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THE MACROECONOMIC CONSEQUENCES OF GOVERNMENT INVESTMENT REVISITED

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The Macroeconomic Consequences of Government Investment Revisited

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Abstract

A large literature over several decades studies government investment through structural DSGE frameworks, empirical fiscal-multiplier estimates, and studies of the long-run productivity effects of public capital. Recent contributions show that investment-specific time-to-build gestation lags and the distortionary fiscal adjustments required for intertemporal government budget balance can compress short-run multipliers despite positive long-run returns. We reassess this mechanism in an estimated medium-scale New Keynesian model that replaces the Cobb–Douglas technology with an empirically supported CES production function. Public-investment expansions are implemented under jointly welfare-maximizing simple monetary and tax rules. The analysis quantifies how the elasticity of substitution between private and public capital, increasing returns to scale and optimal policy interactions shape the dynamic propagation of government-investment shocks and increase both the short-run and long-run productive gains to the government investment fiscal multiplier.

Keywords: public investment, time-to-build, Cobb-Douglas versus CES production function, optimized monetary and tax rules
JEL Codes: D52, E17, E52, E62, H54

Contents

1 Introduction

This paper re-examines the short-run and long-run macroeconomic effects of government investment, a topic that has generated conflicting empirical and theoretical conclusions. It has become particularly topical following the Infrastructure Investment and Jobs Act (IIJA), a bipartisan U.S. federal law enacted in November 2021 that provides long-term funding for transportation (roads, bridges, transit, rail); water infrastructure; broadband deployment; energy and grid resilience and environmental remediation. In a similar vein, the 2024 UK government budget increased infrastructure expenditure which according to the fiscal rule is being funded by borrowing. Figure 1 compares these programmes. Whilst the recent infrastructure programmes in the US and the UK motivate our study,

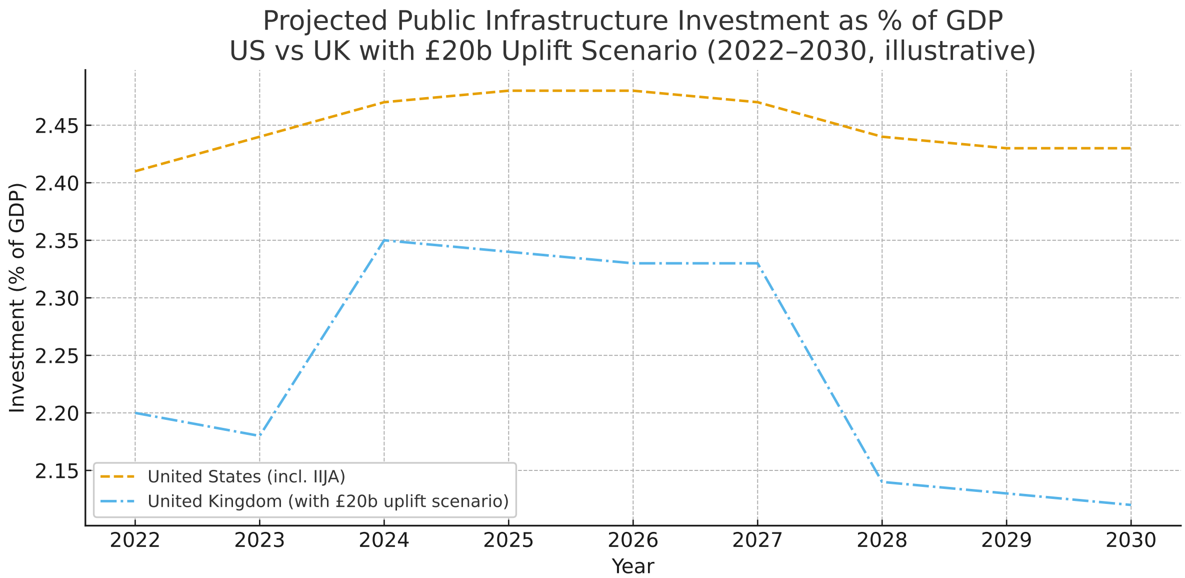

Figure 1: Projected public infrastructure investment as % of GDP, US vs UK with £20b uplift scenario (2022–2030).

the subsequent focus of the paper is on total government investment in the US which, as shown in Figure 2, exceeded infrastructure investment by about a third in recent years.

Although a large literature documents substantial long-run productivity gains from public capital-beginning with Aschauer (1989), Munnell (1990), and Fernald (1999), and surveyed by Bom and Ligthart (2014a)–the short-term output response remains ambiguous. Empirical estimates of fiscal multipliers vary substantially across identification strategies, sample periods, and economic conditions (Blanchard and Perotti (2002); Mountford and Uhlig (2009); Auerbach and Gorodnichenko (2012); Ramey and Zubairy (2018)). Recent

