process, and check if the restrictions are satisfied. If so, the feasibility region for $\boldsymbol{Q}$ is non-empty. Otherwise, go back to Step 2. If If none of the permutations leads to a feasible $\boldsymbol{Q}$, its optimization region is empty.

D Frequency Domain

While the main text illustrates our machinery for the time domain, an extension to frequency domain is fully feasible. First, we introduce the contribution of a shock j to the spectral density of a variable i over the frequency band $[\underline{\omega}, \bar{\omega}]$:

$$\Lambda_j^i(\underline{\omega}, \bar{\omega}) \equiv \int_{\omega \in [\underline{\omega}, \bar{\omega}]} \left(\overline{\boldsymbol{C}^{[i]}(e^{\tilde{i}\omega})\boldsymbol{q}_j} \boldsymbol{C}^{[i]}(e^{\tilde{i}\omega})\boldsymbol{q}_j \right) d\omega, \tag{D.1}$$

$$= \boldsymbol{q}_j' \left(\int_{\omega \in [\underline{\omega}, \bar{\omega}]} \overline{\boldsymbol{C}^{[i]}(e^{\tilde{i}\omega})} \boldsymbol{C}^{[i]}(e^{\tilde{i}\omega})\, d\omega \right) \boldsymbol{q}_j, \tag{D.2}$$

where $\tilde{i}$ is the imaginary unit, $\boldsymbol{C}^{[i]}(L)$ is the i-th row of $\boldsymbol{C}(L) = \sum_{h=0}^{\infty} \boldsymbol{C}_h(\boldsymbol{B})\boldsymbol{\Sigma}_{tr}$, and $\overline{\bullet}$ is the complex conjugate transpose of $\bullet$. The following matrix describes the unexpected fluctuations of i over $[\underline{\omega}, \bar{\omega}]$, and is a function of the reduced-form parameters $\boldsymbol{\phi}$ only:

$$\boldsymbol{\Delta}^i(\underline{\omega}, \bar{\omega}, \boldsymbol{\phi}) \equiv \int_{\omega \in [\underline{\omega}, \bar{\omega}]} \overline{\boldsymbol{C}^{[i]}(e^{\tilde{i}\omega})} \boldsymbol{C}^{[i]}(e^{\tilde{i}\omega})\, d\omega. \tag{D.3}$$

Note that in the frequency domain $\boldsymbol{\Delta}^i(\underline{\omega}, \bar{\omega}, \boldsymbol{\phi})$ is the counterpart of $\boldsymbol{\Upsilon}_i^{\tilde{h}}(\boldsymbol{\phi})$ in the time domain. Thus, we can express the contribution of shock j to the spectral density of variable i as

$$\Lambda_j^i(\underline{\omega}, \bar{\omega}) = \boldsymbol{q}_j' \boldsymbol{\Delta}^i(\underline{\omega}, \bar{\omega}, \boldsymbol{\phi}) \boldsymbol{q}_j. \tag{D.4}$$

The above equation is the counterpart of the FEV in equation (2.6) in the main text. As a result, the maximization of the objective function looks like

$$\boldsymbol{Q}^*_{1:k} = \arg\max_{\boldsymbol{Q}_{1:k}} \sum_{i=1}^{k} \boldsymbol{q}_i' \boldsymbol{\Delta}^i(\underline{\omega}, \bar{\omega}, \boldsymbol{\phi}) \boldsymbol{q}_i. \tag{D.5}$$

This is the counterpart of equation (2.7). Constraints, uniqueness and solution of the optimization naturally follow.

References

Abel, A. B. (1983). Optimal Investment under Uncertainty. *The American Economic Review*, *73*(1), 228–233.

Alfaro, I., Bloom, N., & Lin, X. (2018). The Finance Uncertainty Multiplier. *NBER WP*.

Amir-Ahmadi, P., & Drautzburg, T. (2021). Identification and inference with Ranking Restrictions. *Quantitative Economics*, *12*(1), 1–39.

