SPACE
SPATIAL COGNITION TO ENHANCE MATHEMATICAL LEARNING

Spatial Cognition to Enhance Mathematics (SPACE)

University of Surrey · Education Endowment Foundation · Centre for Evidence and Implementation · Learnus (Understanding Learning) · Birkbeck University of London · Loughborough University · UCD Dublin

Agenda

9 am to 10 am

- Spatial Reasoning and Mathematics
- Year 2 Mathematics Curriculum and Spatial Reasoning

10 am to 10.30

- How to administer the spatial and mathematics assessments

10.30 am to 10.45 am

- Break

10.45 am to 12.15 am

- SPACE LEGO sessions – how to run the sessions
- LEGO models and spatial skills
- SPACE LEGO sessions - how to use the prompt cards

12.15 pm to 12.30 pm

- Q & A

SPACE

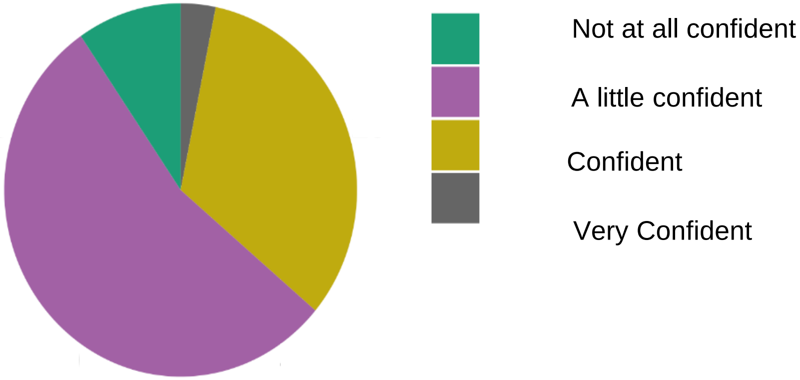
SPACE
SPATIAL COGNITION TO ENHANCE MATHEMATICAL LEARNING

Spatial Reasoning and Mathematics

Spatial Reasoning: Practitioners' Perspectives

Bates, Williams, Gilligan-Lee, Gripton, Lancaster, Williams, Borthwick, Gifford, Farran (2022) https://doi.org/10.31234/osf.io/m8nfv

If you were asked to explain what spatial reasoning is to someone else, how confident would you be in your definition?



e.farran@surrey.ac.uk @ekfarran

SPACE

Spatial Reasoning

Spatial reasoning provides one with the ability to:

- Understand the location and shape of objects and the relations between them.
- Mentally represent and manipulate objects (including parts and wholes)
- Use tools to spatialise thought (e.g., language, graphs, maps)

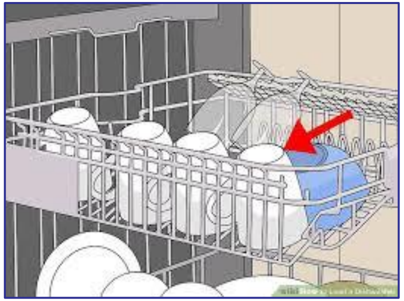


photo by Alan Levine

e.farran@surrey.ac.uk @ekfarran

SPACE

The importance of spatial cognition

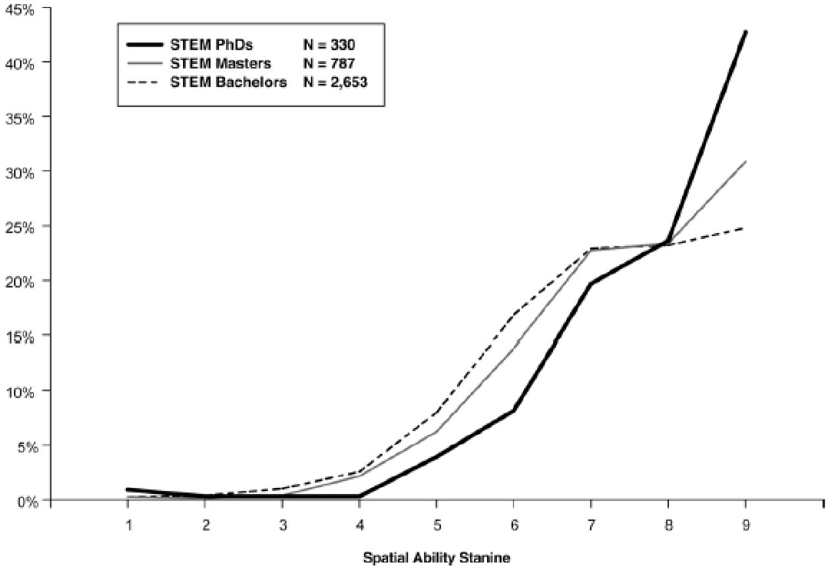


Figure 7. This figure includes the proportion of each degree group (bachelors, masters, and PHDs) as a function of spatial ability. Along the *x*-axis are the spatial ability stanines (numbered 1 through 9). STEM = science, technology, engineering, and mathematics.

Data from 400,000 randomly sampled students in the USA (Wai, Lubinski & Benbow, 2009)

The importance of spatial cognition

Strong spatial reasoning skills:

- more likely to be interested in science and maths
- more likely to choose degrees in STEM subjects
- more likely to be good at STEM research / STEM careers

e.farran@surrey.ac.uk @ekfarran

The importance of spatial cognition

Meta-analyses - spatial training: spatial skills are highly malleable; spatial training is effective, durable, and transferable (Uttal *et al*., 2013; Yang ... 2020).

Meta-analysis - spatial-maths association ... stent relationship between spatial ski... sistent across gender and age (Atit et al., 2021).

Meta-anal... mathematics: 29 studies; spatial training is effective ... ial skills and mathematics. This is most effective when the spatial ... udes physical manipulatives (Hawes et al., 2022).

“the connection between space and math may be one of the most robust and well established findings in cognitive psychology” (Mix & Cheng, 2012)

e.farran@surrey.ac.uk @ekfarran

SPACE

Spatial reasoning and attainment gaps

- Children from disadvantaged backgrounds have lower spatial skills (Verdine et al., 2014), lower spatial language (Bower et al., 2020), reduced access to spatial toys (Levine et al., 2012) and are at risk of experiencing lower quality parent-child interaction during spatial play (Jirout & Newcombe, 2015).

- Spatial training with children consistently report larger gains in mathematics competence for children from disadvantaged backgrounds compared to their peers (Bower et al., 2020; 2021; Schmitt et al., 2018).

In summary, children from disadvantaged backgrounds show more benefit from spatial training and a spatialised curriculum than their peers. They have the biggest room for growth.

Defining spatial reasoning

Spatial Reasoning involves:

Spatial relations

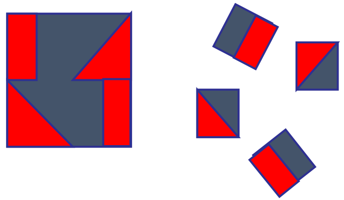
- **Language of position –** *Where?* in relation to one or two things e.g. *next to, between*; relative to the viewer, e.g. *in front of, behind*.
- **Distance –** *How far away?* Length and area, e.g. *near, in the middle*.
- **Direction –** *Which way?* Moving around, e.g. *up/down, forwards/ backwards, left, right.*
- **Changed orientation -** *Which way up (or round)? Upside down, back to front, tipped over, this way up*.
- **Composing** - fitting together 2D and 3D shapes, using interrelationships between properties e.g. with jigsaw puzzle pieces, pattern blocks, nesting containers and construction.
- **Movement and rotatio**n - e.g. turning, sliding or flipping a shape or jigsaw puzzle piece to fit or match.
- **Symmetry recognition** - in 2D and 3D, reflecting, pattern making, block-building.
- **Perspective-taking** – appearance from different viewpoints.
  - Visibility (*what* can be seen, e.g. hidden or partially visible).
  - Size and distance (*how* things far away look smaller).
  - Position (*where* objects are in relation to each other, e.g. things behind each other appear to overlap).
  - Appearance (e.g. *how* circles can look like ovals from certain viewpoints).
- **Scaling** - zooming in and out, e.g. small-world play (toy farms, dolls houses, toy train tracks) and map-making.
- **Navigation** – e.g. way finding and routes.