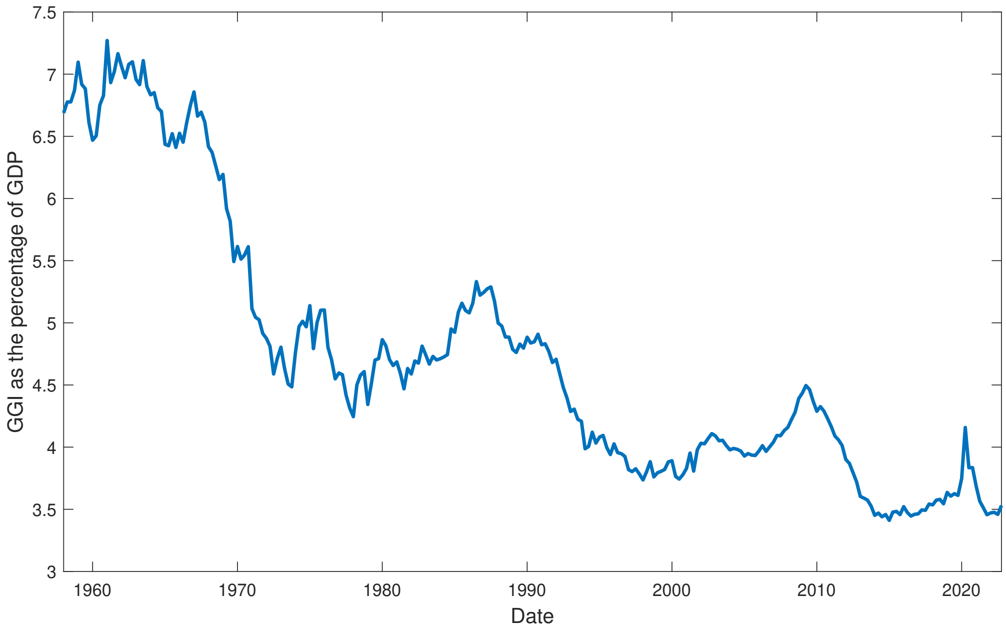

Figure 2: The Gross Government Investment as the percentage of the GDP for the US. The Gross Government Investment includes investments in infrastructure, such as highways and public buildings, and purchases of military equipment.

evidence summarized by Ramey (2020) stresses that public investment, unlike government consumption, generates durable productive capacity, which raises the central question of how these long-run gains map into short-run multipliers.

The theoretical literature highlights several mechanisms that may weaken the near-term effects of government investment. Time-to-build frictions (Kydland and Prescott (1982)), adjustment costs (Altig et al. (2001)), and financial frictions (Khan and Thomas (2013)) delay the translation of government investment into productive capital. DSGE models with public capital (Baxter and King (1993); Turnovsky (1997); Leeper et al. (2010a)) show that distortionary taxation required for debt stabilization can further depress short-run multipliers. These mechanisms interact strongly with fiscal–monetary policy, as demonstrated in Ramsey analyzes (Chari and Kehoe (1999); Benigno and Woodford (2004); Schmitt-Grohé and Uribe (2004); Leeper (2017)). Despite these insights, most models assume a Cobb–Douglas production function that restricts the elasticity of substitution between private and public capital to unity, limiting the ability to assess how the production technology influences fiscal transmission.

An influential recent survey by Ramey (2020) brings together theoretical and empirical strands, highlighting why naive estimates can overstate short-term multipliers and stressing the importance of careful identification and concludes: first, implementation lags are central: large projects take time to plan and execute, muting short-run multipliers while producing larger multi-year and long-run benefits; second results are heterogeneous by sector, country, and project quality. Measurement and endogeneity issues can bias estimates; third, Policy implication: prioritize high-productivity projects and strengthen delivery capacity to realize potential gains; fourth, fiscal policy operates via tax and wealth channels; multipliers for consumption-type government spending are typically modest and highly parameter dependent.

The main contribution of our paper is to quantify how production structure, gestation lags, adjustment costs, and optimal fiscal–monetary policy jointly determine the dynamic effects of government investment. We estimate a medium-scale New Keynesian model with a CES production function disciplined by recent empirical work on factor substitutability (Antràs (2004); Klump et al. (2007); León-Ledesma et al. (2010)). Within this environment, government investment is evaluated under Ramsey-optimal policy, allowing taxes, debt, and monetary policy to adjust endogenously. This framework enables us to isolate the mechanisms that govern both the short-run multiplier and the long-run productivity effect.

Our results speak directly to the mixed conclusions in the literature. When public and private capital are complements, government investment can generate larger short-run multipliers even in the presence of distortionary taxation. When substitutability is high, short-run multipliers remain small despite sizable long-run returns. These findings show how production structure and policy design condition the stabilization role of government investment and clarify when conventional results on small short-run multipliers apply.

The paper proceeds as follows. Section 2 follows up points from the Introduction. Section 3 presents the model. Section 4 estimates the model by Bayesian methods and highlights the superior fit of the CES over CD production. Section 5 reports the main results. Section 6 concludes the paper. Appendices provide full details of the model, data, Bayesian estimation and, for comparison, a reproduction of the multipliers found by Ramey (2020) using a CD production function.

2 Main Issues in Detail

This section follows up from our discussion by discussing in detail the four issues that underlie our reassessment of the size of the public investment multiplier.

2.1 Production Function

The specification of the **production function** is of central importance and is a major focus of our paper. With a slight change of notation and choice of technology shock to align with our paper, BK choose a Cobb-Douglas specification:

$$Y_t = (A_t H_t)^{\gamma_H} K_{t-1}^{\gamma_K} (K_{t-1}^G)^{\gamma_{KG}} \tag{1}$$

where A_t is labour productivity, H_t is the labour input, K_{t-1} is end-of-period $t-1$ private capital stock and K_{t-1}^G is end-of-period $t-1$ public capital. BK assumes $\gamma_H + \gamma_K = 1$ so **constant returns to scale (CRS) prevails in the the private sector.**

2.2 Time to Build

Now let public capital accumulate with gestation:

$$K_{t+1}^g = (1-\delta_g)K_t^g + \sum_{j=0}^{J} \theta_j I_{t-j}^g, \qquad \sum_{j=0}^{J} \theta_j = \eta,$$

where J is the **time-to-build (TTB)**, $\{\theta_j\}$ distributes completion over time, and $\eta \leq 1$ captures implementation efficiency. With longer TTB (mass of θ_j at higher j), the output response to a given investment outlay becomes more **back-loaded**. For horizons $H < J$, the cumulative multiplier is smaller because there is little or no contemporaneous service flow from K^g; demand reallocation into construction may then crowd out private uses when monetary policy reacts to demand. Over longer horizons, the **response is hump-shaped** as projects complete; however, discounting and possible depreciation during the pipeline typically reduce the present-value multiplier unless the output elasticity of public capital is high and η is close to one - see Bom and Ligthart (2014b).