Angeletos, G.-M., Collard, F., & Dellas, H. (2020). Business-cycle Anatomy. *American Economic Review*, *110*(10), 3030–70.

Angelini, G., Bacchiocchi, E., Caggiano, G., & Fanelli, L. (2019). Uncertainty across Volatility Regimes. *Journal of Applied Econometrics*, *34*(3), 437–455.

Arellano, C., Bai, Y., & Kehoe, P. (2018). Financial Frictions and Fluctuations in Volatility. *Journal of Political Economy*, *forthcoming*.

Atkinson, T., Plante, M., Richter, A. W., & Throckmorton, N. A. (2021). Complementarity and Macroeconomic Uncertainty. *Review of Economic Dynamics*.

Bachmann, R., Elstner, S., & Sims, E. R. (2013). Uncertainty and Economic Activity: Evidence from Business Survey Data. *American Economic Journal: Macroeconomics*, *5*(2), 217–249.

Bachmann, R., & Moscarini, G. (2011). Business Cycles and Endogenous Uncertainty. *Unpublished Manuscript*.

Baker, S. R., Bloom, N., & Davis, S. J. (2016). Measuring Economic Policy Uncertainty. *The Quarterly Journal of Economics*, *131*(4), 1593–1636.

Bar-Ilan, A., & Strange, W. C. (1996). Investment Lags. *The American Economic Review*, *86*(3), 610–622.

Barsky, R. B., & Sims, E. R. (2011). News Shocks and Business Cycles. *Journal of monetary Economics*, *58*(3), 273–289.

Basu, S., & Bundick, B. (2017). Uncertainty Shocks in a Model of Effective Demand. *Econometrica*, *85*(3), 937–958.

Basu, S., Fernald, J. G., & Kimball, M. S. (2006). Are Technology Improvements Contractionary? *American Economic Review*, *96*(5), 1418–1448.

Baumeister, C., & Hamilton, J. D. (2015). Sign Restrictions, Structural Vector Autoregressions, and Useful Prior Information. *Econometrica*, *83*(5), 1963–1999.

Berger, D., Dew-Becker, I., & Giglio, S. (2020). Uncertainty Shocks as Second-Moment News Shocks. *The Review of Economic Studies*, *87*(1), 40–76.

Bernanke, B. S. (1983). Irreversibility, Uncertainty, and Cyclical Investment. *The Quarterly Journal of Economics*, *98*(1), 85–106.

Bijsterbosch, M., & Falagiarda, M. (2014). *Credit Supply Dynamics and Economic Activity in Euro Area Countries: A Time-Varying Parameter VAR Analysis* (Tech. Rep.). European Central Bank.

Bloom, N. (2009). The Impact of Uncertainty Shocks. *Econometrica*, *77*(3), 623–685.

Bloom, N. (2014). Fluctuations in Uncertainty. *Journal of Economic Perspectives*, *28*(2), 153–76.

Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., & Terry, S. J. (2018). Really Uncertain Business Cycles. *Econometrica*, *86*(3), 1031–1065.

Bollerslev, T., Tauchen, G., & Zhou, H. (2009). Expected stock returns and variance risk premia. *The Review of Financial Studies*, *22*(11), 4463–4492.

Bonciani, D., & Oh, J. (2022). Uncertainty Shocks, Innovation, and Productivity. *The BE Journal of Macroeconomics*.

Brianti, M. (2021). Financial Shocks, Uncertainty Shocks, and Monetary Policy Trade-Offs. *Unpublished Manuscript*.

Brunnermeier, M. K., & Sannikov, Y. (2014). A Macroeconomic Model with a Financial Sector. *American Economic Review*, *104*(2), 379–421.

Caggiano, G., Castelnuovo, E., Delrio, S., & Kima, R. (2021). Financial uncertainty and real activity: The good, the bad, and the ugly. *European Economic Review*, *136*, 103750.