Objects and images

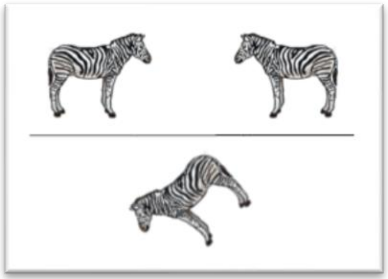
- **Identifying–** *What?* 2D and 3D including the shape of everyday objects such as cups, clothes, jigsaw pieces, leaves and clouds, eg. *circle, rectangle, triangle, heart-shaped; cuboid, cone, ball, roof-shaped*.
- **Properties** including:
  - Size, e.g. *big, tall, wide*.
  - Sides, faces, edges, lines; e.g. *straight/ curved, wiggly, zig-zag*.
  - Corners and angles e.g. *points, vertices, right angle, square corner, sharp*.
- **Cutting and decomposing shapes** – to make new shapes, parts within wholes, bending and folding (e.g. making cylinders with paper strips, unfolding boxes to make nets and then refolding, halving shapes, creating symmetries).
- **Structure** - symmetry, cross-sections, 2D to 3D.
  - **Scaling** – identifying the same item in different sizes, enlarging and shrinking.

SPACE

Spatial and mathematical reasoning



- British Ability Scale III pattern construction task at age 5 predicted classroom-based mathematics (NFER progress in maths) at age 7 (Gilligan, Flouri & Farran, 2017).



- Mental rotation is associated with both calculation and arithmetic in children aged 6 to 8 years (Cheng & Mix, 2014; Hawes et al., 2015).

Spatial reasoning is an important foundation for the development of number and maths skills.

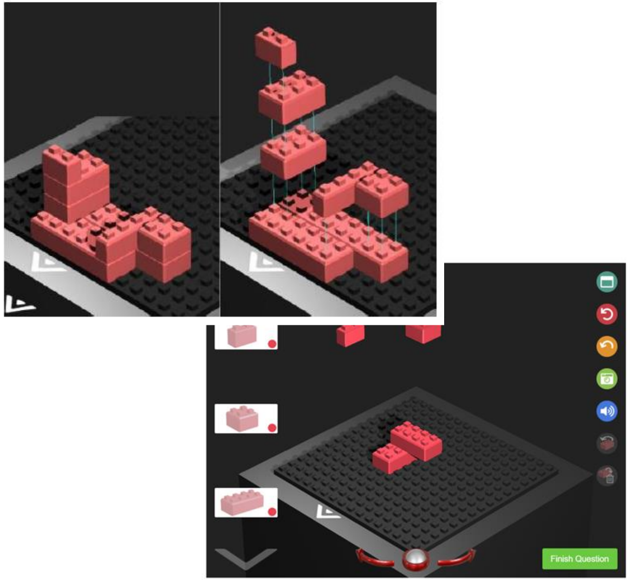
e.farran@surrey.ac.uk @ekfarran

Spatial and mathematical reasoning: BLOCs

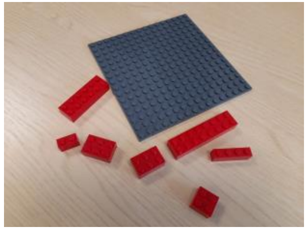
https://www.surrey.ac.uk/block-construction-skills-mathematics-blocs
Funded by the Leverhulme Trust
McDougal, Silverstein, Treleaven, Jerrom, Gilligan-Lee, Gilmore & Farran (2023).

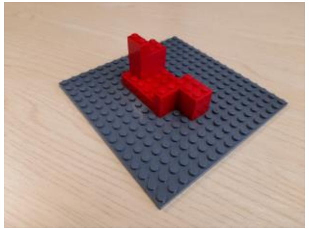
Research consistently shows relationships between LEGO® skills and maths skills

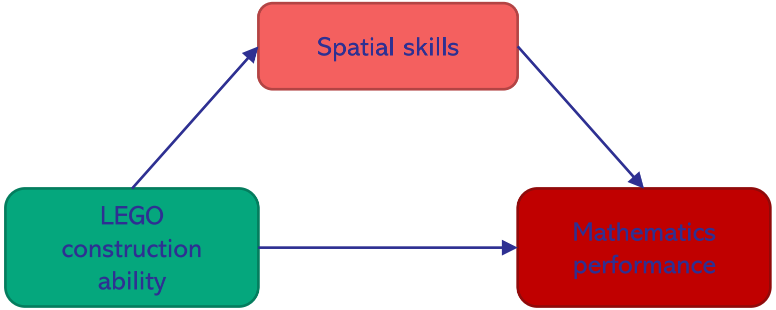
To what extent do spatial skills explain the relationship between LEGO construction and mathematics performance in 7 to 9 year olds?



Block Construction Skills For Mathematics
BLOCS





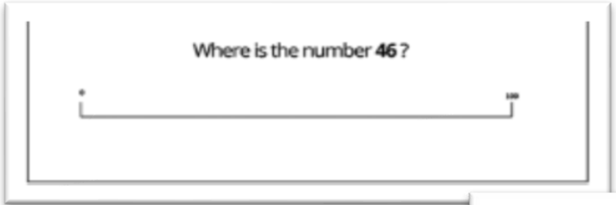


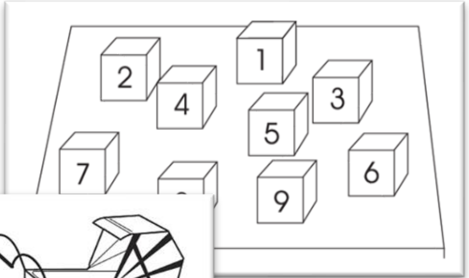
e.farran@surrey.ac.uk @ekfarran

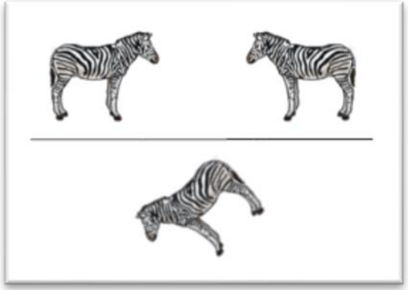
SPACE

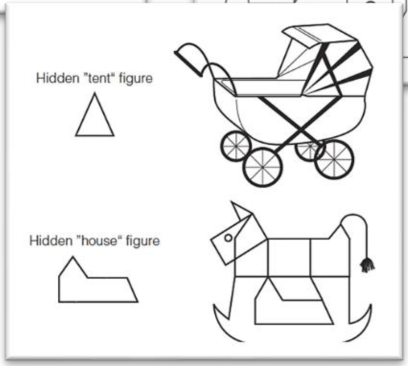
Spatial and mathematical reasoning: BLOCs

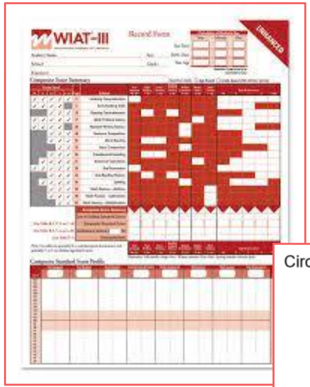
McDougal, Silverstein, Treleaven, Jerrom, Gilligan-Lee, Gilmore & Farran (2023) https://doi.org/10.31234/osf.io/5hvpx