There are also implications of TTB for the **monetary regime.** At the zero lower bound (ZLB), TTB can raise peak multipliers because the disinflationary supply effects arrive later, after liftoff, mitigating a near-term monetary offset - see Bouakez et al. (2017). Away from the ZLB under active Taylor rules, the absence of immediate productivity gains makes short-run multipliers smaller relative to the $J = 0$ case - see Leeper et al. (2010a).

Policy implications of TTB include shortening TTB via permitting/procurement reform and modular designs; staging projects to deliver partial service flows early ($\theta_0, \theta_1 > 0$); targeting slack sectors to minimize crowding out; and prioritizing high-productivity public capital - see Koeva (2000) and Li and Li (2018).

2.3 Cobb-Douglas versus Constant Elasticity of Substitution

We now generalize the Cobb-Douglas production (CD) function (1) to the following Constant Elasticity of Substitution (CES) form

$$Y_t^W = \left[\theta\left((A_t H_t)^{\alpha}(K_{t-1})^{1-\alpha}\right)^{\vartheta} + (1-\theta)\left(K_{t-1}^G\right)^{\vartheta}\right]^{\frac{\rho}{\vartheta}} \tag{2}$$

where θ is the share parameter of the private final output, ϑ is the degree of substitutability of the private finals and public capitals and ρ is the degree of homogeneity, with constant returns to scale (CRS) if $\rho = 1$, increasing returns (IRS) if $\rho > 1$ and decreasing returns (DRS) if $\rho < 1$.

When $\rho = 1$ we have the following relation on the elasticity of factors of production: $\alpha\theta + (1-\alpha)\theta + 1 - \theta \equiv 1$, this confirms CRS of the single factor of production. Then CRS prevails in the private sector ($\gamma_H + \gamma_K = 1$), as in the papers reviewed up to now, if we choose $\rho = \frac{1}{\theta}$. We have CRS overall ($\gamma_H + \gamma_K + \gamma_G = 1$) if $\rho = 1$ as in the UK Office of Budget Responsibility report Ghaw et al. (2024).

It is useful to write $\vartheta = \frac{\sigma-1}{\sigma}$ where the **elasticity of substitution** $\sigma = \frac{1}{1-\vartheta}$ measures how easily one input can substitute for another. Then

- $\sigma > 1$: Inputs are easily substitutable. Public capital may crowd out private capital.
- $\sigma = 1$: Cobb–Douglas case with moderate substitutability.
- $\sigma < 1$: Inputs are complements. Public capital strongly crowds in private investment, leading to high multipliers.

Mechanism	**Cobb-Douglas**	**CES** ($\sigma > 1$)	**IRS** ($\rho > 1$)
Growth from public investment	Modest	Stronger with high σ	Still Stronger
Tax revenue potential	Limited	Higher	Very high
Fiscal limit level	Tight	Relaxed	Much higher

2.4 Fiscal and Monetary Optimized Simple Rules

We draw upon Kliem and Kriwoluzky (2014) who examine systematic fiscal rules (analogous to monetary Taylor rules) and their implications for stability, volatility and investment smoothing. They find fiscal rules that stabilise and smooth public investment over the cycle (protecting multi-year envelopes) improve long-run productivity payoffs by reducing stop-start inefficiencies and increasing delivery credibility. The interaction of fiscal rules with monetary policy matters for macro stability and multipliers.

A related general literature compares optimized constrained simple rules with their optimal unconstrained counterparts (see, for example, Levine and Currie (1987) and , Schmitt-Grohe and Uribe (2007). A common finding in this literature is that optimized simple rules can closely mimic optimal policies and perform well in a wide variety of models. By contrast optimal policy can perform very poorly if the policymaker's reference model is mis-specified. The reason for this is that optimal policies can be overly fine-tuned to the particular assumptions of the reference model. If the model is the correct one all is well; but if not, the costs can be high. In contrast, our chosen simple fiscal and monetary policy rules are designed to take account of only the most basic principle of leaning against the wind of inflation and output movements. Because they are not fine-tuned to specific model assumptions, they are more robust to mistaken assumptions regarding the parameters of the model ('within-model robustness') or to basic modelling features ('between-model robustness').

3 Monetary and Fiscal Policy in a DSGE NK Model

Most papers using the Smets and Wouters (2007) model use the linearized form about a balanced-non-zero growth and effectively zero-net-inflation steady-state.[1] The non-linear form of the model with a trend net inflation is relatively unexplored, but is essential for the welfare-analysis of this paper which is based on a second-order perturbation solution. The properties of the model in a non-zero-net inflation rate steady state, set out in this Section are crucial in this set-up. This section therefore sets the full non-linear form to be solved in the vicinity of a trend net inflation deterministic steady state.

There are four sets of representative agents: households, final goods producers, trade unions and intermediate goods producers. The latter two produce differentiated labour services and goods respectively and, in each period of time, consist of a group that is locked into an existing contract and another group that can re-optimize.[2]

[1] This is achieved by assuming that Calvo price and wage contracts are fully indexed in the steady state, but only partially away from the steady state. A zero net inflation steady is convenient for linearization as it removes the steady state distortion from dispersion, but abstracts from the trend inflation rate effects that are central to this paper. Moreover the convenient indexing assumption is inconsistent with microevidence on price setting - see, for example, Linde and Trabandt (2018).