Caggiano, G., Castelnuovo, E., & Groshenny, N. (2014). Uncertainty Shocks and Unemployment Dynamics in US Recessions. *Journal of Monetary Economics*, *67*, 78–92.

Caldara, D., Fuentes-Albero, C., Gilchrist, S., & Zakrajšek, E. (2016). The Macroeconomic Impact of Financial and Uncertainty Shocks. *European Economic Review*, *88*, 185–207.

Carrière-Swallow, Y., & Céspedes, L. F. (2013). The Impact of Uncertainty Shocks in Emerging Economies. *Journal of International Economics*, *90*(2), 316–325.

Carriero, A., Clark, T. E., & Marcellino, M. (2018b). Measuring Uncertainty and its Impact on the Economy. *Review of Economics and Statistics*, *100*(5), 799–815.

Carriero, A., Clark, T. E., & Marcellino, M. (2021). *The Identifying Information in Vector Autoregressions with Time-Varying Volatilities: An Application to Endogenous Uncertainty* (Tech. Rep.). FRB of Cleveland.

Carriero, A., Clark, T. E., & Marcellino, M. G. (2018a). *Assessing International Commonality in Macroeconomic Uncertainty and its Effects* (Tech. Rep.). FRB of Cleveland.

Cascaldi-Garcia, D., & Galvão, A. B. (2020). News and uncertainty shocks. *Journal of Money, Credit and Banking*.

Cerra, V., & Saxena, S. C. (2008). Growth Dynamics: The Myth of Economic Recovery. *American Economic Review*, *98*(1), 439–57.

Cesa-Bianchi, A., Pesaran, M. H., & Rebucci, A. (2018). *Uncertainty and Economic Activity: A Multi-Country Perspective* (Tech. Rep.). NBER.

Christiano, L. J., Motto, R., & Rostagno, M. (2014). Risk Shocks. *American Economic Review*, *104*(1), 27–65.

Eickmeier, S., & Ng, T. (2015). How Do US Credit Supply Shocks Propagate Internationally? A GVAR Approach. *European Economic Review*, *74*, 128–145.

Fajgelbaum, P. D., Schaal, E., & Taschereau-Dumouchel, M. (2017). Uncertainty Traps. *The Quarterly Journal of Economics*, *132*(4), 1641–1692.

Fernández-Villaverde, J., Guerrón-Quintana, P., Kuester, K., & Rubio-Ramírez, J. (2015). Fiscal Volatility Shocks and Economic Activity. *American Economic Review*, *105*(11), 3352–84.

Fernández-Villaverde, J., Guerrón-Quintana, P., Rubio-Ramírez, J. F., & Uribe, M. (2011). Risk Matters: The Real Effects of Volatility Shocks. *American Economic Review*, *101*(6), 2530–61.

Fernández-Villaverde, J., & Guerrón-Quintana, P. A. (2020). Uncertainty Shocks and Business Cycle Research. *Review of economic dynamics*, *37*, S118–S146.

Francis, N., Owyang, M. T., Roush, J. E., & DiCecio, R. (2014). A Flexible Finite-Horizon Alternative to Long-Run Restrictions with an Application to Technology Shocks. *Review of Economics and Statistics*, *96*(4), 638–647.

Furlanetto, F., Ravazzolo, F., & Sarferaz, S. (2017). Identification of Financial Factors in Economic Fluctuations. *The Economic Journal*, *129*(617), 311–337.

Gafarov, B., Meier, M., & Olea, J. L. M. (2018). Delta-Method Inference for a Class of Set-Identified SVARs. *Journal of Econometrics*, *203*(2), 316–327.

Gambetti, L., & Musso, A. (2017). Loan Supply Shocks and the Business Cycle. *Journal of Applied Econometrics*, *32*(4), 764–782.

Gertler, M., & Gilchrist, S. (1994). Monetary Policy, Business Cycles, and the Behavior of Small Manufacturing Firms. *The Quarterly Journal of Economics*, *109*(2), 309–340.