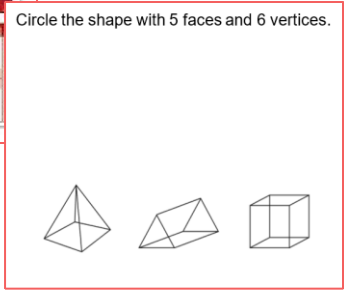










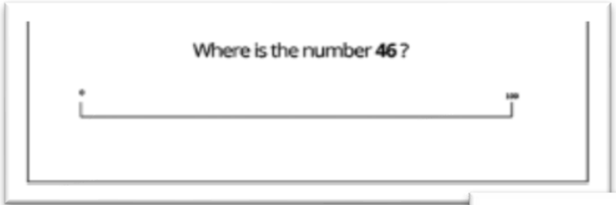


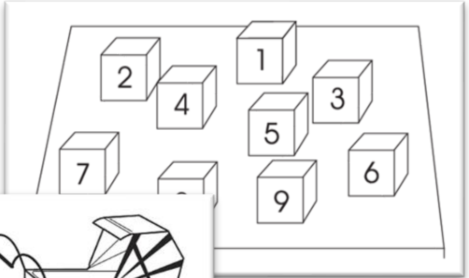
- Strong and consistent relationship between Lego construction ability and maths competence (numeracy, geometry and mathematics problem solving).
- Mediated by part/whole understanding, spatial-numerical representation, mental rotation, visuo-spatial working memory

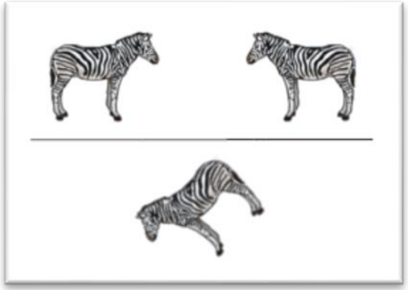
e.farran@surrey.ac.uk @ekfarran

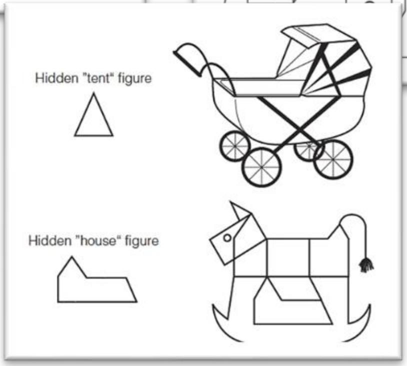
Spatial and mathematical reasoning: BLOCs

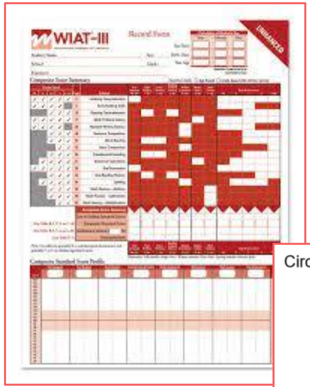
McDougal, Silverstein, Treleaven, Jerrom, Gilligan-Lee, Gilmore & Farran (2023) https://doi.org/10.31234/osf.io/5hvpx

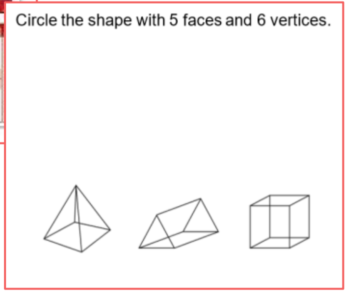












Lego represents a tool for training a range of important spatial skills (mental rotation, part-whole relationships, visuo-spatial working memory as well as problem-solving skills) in a fun and accessible way with proven positive gains in mathematics competence.

e.farran@surrey.ac.uk @ekfarran

SPACE

Spatial reasoning training

(Hawes et al., 2017)

- Integrating spatial skills into mathematical training and instruction over 32 weeks
- Co-created training
- Four days of PD
- 4 to 7 years
- 2 Components of classroom-based training:
- Five 1-hour geometry lessons
- Quick challenge activities (15-20 mins)
- Teachers didn't spend more time on maths they integrated these activities into their teaching (spent approx. 44 hours of in class time on the intervention materials)

Spatial reasoning training

(Hawes et al., 2017)

- Visualisation
- Visuo-spatial memory
- Mental Transformations
- Spatial Language
- Composing/decomposing 2D shapes/3D figures

3. Building with the Mind's Eye

Symmetry Game

4. Shape Transformer

Gains in mental rotation, spatial language, visuospatial reasoning and number processing compared to control.

Spatial reasoning training

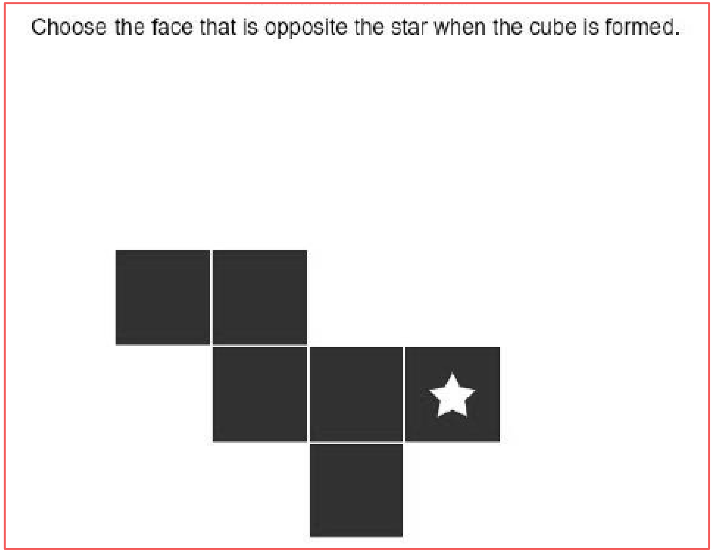
(Lowrie & Logan, 2023)

- Professional development: benefits of spatial reasoning (6 hours).
- Spatial training (14 weeks) integrated into the maths timetable for 40 to 60 mins a week (replacing geometry and measurement).
- 9 years old

Conditions

- Isolated training condition: digital activities (reflection and 3D mental folding - nets)
- Embedded training condition: lessons aligned with curriculum outcomes (reflection, 2D and 3D shapes). Students encouraged to use visualisation to make predictions.

Isolated training



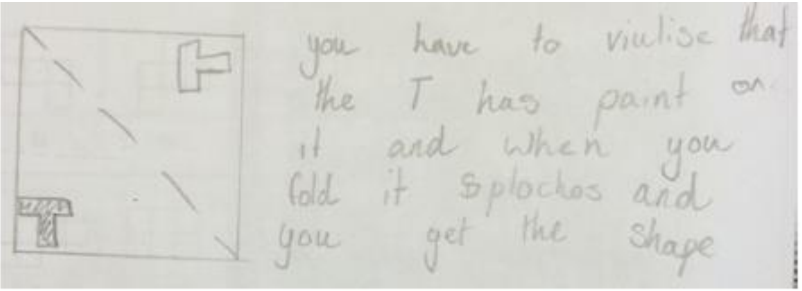
Spatial reasoning training

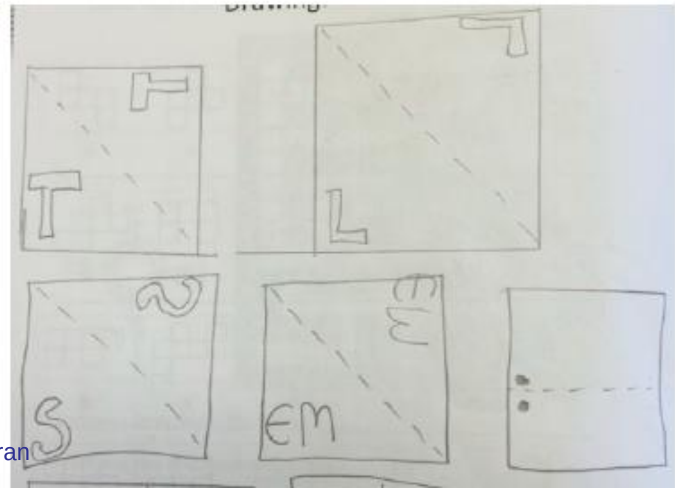
(Lowrie & Logan, 2023)