[2] Our model is a slightly slimmed down version of Smets and Wouters (2007) in one respects, we employ a

3.1 The Monetary Rule

A monetary policy rule for the nominal interest rate is given by the following Taylor-type rule

$$\begin{aligned}\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) \\ &+ (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\right) \\ &+ \log(MPS_t), \end{aligned} \tag{3}$$

where MPS_t is a monetary policy shock. Our rule is of the implementable form as proposed by Schmitt-Grohe and Uribe (2007) in that the nominal interest rate responds to deviations of output about its steady state rather than deviations about the flexi-price level of output (i.e., the output gap). The latter would encompass the original rules proposed by Taylor (1993) and Taylor (1999) for which there is no interest-rate smoothing ($\rho_r = 0$) and $\theta_{dy} = 0$. In the more recent of these papers, parameter values $\theta_\pi = 1.5$ and $\theta_y = 1.0$ are proposed.[3]

Nominal and real interest rates are related by the Fischer equation

$$R_t = \frac{R_{n,t-1}}{\Pi_t}$$

3.2 The Government Budget Constraint

Corresponding to (B.4), real government debt D_t accumulates according to:

$$D_t = RP_{t-1}R_{t-1}D_{t-1} + G_t^G + I_t^G + Z_t - \text{Tax}_t \tag{4}$$

where R_t remains the riskless real rate, $R_t\, RP_t \geq R_t$ is the bond rate, RP_t is the risk premium, $G_t = G_t^G + I_t^G$ is real spending on government services consisting of normal government spending and government investment, and total tax revenue from capital, labour and monopolistic profits is given by

$$\text{Tax}_t = \tau_{k,t} r_t^K u_t K_{t-1} + \tau_{w,t} W_t H_t^d + \tau_{k,t}\Gamma_t \tag{5}$$

Dixit-Stiglitz rather than Kimball aggregators over differentiated goods and labour types. We discuss this simplification later.

[3]Note forward-looking 'inflation-forecasting rules could also be considered but these are prone to a severe indeterminacy constraint that results in welfare-inferior outcomes (see Batini et al. (2006)).

Note that $\text{FD}_t \equiv G_t^G + I_t^G + Z_t - \text{Tax}_t$ is the government spending on services and transfers unfunded by taxes termed the **fiscal deficit**.

For the analysis of fiscal policy it is useful to re-parameterize variables in per output terms. Thus we denote by lower case $d_t \equiv \frac{D_t}{Y_t}$, $\text{tax}_t \equiv \frac{\text{Tax}_t}{Y_t}$, $z_t \equiv \frac{Z_t}{Y_t}$, $iy_t \equiv \frac{I_t^G}{Y_t}$ and $gy_t \equiv \frac{G_t^G}{Y_t}$. Define $\Delta Y_t \equiv \frac{Y_t - Y_{t-1}}{Y_{t-1}}$ as the growth rate of output in the interval $[t-1, t]$. Then (4) becomes

$$d_t = \frac{RP_{t-1}R_{t-1}}{1+\Delta Y_t} d_{t-1} + gy_t + iy_t + z_t - \text{tax}_t \tag{6}$$

noting that $\frac{D_{t-1}}{Y_t} = \frac{D_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t} = \frac{d_{t-1}}{1+\Delta Y_t}$. The deterministic steady state is therefore given by

$$d\left(1 - \frac{(1+r)RP}{1+\Delta Y}\right) = gy + iy + z - \text{tax} = \text{fd} \Rightarrow d = \frac{(1+\Delta Y)(gy + iy + z - \text{tax})}{1 + \Delta Y - (1+r)RP} \tag{7}$$

We take the following formula for the risk premium:

$$RP_t = (1 + \phi_d(e^{d_t} - 1))RPS_t \equiv (1 + f(d_t))RPS_t \tag{8}$$

say where RPS_t is a risk premium shock as we usually have in the standard SW model and $f(0) = 0$ and $f'(d_t) > 0$

3.3 Tax Rules

We use separate tax rules for labour and capital taxes in the estimation as follows:

$$\log\left(\frac{\tau_{k,t}}{\tau_k}\right) = \rho_{\tau_k}\, log\left(\frac{\tau_{k,t-1}}{\tau_k}\right) + (1-\rho_{\tau_k})\left(\theta_{\tau_k,b}\log\left(\frac{d_{t-1}}{d}\right) + \theta_{\tau_k,y}\log\left(\frac{Y_t}{Y}\right) + \theta_{\tau_k,dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \epsilon_{\tau_k,t} \tag{9}$$

$$\log\left(\frac{\tau_{w,t}}{\tau_w}\right) = \rho_{\tau_w}\, log\left(\frac{\tau_{w,t-1}}{\tau_w}\right) + (1-\rho_{\tau_w})\left(\theta_{\tau_w,b}\log\left(\frac{d_{t-1}}{d}\right) + \theta_{\tau_w,y}\log\left(\frac{Y_t}{Y}\right) + \theta_{\tau_w,dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \epsilon_{\tau_w,t} \tag{10}$$

Lags in Tax Rules: According to Leeper et al. (2010b) "The federal government is not subject to year-to-year balanced budget rules, therefore delayed financing is more empirically plausible than immediate financing."