Gertler, M., & Karadi, P. (2011). A Model of Unconventional Monetary Policy. *Journal of Monetary Economics*, *58*(1), 17–34.

Gertler, M., & Karadi, P. (2015). Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, *7*(1), 44–76.

Giacomini, R., & Kitagawa, T. (2020). Robust Bayesian Inference for Set-Identified Models. *Econometrica*, *forthcoming*.

Giacomini, R., Kitagawa, T., & Read, M. (2021). Identification and Inference under Narrative Restrictions. *arXiv preprint arXiv:2102.06456*.

Giacomini, R., Kitagawa, T., & Read, M. (2022). Robust Bayesian Inference in Proxy SVARs. *Journal of Econometrics*, *228*(1), 107–126.

Giacomini, R., Kitagawa, T., & Volpicella, A. (2022). Uncertain Identification. *Quantitative Economics*, *13*(1), 95–123.

Gilchrist, S., Sim, J. W., & Zakrajšek, E. (2014). *Uncertainty, Financial Frictions, and Investment Dynamics* (Tech. Rep.). NBER.

Gilchrist, S., & Zakrajšek, E. (2012). Credit Spreads and Business Cycle Fluctuations. *American Economic Review*, *102*(4), 1692–1720.

Granziera, E., Moon, H. R., & Schorfheide, F. (2018). Inference for VARs Identified with Sign Restrictions. *Quantitative Economics*, *9*(3), 1087-1121.

Hartman, R. (1972). The effects of price and cost uncertainty on investment. *Journal of Economic Theory*, *5*(2), 258–266.

Jeyakumar, V., Lee, G. M., & Li, G. Y. (2009). Alternative theorems for quadratic inequality systems and global quadratic optimization. *SIAM Journal on Optimization*, *20*(2), 983–1001.

Johannsen, B. K. (2014). *When Are the Effects of Fiscal Policy Uncertainty Large?* (Tech. Rep.). FEDS.

Jurado, K., Ludvigson, S. C., & Ng, S. (2015). Measuring Uncertainty. *American Economic Review*, *105*(3), 1177–1216.

Kashyap, J. C. S., A. K., & Wilcox, D. W. (1993). Monetary Policy and Credit Conditions: Evidence from the Composition of External Finance. *American Economic Review*, *83*, 78–98.

Katayama, M., & Kim, K. H. (2018). Uncertainty Shocks and the Relative Price of Investment Goods. *Review of Economic Dynamics*, *30*, 163–178.

Kilian, L., & Murphy, D. (2012). Why Agnostic Sign Restrictions Are Not Enough: Understanding the Dynamics of Oil Market VAR Models. *Journal of the European Economic Association*, *10*(5), 1166-1188.

Kraft, H., Schwartz, E., & Weiss, F. (2018). Growth Options and Firm Valuation. *European Financial Management*, *24*(2), 209–238.

Leduc, S., & Liu, Z. (2016). Uncertainty Shocks are Aggregate Demand Shocks. *Journal of Monetary Economics*, *82*, 20–35.

Leeper, E. M., Walker, T. B., & Yang, S.-C. S. (2013). Fiscal Foresight and Information Flows. *Econometrica*, *81*(3), 1115–1145.

Lettau, M., Ludvigson, S., & Steindel, C. (2002). Monetary Policy Transmission through the Consumption-Wealth Channel. *FRBNY Economic Policy Review*, *5*, 117–133.

Lown, C. S., & Morgan, D. P. (2006). The Credit Cycle and the Business Cycle: New Findings Using the Loan Officer Opinion Survey. *Journal of Money, Credit, and Banking*, *38*(6), 1575–1597.

Ludvigson, S., Ma, S., & Ng, S. (2021). Uncertainty and business cycles: Exogenous impulse or endogenous response? *Forthcoming American Economic Journal: Macroeconomics*.