Findings

- Spatial skills improved for the isolated (untrained spatial skills) and the Embedded condition (trained and untrained spatial skills)
- Maths improved for both conditions

Embedded training





Interpretation

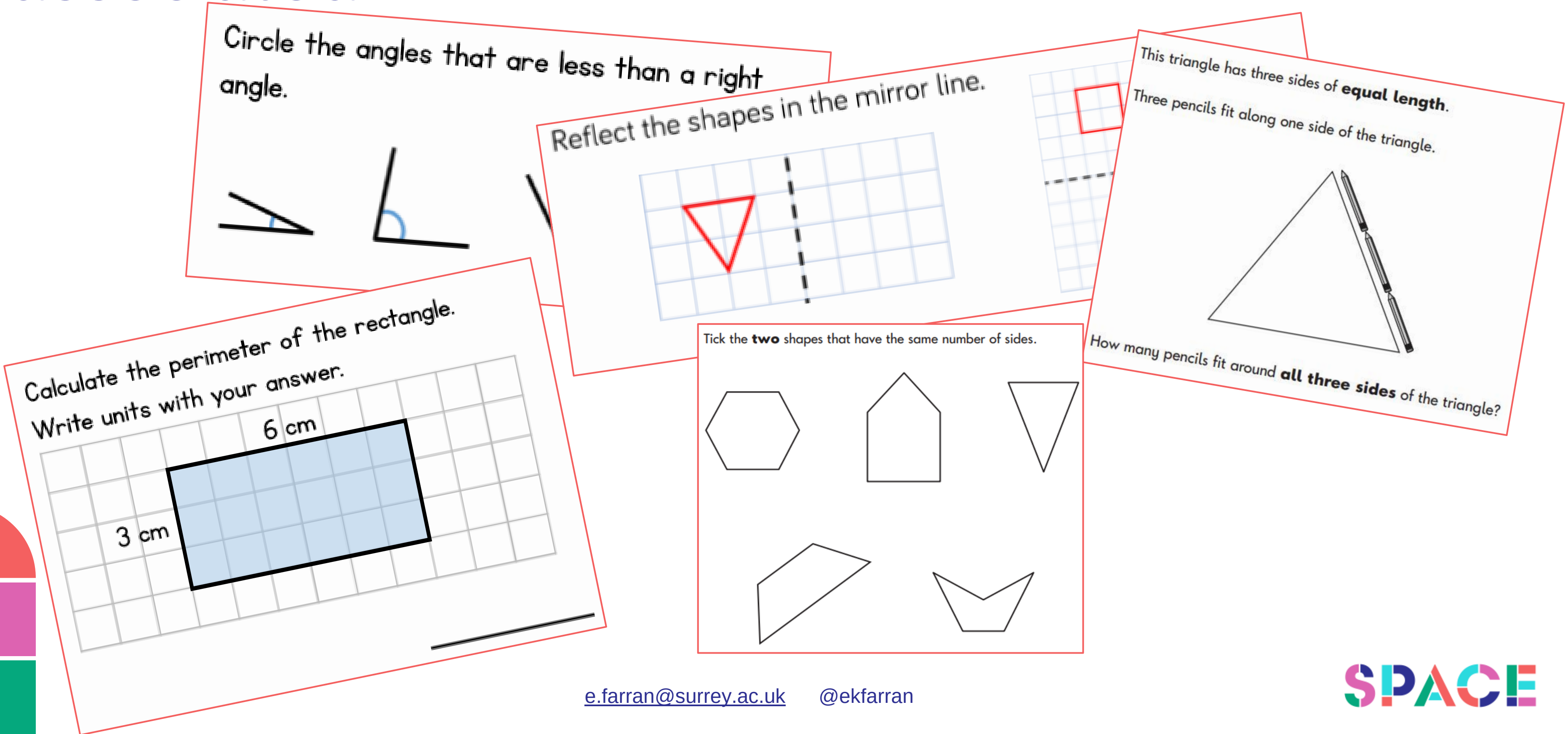
- **Training embedded into classroom curricula is meaningful in terms of student learning and thus more impactful and transferable**
- **Importance of the teacher to build and support the skill development**

Why are spatial reasoning & mathematics associated?

Meta-analysis of maths and spatial research on the brain (Hawes et al., 2019)

- Symbolic number processing, arithmetic and mental rotation activate the same areas of the brain. Conclude that these shared brain areas play a role in general mathematical cognition including spatial reasoning.

Why are spatial reasoning & mathematics associated?



e.farran@surrey.ac.uk @ekfarran

Why are spatial reasoning & mathematics associated?

Gilligan, Hodgkiss, Thomas & Farran (2018)
Gilligan-Lee et al. (2023)

- **Spatial visualisation** is a shared underlying cognitive mechanism between many spatial tasks and mathematics – a mental black board for solving mathematical problems.

- Word problems can be supported using spatial visualisation.

- Spatial visualisation is particularly important for novel and unfamiliar problems.

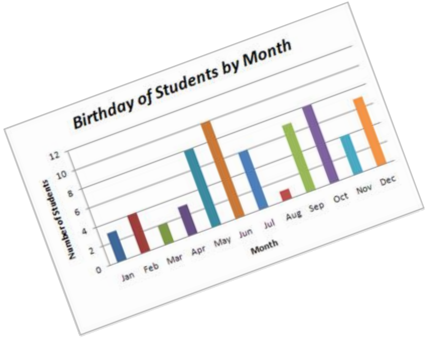
e.farran@surrey.ac.uk @ekfarran

SPACE

Why are spatial reasoning & mathematics associated?

Gilligan, Hodgkiss, Thomas & Farran (2018)
Gilligan-Lee et al. (2023)

- **Mathematical structure is represented spatially**. e.g., tens frame, rekenrek, number line, graphs. Being able to spatially represent number helps many mathematic activities – composition and decomposition, fractions, statistics, judgements about size, proportions, lengths.



- Spatial reasoning also helps children to learn the spatial arrangement of symbols such as the spatial position of tens and units (see Mix, 2019).

Spatial reasoning in the classroom

Spatial language and gesture

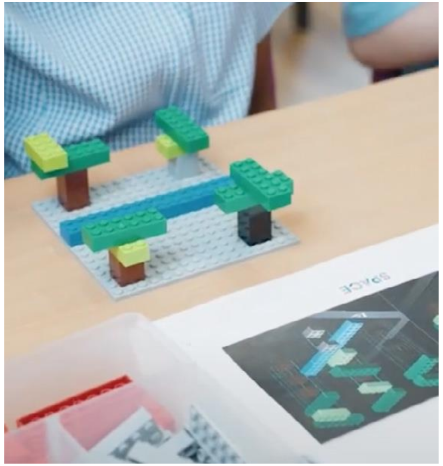
- Terms like “between”, “through” and “separate”, “slope” or “parallel” are difficult concepts within the primary school years, and the learning of these words can be embedded within mathematics teaching. Children with stronger spatial language demonstrate stronger maths performance (Gilligan-Lee et al., 2021).

- Teachers can also support spatial word acquisition with gesture to enable children to visualise the concept. Gesture provides an additional representation of the concept. When teachers use gesture, children show a learning benefit over and above teaching using speech alone (Singer & Goldin-Meadow, 2005).