This is not considered in Kliem and Kriwoluzky (2014) who, as we do, allow tax rates to adjust every period (quarterly). Instead suppose that tax rates are twice a year. Tax rules consistent with biannual budget statements are:

$$\log\left(\frac{\tau_{k,t}}{\tau_k}\right) = \rho_{\tau_k}\, log\left(\frac{\tau_{k,t-1}}{\tau_k}\right)+(1-\rho_{\tau_k})\left(\theta_{\tau_k,b}\log\left(\frac{d_{t-3}}{d}\right)+\theta_{\tau_k,y}\log\left(\frac{Y_{t-2}}{Y}\right)+\theta_{\tau_k,dy}\log\left(\frac{Y_{t-2}}{Y_{t-3}}\right)\right)+\epsilon_{\tau_k,t} \tag{11}$$

and similarly for the wage rate tax rate. Then every two quarters a contingent tax rate can be set for each quarter of the year given observations of $d_{t-3}, d_{t-2}, d_{t-1}$ and Y_{t-2}, Y_{t-1}.

To be consistent with this reasoning, the monetary rule (3) should be modified to

$$\begin{aligned}\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right)\\ &+ (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_{t-2}}{\Pi}\right)+\theta_y\log\left(\frac{Y_{t-2}}{Y}\right)+\theta_{dy}\log\left(\frac{Y_{t-2}}{Y_{t-3}}\right)\right)\\ &+ \log(MPS_t),\end{aligned} \tag{12}$$

4 Estimation

We employ standard estimation methods and report results for the following CD and CES production functions in turn.

$$Y_t^W = \left((A_t H_t^d)^\alpha (u_t K_{t-1})^{1-\alpha}\right)^{\rho\theta}\left(K_{t-1}^G\right)^{\rho(1-\theta)} - F_t \tag{13}$$

$$Y_t^W = \left[\theta\left((A_t H_t)^\alpha (K_{t-1})^{1-\alpha}\right)^{\vartheta} + (1-\theta)\left(K_{t-1}^G\right)^{\vartheta}\right]^{\frac{\rho}{\vartheta}} - F_t \tag{14}$$

where (14) becomes (13) as $\vartheta \to 0$ (see Appendix 2.3). We first calibrate the Fixed cost F, such that the zero-profit condition in the wholesale sector is satisfied. We also fix the labour share $\alpha = 0.67$ for the two models. We estimate ρ and θ for production function equation (13) and ρ, θ and ϑ for the production function equation (14). Table 1 reports the Bayesian estimation results for the crucial parameters. Full details of the data, measurement equations and estimation results are provided in Appendices F and G.

Models	Estimated parameters	Log Likelihood	Results
CD equation (13)	ρ, θ, fix $\alpha = 0.67$	4757.5036;	$\rho = 1.3393; \theta = 0.9150$
CES equation (14)	ρ, θ, ϑ, fix $\alpha = 0.67$	4761.1326;	$\rho = 1.3267; \theta = 0.6198; \vartheta = -1.1021$

Table 1: Bayesian Estimation results

Two important conclusions emerge from these results. First, consider the CD limit as $\vartheta \to 0$. Then from Appendix A, the CES production function (2) reduces to the CD form (1) with $\gamma_H = \alpha\rho\theta$, $\gamma_K = (1-\alpha)\rho\theta$ and $\gamma_G = \rho(1-\theta)$. Hence our estimate of the elasticity of output to public capital in the CD case is $\gamma_G = 0.1139$. This is at the upper end of the range of estimates reported by Ramey (2020) of [0.065, 0.12]. We also find strong evidence of increasing returns to scale ($\rho > 1$). For the CES case which is strongly preferred to CD we find the elasticity of substitution between private and public capital, $\sigma = \frac{1}{1-\vartheta} = 0.4757$.[4] As we discussed in Section 2.3, $\sigma < 1$ implies public and private capital are *complements*. Comparing our CES with a CD production functions, we expect complementarity plus increasing returns to scale to favor a high positive impact of public investment on output in both the short-run and long-run. The following Section explores this prediction in our estimated DSGE NK model.

5 The Government Investment Multiplier

This section examines the effect of a 1% permanent increase in the authorized Government Investment - GDP ratio accompanied by welfare-optimized tax and nominal interest rate rules.

5.1 Optimized Tax and Interest Rate Rules (OSR)

We begin by defining the inter-temporal household welfare at time t in recursive Bellman stationarized form in a symmetric equilibrium form as:

$$\Omega_t = U_t(C_t, C_{t-1}, H_t^s) + \beta_g \mathbb{E}_t[\Omega_{t+1}] \tag{15}$$

where β_g is a growth-adjusted discount factor defined by $\beta_g \equiv \beta(1+g)^{1-\sigma}$.

Starting from the estimated steady state, in a perfect foresight equilibrium, welfare-optimal monetary and tax rules time $t = 0$ are obtained by solving the maximization problem:

$$\max_{\rho \in S} \Omega_t(AU_t, \rho) \Rightarrow \rho = \rho(AU_t) \tag{16}$$

where ρ are the feedback parameters in the monetary and fiscal rules and AU_t is authorized investment at time t as defined as follows. **Without time to build (TTB)** public capital

[4] Our paper is not the only one to question CD technology. Cantore et al. (2015) find evidence for CES technology for private capital and labour with a similar low elasticity of substitution. A possible avenue for future research could examine a nested private sector CES production function inside (2).

accumulation is given by:

$$K_t^G = (1 - \delta_G)K_{t-1}^G + I_t^G \tag{17}$$

where I_t^G is the immediately implemented government investment. **With TTB**, following Leeper et al. (2010b), (17) is replaced with

$$K_t^G = (1 - \delta_G)K_{t-1}^G + AU_{t-n} \tag{18}$$

where AU_{t-n} denotes authorized appropriation at time $t - n$ that takes n periods to build and contribute to capital stock. Then implemented government investment at time t is given by

$$G_t^I = \sum_{i=0}^{n-1} \omega_i AU_{t-i} \tag{19}$$

where $\sum_{i=0}^{n-1} \omega_i = 1$. In our calibration we choose $\omega_i = 1/6$ over a six-period interval. For all the exercises in the main text we consider a perfect foresight equilibrium following a **permanent increase** in authorized investment AU_t of 1% as a percentage of GDP alongside the fiscal and monetary rules.