McDonald, R., & Siegel, D. (1986). The Value of Waiting to Invest. *The Quarterly Journal of Economics*, *101*(4), 707–727.

Mertens, K., & Ravn, M. O. (2011). Understanding the aggregate effects of anticipated and unanticipated tax policy shocks. *Review of Economic Dynamics*, *14*(1), 27–54.

Mertens, K., & Ravn, M. O. (2013). The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States. *American economic review*, *103*(4), 1212–47.

Mumtaz, H., Pinter, G., & Theodoridis, K. (2018). What do VARs tell US about the Impact of a Credit Supply Shock? *International Economic Review*, *59*(2), 625–646.

Mumtaz, H., & Theodoridis, K. (2015). The international transmission of volatility shocks: An empirical analysis. *Journal of the European Economic Association*, *13*(3), 512–533.

Mumtaz, H., & Theodoridis, K. (2018). The Federal Reserve's Implicit Inflation Target and Macroeconomic Dynamics: A SVAR Analysis. (E2018/1).

Ng, S., & Wright, J. H. (2013). Facts and challenges from the great recession for forecasting and macroeconomic modeling. *Journal of Economic Literature*, *51*(4), 1120–54.

Oi, W. Y. (1961). The Desirability of Price Instability under Perfect Competition. *Econometrica*, 58–64.

Pástor, L., & Veronesi, P. (2006). Was there a Nasdaq bubble in the late 1990s? *Journal of Financial Economics*, *81*(1), 61–100.

Peersman, G. (2011). Bank Lending Shocks and the Euro Area Business Cycle. *Unpublished Manuscript*.

Piffer, M., & Podstawski, M. (2018). Identifying Uncertainty Shocks Using the Price of Gold. *The Economic Journal*, *128*(616), 3266–3284.

Plante, M., Richter, A. W., & Throckmorton, N. A. (2018). The Zero Lower Bound and Endogenous Uncertainty. *The Economic Journal*, *128*(611), 1730–1757.

Ramey, V. A. (2016). Macroeconomic Shocks and Their Propagation. *Handbook of macroeconomics*, *2*, 71–162.

Romer, C. D., & Romer, D. H. (1989). Does Monetary Policy matter? A New Test in the Spirit of Friedman and Schwartz. *NBER macroeconomics annual*, *4*, 121–170.

Rossi, B., & Sekhposyan, T. (2015). Macroeconomic Uncertainty Indices Based on Nowcast and Forecast Error Distributions. *American Economic Review*, *105*(5), 650–55.

Rubio-Ramirez, J., Waggoner, D., & Zha, T. (2010). Structural Vector Autoregressions: Theory of Identification and Algorithm for Inference. *The Review of Economic Studies*, *77*(2), 665-696.

Segal, G., Shaliastovich, I., & Yaron, A. (2015). Good and Bad Uncertainty: Macroeconomic and Financial Market Implications. *Journal of Financial Economics*, *117*(2), 369–397.

Shin, M., & Zhong, M. (2020). A new approach to identifying the real effects of uncertainty shocks. *Journal of Business & Economic Statistics*, *38*(2), 367–379.

Uhlig, H. (2004). What Moves GNP? In *Econometric Society 2004 North American Winter Meeting.*

Uhlig, H. (2005). What Are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure. *Journal of Monetary Economics*, *52*(2), 381-419.

Uhlig, H. (2017). Shocks, Sign Restrictions, and Identification. In M. B. Honore A. Pakes & L. Samuelson (Eds.), *Advances in Economics and Econometrics* (Vol. 2, pp. 95–127). Cambridge University Press.

Van Nieuwerburgh, S., & Veldkamp, L. (2006). Learning Asymmetries in Real Business Cycles. *Journal of Monetary Economics*, *53*(4), 753–772.

Volpicella, A. (2022). SVARs Identification through Bounds on the Forecast Error Variance. *Journal of Business* & *Economic Statistics*, *40*(3), 1291-1301.