Spatial reasoning in the classroom

Visualisation

- Teachers can point out to children when visualisation would be useful (i.e., imagining a process in your head). For example, visualising the content of maths word problems, mentally keeping track of the steps of the problem being solved. Children with stronger visualisation skills have stronger maths performance (Gilligan et al., 2019).



Composition and Decomposition

- Teachers can emphasise parts and wholes, and their relationships. Children with stronger Lego performance have stronger maths, and this is partly explained by their understanding of parts and wholes (McDougal et al., 2023)

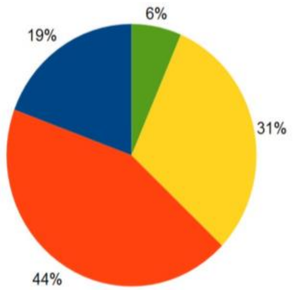
e.farran@surrey.ac.uk @ekfarran

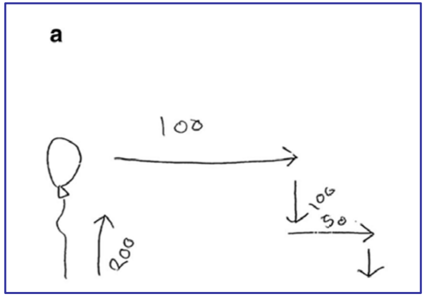
SPACE

Spatial reasoning in the classroom

Representation

- Diagrams use space to show information simultaneously. This contrasts to words, which are sequential in nature. Diagrams can also make an otherwise abstract concept more concrete, such as when number lines are used to depict negative numbers (Newcombe, 2016).
- Teachers can encourage children to create their own diagrams in the form of sketches. Sketching helps children to actively learn a concept in a spatial manner (Newcombe, 2016).





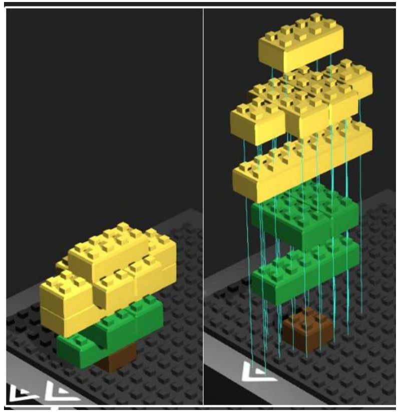
From Hawes & Ansari, 2020

e.farran@surrey.ac.uk @ekfarran

SPACE

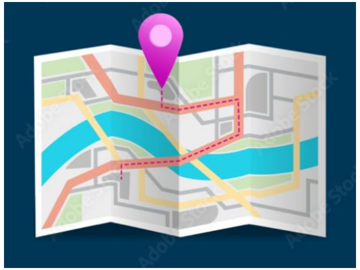
Spatial reasoning in the classroom

Spatial memory



- Encourage children to consult pictorial instructions when using construction toys. Individuals with higher spatial skills look more to the guiding picture (Verdine et al., 2008). This helps to create a rich mental representation of their task and nurtures skills such as spatial memory, mental rotation and part/whole relationships, all of which are associated with mathematics proficiency (McDougal et al., 2023).

Spatial scaling and perspective taking



- Encourage children to use and draw scaled diagrams. Models, maps and pictorial instructions help children to develop spatial skills such as spatial scaling, distance estimation, spatial relationships and perspective taking (Lowrie et al., 2017).

e.farran@surrey.ac.uk @ekfarran

SPACE

Year 2 Mathematics Curriculum and Spatial Reasoning

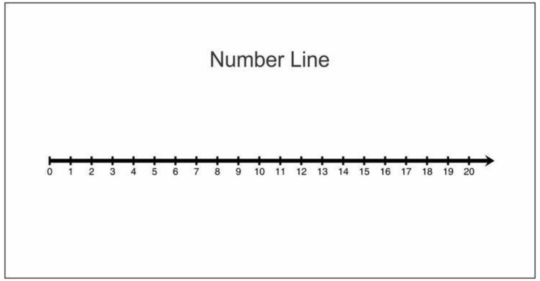
Spatial Skills

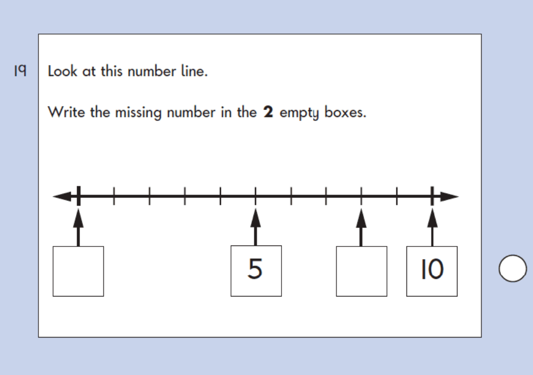
Spatial skills	Definition
Visualisation	Imagining and manipulating spatial information in the mind's eye, involving memory and prediction.
Visual and Spatial memory	The ability to maintain an image in memory for a small amount of time.
Composing and Decomposing	Understanding of structure, parts, and wholes.
Spatial Scaling	Working between different size versions of the same thing. Understanding the spatial relationships represented by diagrams of real objects.
Perspective Taking	Things appear differently depending on where we are (position) and what we can see from where we are (visibility).
Representation	Representations help children to make sense of spatial and mathematical structures and relationships, for problem solving. Examples include gesture, language, physical manipulatives, graphs and diagrams.

SPACE

Mathematics Curriculum and Spatial Reasoning

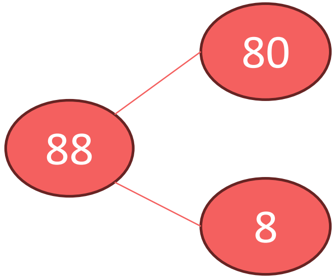
- **Number and place value**

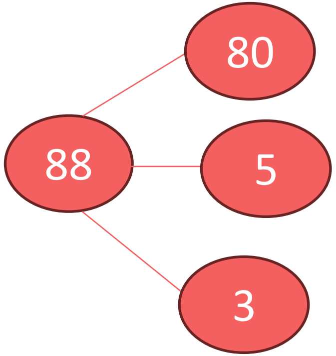




Mathematics Curriculum and Spatial Reasoning

- **Addition and Subtraction**





□ + 8 = 88

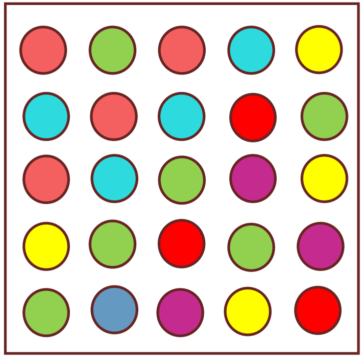
88 - □ = 80

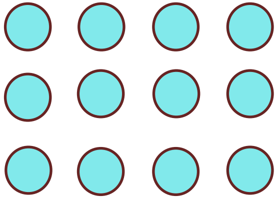
□ + 5 + □ = 88

SPACE

Mathematics Curriculum and Spatial Reasoning

- **Multiplication and Division**

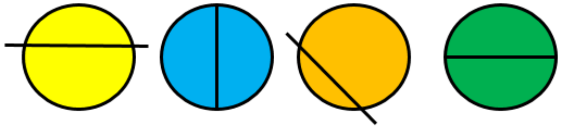


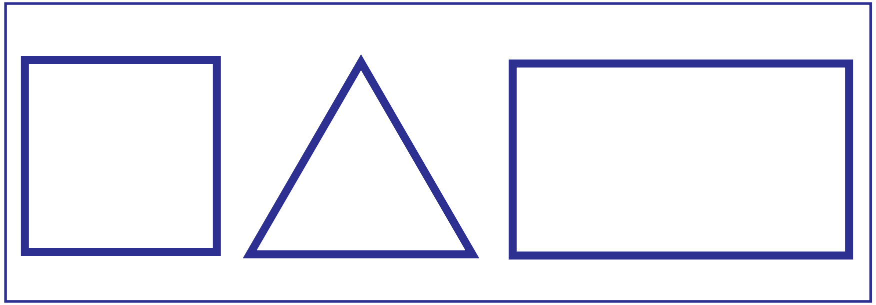


Teddy and Bear have some sweets. Teddy has twice as many as Bear. If Bear has 3 sweets how many sweets does Bear have?