Table 2 reports welfare-optimal rules for the capital and wage rate tax rules (Fiscal Policy - FP, (9) and (10)) and the interest rate rule (Monetary Policy - MP, (3)) for the estimated model with the CES production function. The table shows the estimated rules, the welfare optimized fiscal rules alongside the estimated interest rate rule (regime 1) and then regime 2 with both fiscal and interest rate rules at their welfare optimized values.

First consider the estimated rules. For FP these are significantly different for capital and wage rate rules and indicate high persistence with ρ_{τ_k} and ρ_{τ_w} close to 0.9. Feedback coefficients on output and the change in output are moderate and the rules are dominated by the debt feedback coefficients that stabilize the debt-income ratio. This is broadly consistent with the empirical literature discussed earlier. The estimated MP rule displays moderate persistence and satisfies the Taylor principle with $\theta_\pi > 1$. Feedback on output is negative, but close to zero, but there is a strong response to the change in output. Again, this result is within the bounds of estimated Taylor rules found in the literature.

Now turn to regime 1 which combines an welfare-optimized FP rule with the empirical MP rule. The main change in the FP rule lies in the capital tax response to output which reaches the upper bound we have imposed and the wage rate large response to debt. These together will respond to the demand stimulus that raises the real wage by raising tax revenue, which in turn stabilizes the debt-GDP ratio.

Estimated FP	ρ_{τ_k}	$\theta_{\tau_{k,b}}$	$\theta_{\tau_{k,y}}$	$\theta_{\tau_{k,dy}}$	ρ_{τ_w}	$\theta_{\tau_{w,b}}$	$\theta_{\tau_{w,y}}$	$\theta_{\tau_{w,dy}}$
	0.9264	1.3278	0.1420	0.1528	0.8781	0.3944	0.2027	0.1469
Estimated MP	ρ_r	θ_π	θ_y	θ_{dy}				
	0.3296	1.5168	-0.0058	0.1348	Welfare	-292.3662	CEV	0
Optimized FP	ρ_{τ_k}	$\theta_{\tau_{k,b}}$	$\theta_{\tau_{k,y}}$	$\theta_{\tau_{k,dy}}$	ρ_{τ_w}	$\theta_{\tau_{w,b}}$	$\theta_{\tau_{w,y}}$	$\theta_{\tau_{w,dy}}$
	0.9853	0.1007*	14.9991*	0.0230	0.1971	8.4250	0.0417	0.1260
Estimated MP	ρ_r	θ_π	θ_y	θ_{dy}				
Regime 1	0.3296	1.5168	-0.0058	0.1348	Welfare	-287.5534	CEV_1	3.75%
Optimized FP	ρ_{τ_k}	$\theta_{\tau_{k,b}}$	$\theta_{\tau_{k,y}}$	$\theta_{\tau_{k,dy}}$	ρ_{τ_w}	$\theta_{\tau_{w,b}}$	$\theta_{\tau_{w,y}}$	$\theta_{\tau_{w,dy}}$
	0.9820	0.1615	14.5526	0.8709	0.4922	3.3124	10.0838	1.6104
Optimized MP	ρ_r	θ_π	θ_y	θ_{dy}				
Regime 2	0.5326	8.7006	0.1810	0.3029	Welfare	-287.4334	CEV_2	3.84%

Table 2: OSR with the **CES Production Function**. * means the optimal parameters are at the lower or upper bounds. CE=1.2834

The consumption equivalent measures for the two regimes, CEV_i, i=1,2, are computed as follows. First define the consumption equivalent 1% variation at time t, CE_t, by:

$$CE_t = U_t(1.01C_t, 1.01C_{t-1}, H_t^s) + \Gamma(G_t, I_t^G) - \left[U_t(C_t, C_{t-1}, H_t^s) + \Gamma(G_t, I_t^G)\right] + \beta(1+g)^{(1-\sigma_c)}CE_{t+1}$$

which represents the intertemporal welfare gain when consumption increases permanently by 1%. This is equivalent to:

$$CE_t = \left[1.01^{1-\sigma_c} - 1\right] U_t(C_t, C_{t-1}, H_t^s) + \beta(1+g)^{(1-\sigma_c)}CE_{t+1} \tag{20}$$

For the estimated model with the CES production function CE=1.2834. The consumption equivalent variation (CEV) is then calculated from the table as follows:

$$CEV_i = \frac{\Omega(regime_i) - \Omega(\text{Estimated Rules})}{CE} \tag{21}$$

where CE is the steady state consumption equivalent at the OSR economy. Hence, the CEV_i is the welfare gain(loss) with different fiscal and monetary regimes compared to when the central bank pursues the estimated rules.

From the table, we see a substantial welfare consumption equivalent gain for regime 1 of 3.75% and for regime 2 of 3.84%.

To understand what drives these welfare improvements on the estimated rule, we turn to the impulse responses to the 1% permanent increase in authorized investment as a percentage of GDP.