Mathematics Curriculum and Spatial Reasoning

- **Fractions**

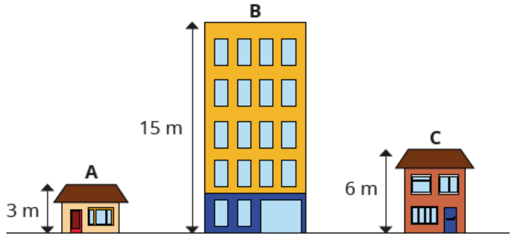




Mathematics Curriculum and Spatial Reasoning

- **Measurement**

- The height of three buildings is shown.

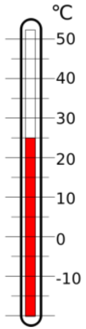


  - Which building is the tallest?
  - Which building is the shortest?
  - Put the buildings in order, from tallest to shortest.

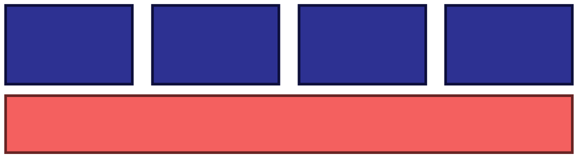
- Glass A has more water than glass B.

  Glass C has less water than glass B.

  Show the volume of water that could be in glasses A and C.



How long is a ribbon in blocks?



SPACE

Mathematics Curriculum and Spatial Reasoning

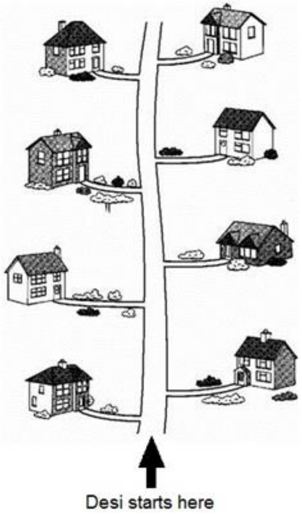
- **Geometry – properties of shape**

- **Geometry – position & directions**

Look at this map.

Desi's house is the **2nd** on the **left.**

Tick (✓) it.



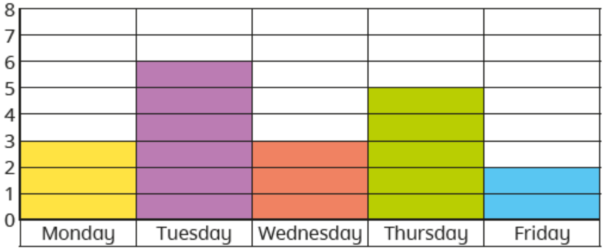
SPACE

Mathematics Curriculum and Spatial Reasoning

- **Statistics**

Days	Number of children
Monday	3
Tuesday	6
Wednesday	3
Thursday	5
Friday	2

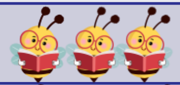
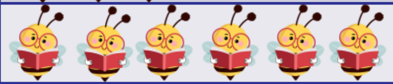
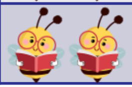
- The block diagram shows how many children went to after-school club each day.



▸ On Monday, ______ children went to after-school club.

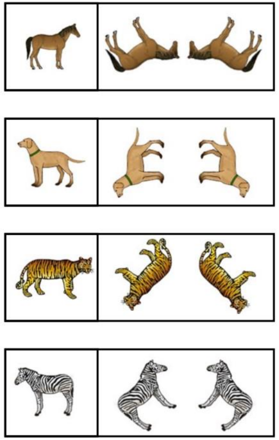
▸ The day with the most children was ______

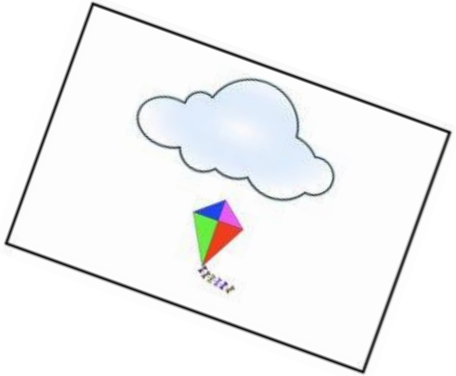
Days	Tally
Monday	III
Tuesday	𝍸 I
Wednesday	III
Thursday	𝍸
Friday	II

Days	**Number of children**
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	

Spatial and Mathematics assessments

Why we need it and how to run it?





ASSESSMENTS (two sessions)

Mathematics measure

- Mathematics task (Form A and Form B)

Spatial measures

- Spatial language tasks (production and comprehension)
- Mental Rotation task (Form A and Form B)

For each session

Children

✓Assessment booklet

✓Pencil

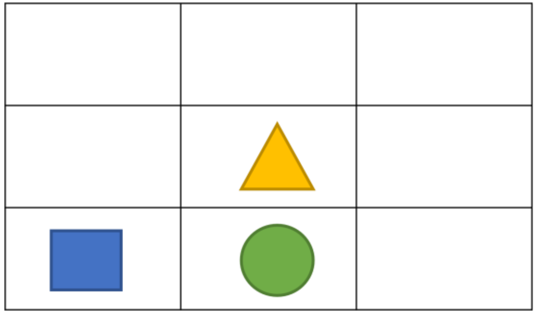
Teacher

✓ Instruction booklet

✓Spare assessment booklet

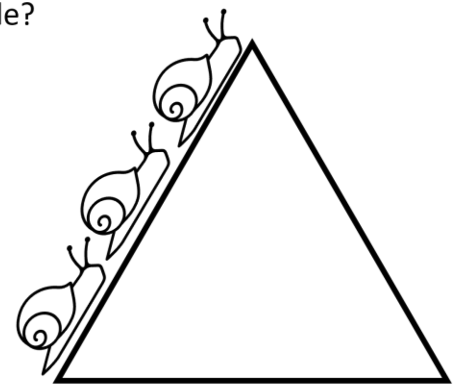
Mathematics task

Ella says: “The square is to the right of the circle. “



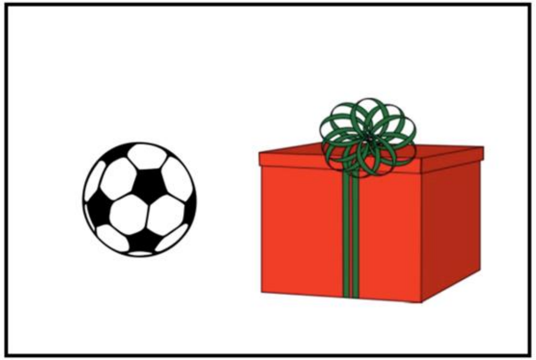
Is Ella correct? YES ☐ NO ☐

This triangle has three sides of equal length.
Three snails fit along one side of the triangle.
How many snails fit around all three sides of the triangle?