5.2 The Short-Run and Long-Run effect of an Increase in Authorized Investment: the CES Production Function Case

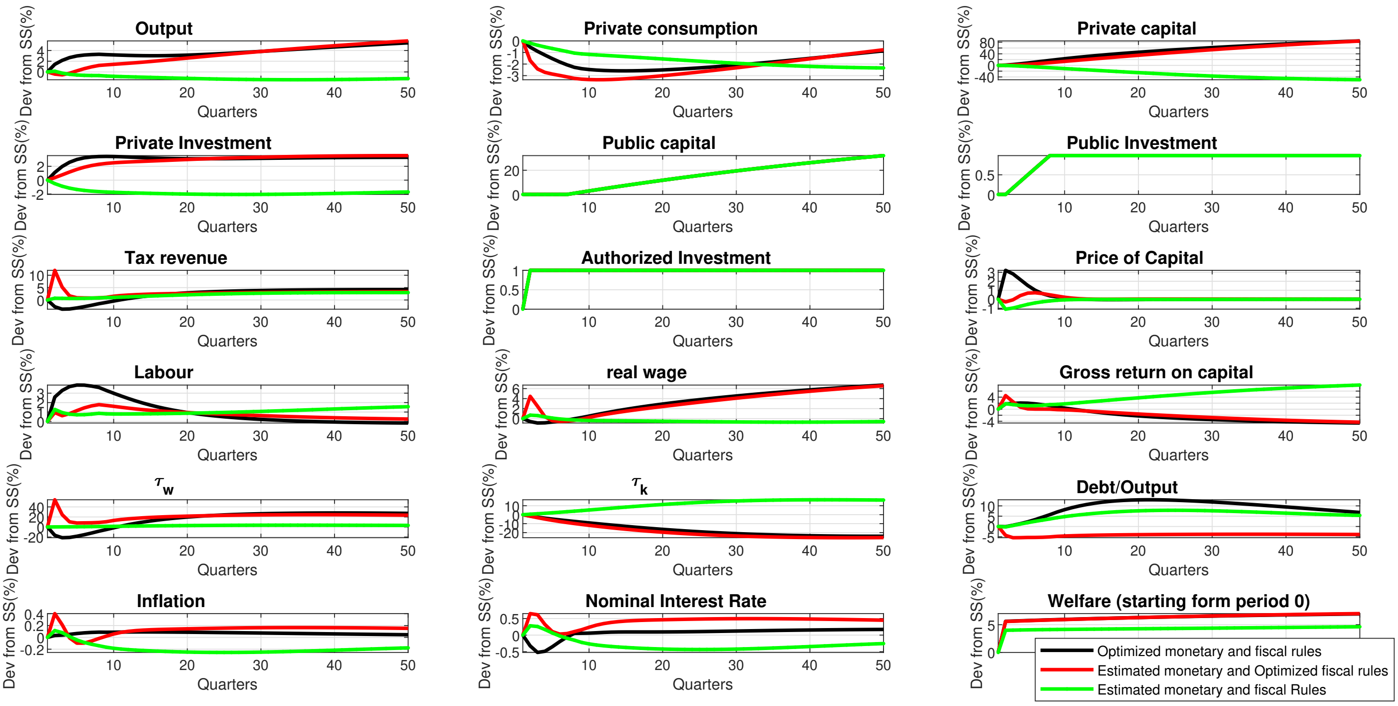

Figure 3: Short-Term Effect of Increases in Authorized Government Investment with the **CES production function**. Comparison between the Optimized simple rules and the estimated rules for $\beta = 0.995$. Rules are in form of equations (9) and (10). Monetary policy is in the implementable form. The **Output, Private consumption, Private capital, Private investment, Public capital, Public investment, Tax revenue, and Authorized Investment** are the deviations from their own steady state as a percentage of steady state output, other variables are the deviations from their own steady state as a percentage of their own steady state.

Figure 3 shows the short-run impact. It compares the impulse responses with three possible combinations of monetary (interest rate) and tax rules: (i) the estimated rules (green); (ii) the estimated monetary rule combined with the optimized simple tax rules (red) and (iii) both monetary and tax optimized rules (black). Figure 4 shows the long-run impact and confirms that all trajectories converge to the same new steady state.

With the estimated rules we see a substantial **crowding out effect** on private investment of the increase in public investment in the short term. Eventually in the very long term, private investment recovers and output rises to a very long-run increase of around 7%. This is facilitated by an increase in household savings brought about by reduced consumption. With the welfare-optimal FP the short-term crowding out disappears and this happens with or without optimal MP. This is one of the main results from Figure 2. It is of note that optimal FP is conducted through an increase in the wage rate but a decrease in the capital rate and it is this latter feature that results in the increase in private investment. The effect of adding optimal MP is to make fiscal consolidation much slower, so that the debt-income ratio is allowed to increase for about 20 quarters and then fall. Then tax revenue falls at first and the real interest rate becomes negative, adding a demand-side push increasing the size of the short-run multiplier.