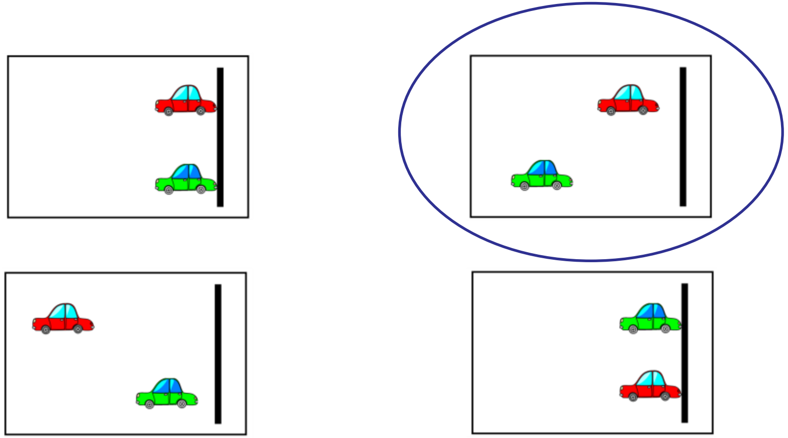


SPACE

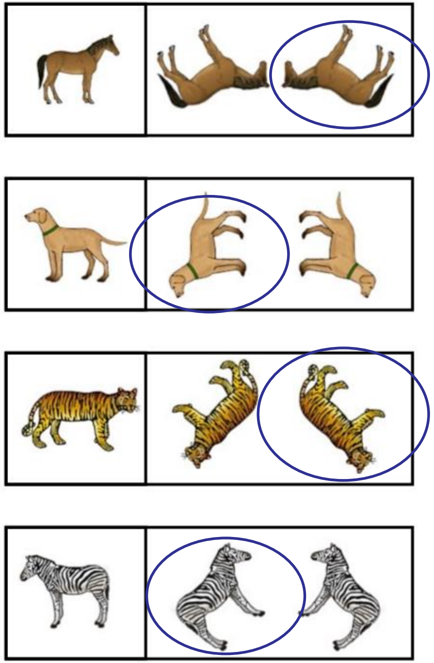
Spatial Language Tasks



The ball is to the _________ of the present.

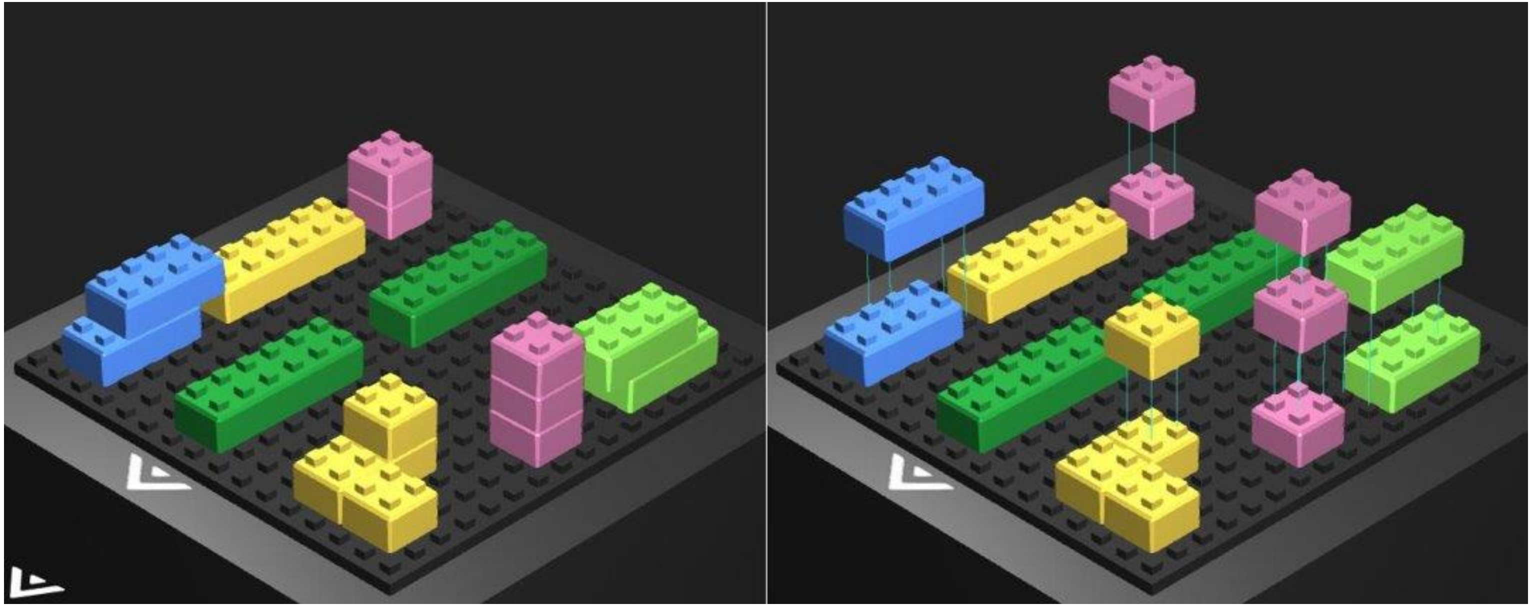


Mental Rotation Task



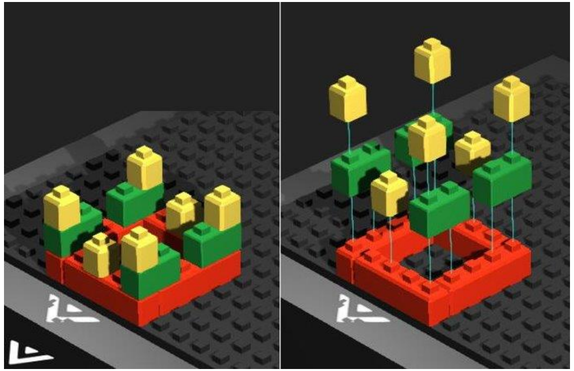
WHEN YOU FINISH EACH ASSESSMENT, PLEASE RETURN ALL BOOKLETS TO OUR TEAM WHEN WE VISIT.

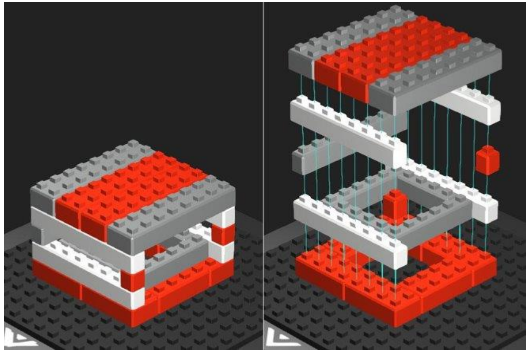
SPACE LEGO activity

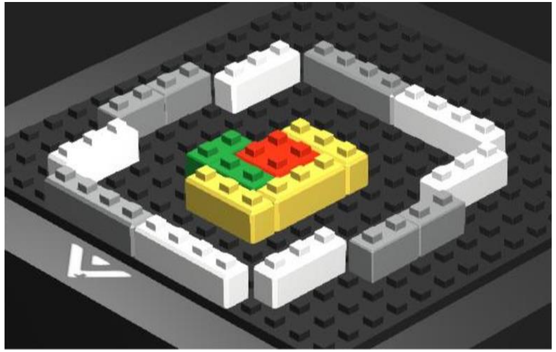


BREAK

SPACE LEGO sessions – how to run the sessions







SPACE

1

We would like you to include:

- **two 30-minutes** SPACE Lego sessions in place of your normal Mathematics lessons
- **6 weeks**, to a total of **12 sessions**.

2

What you will need for the sessions:

- SPACE **training manual**,
- the **video link** for that week,
- the **prompt cards** and
- a **timer** or **clock** to time the session length.

SPACE

3

Children will need:

- a **Lego box** and
- the **correct booklet** for that session.

SPACE

4

- At the beginning of each session **play the YouTube video** for that week on the screen in the classroom and ask the children to watch it.
- The link for each week is in your **SPACE training manual**.
- We will also **email** the link to you each week.

SPACE

5

- **Children will build** up to **6 Lego models per session** using the pictures in their booklet.
- Children should complete the models **in the order provided** in their booklet. Please **encourage** them to **attempt** every model before moving on.
- The children should **work individually**.

SPACE

6

After they have built each model, they need to tick it off in their booklet, and then break the model up putting the Lego back in their Lego box and move to the next model.

7

At the end of the session ask them **to pack away** their Lego bricks into their **Lego Tray**.
Collect the booklets ready for us to collect them from you.

SPACE

SPACE resources

https://www.surrey.ac.uk/spatial-cognition-enhance-mathematical-learning-space/space-programme-resources

SPACE prompt cards

SPACE videos

SPACE session training manual
SPACE assessment instructions
LEGO inventory sheet
SPACE training session material

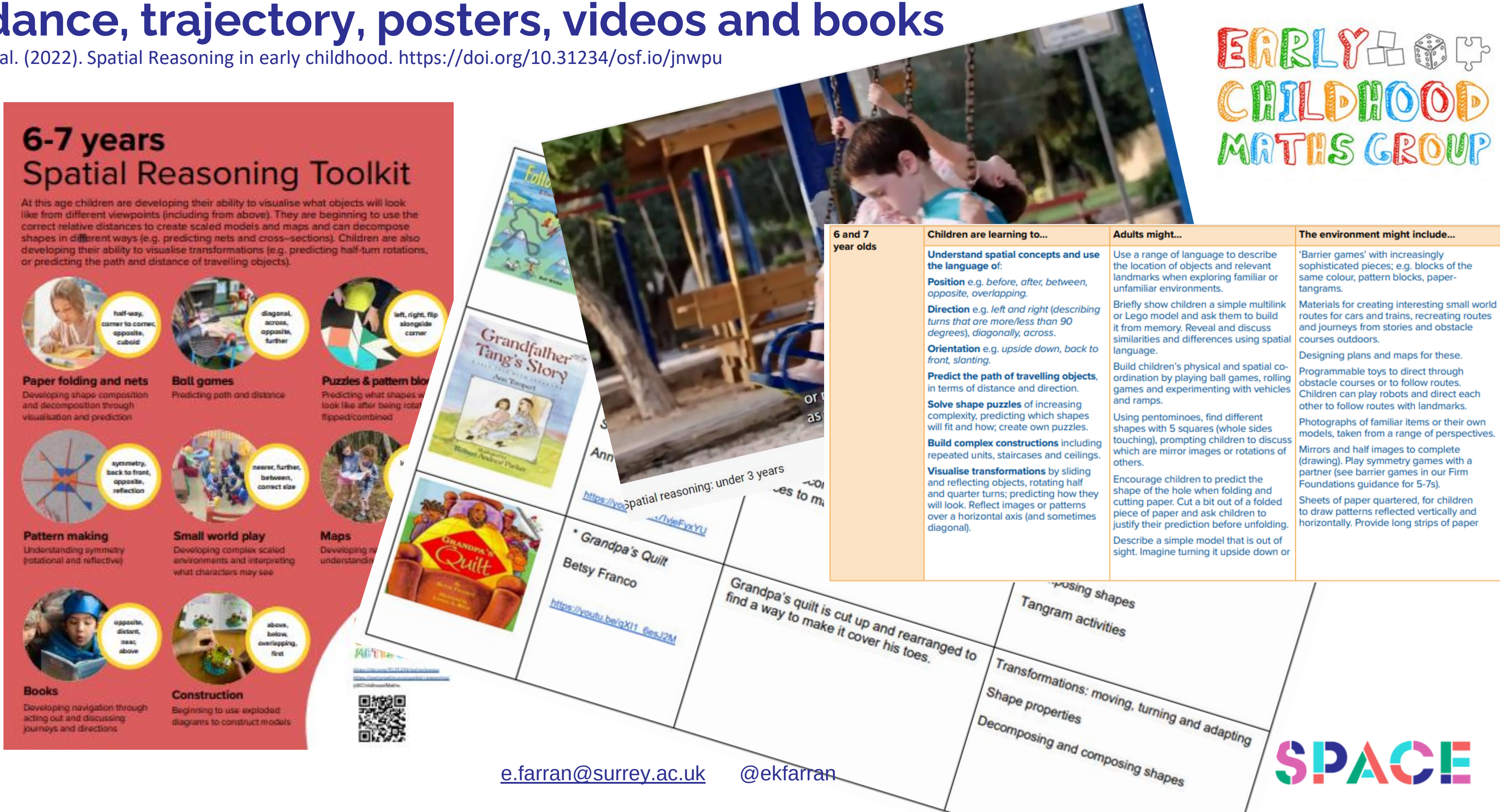
e.farran@surrey.ac.uk @ekfarran

Spatial reasoning toolkit @EChildhoodMaths

Guidance, trajectory, posters, videos and books

Gifford, et al. (2022). Spatial Reasoning in early childhood. https://doi.org/10.31234/osf.io/jnwpu

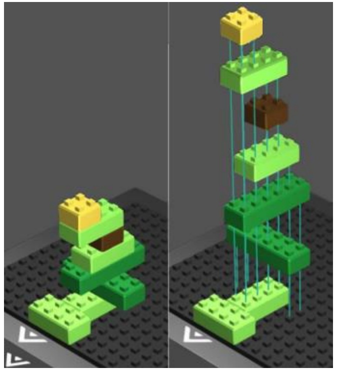
EARLY CHILDHOOD MATHS GROUP



e.farran@surrey.ac.uk @ekfarran

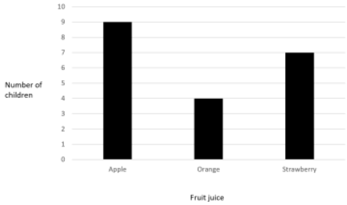
SPACE

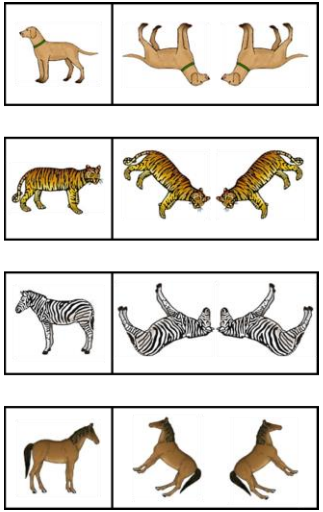
Q & A

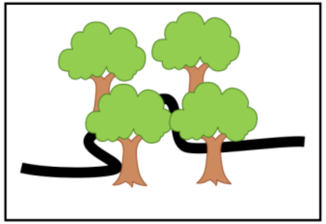


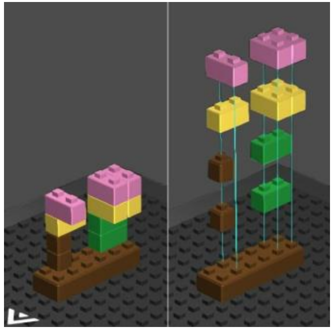
26 20 children choose their favourite fruit juice.

The chart shows the results.









SPACE
SPATIAL COGNITION TO ENHANCE MATHEMATICAL LEARNING

For any queries, please contact Marija or Anna

m.zivkovic@surrey.ac.uk
anna.korzeniowska@surrey.ac.uk

We hope that you will enjoy in SPACE activities as much as we enjoyed developing it.

SPACE

SPATIAL COGNITION TO ENHANCE MATHEMATICAL LEARNING

University of Surrey · Education Endowment Foundation · Centre for Evidence and Implementation · Learnus (Understanding Learning) · Birkbeck University of London · Loughborough University · UCD Dublin