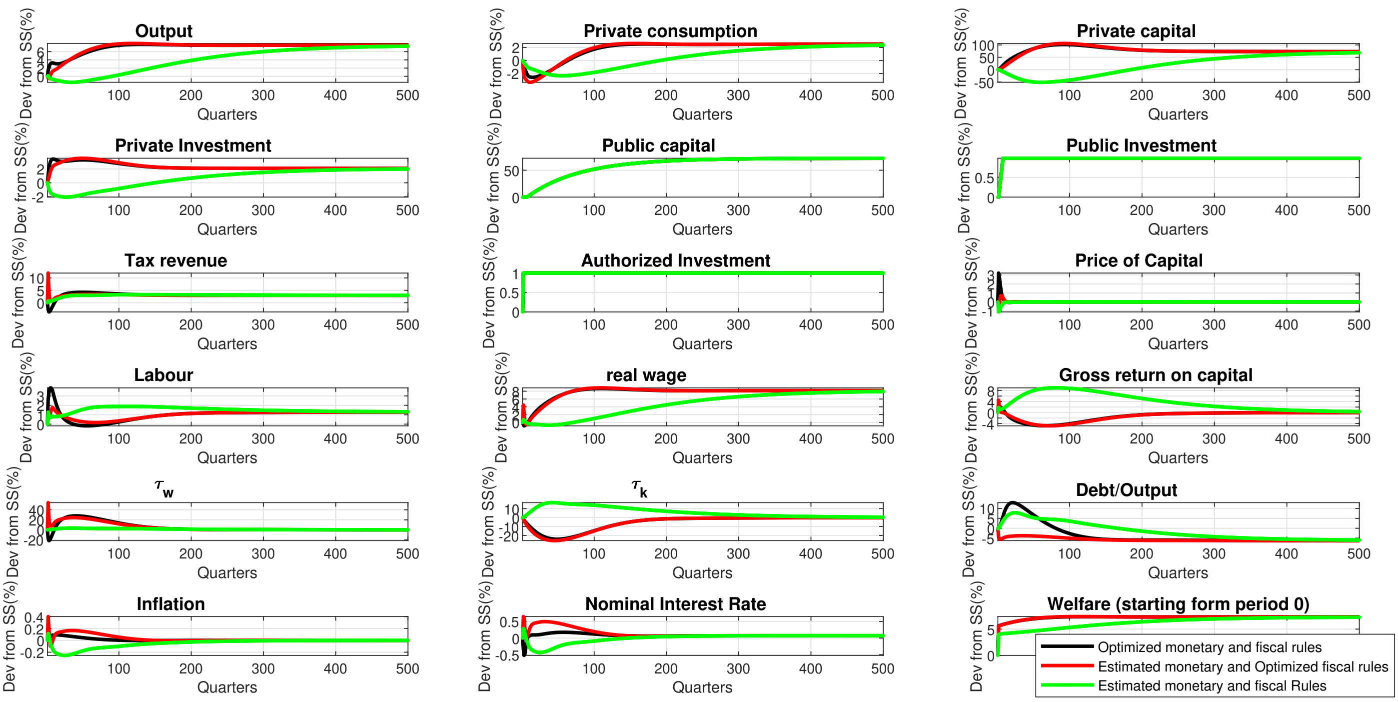

Figure 4: Long-run Effect of Increases in Authorized Government Investment with the **CES production function**. Comparison between the Optimized simple rules and the estimated rules for $\beta = 0.995$. Rules are in form of equations (9) and (10). Monetary policy is in the implementable form. The **Output, Private consumption, Private capital, Private investment, Public capital, Public investment, Tax revenue, and Authorized Investment** are the deviations from their own steady state as a percentage of steady state output, other variables are the deviations from their own steady state as a percentage of their own steady state.

5.3 The Importance of Commitment

Figure 5 examines more closely the initial choice of the nominal interest rate in Figure 3 and how, given the inflation rate and output, it is consistent with the interest rate rule (3). Whereas in Figure 3 the trajectories are deviations from their own steady state as a percentage of their own steady state, in Figure 5 they are the actual state values. It is now clear to see that, at time t=0, the gross nominal interest rate instrument is set at close to 1.007 which is below its steady state of 1.01313; in net terms this is a drop of 0.65%.

Figure 6 repeats this exercise for the wage and capital tax rates. In that case in regime 3 we see an initial fall at t=0 of τ_w to about 0.18 then a rise to 0.3 before falling to its steady state of 0.235. This initial fall then persists through the persistence parameter in the Taylor rule. Persistent falls in both tax rates boost both labour supply and private investment at the expense of a rise in the debt-GDP ratio that peaks after 20 periods before gradually returning its steady state.

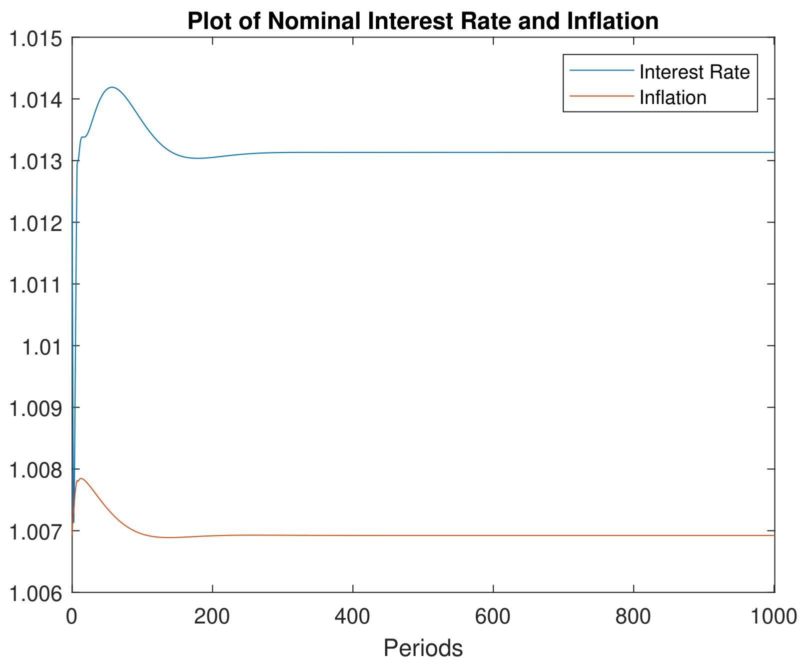

Figure 5: Short-Term Effect of Increases in Authorized Government Investment with the **CES production function**. Focus on the inflation and nominal interest rate changes in regime 2.

Welfare-optimal MP and FP planned trajectories in regime 3, at t=0, then sees an immediate substantial drop in the nominal interest rate and the wage rate. This is time-inconsistent because given the opportunity to re-optimize at a future, even in a perfect foresight equilibrium, would see a repeat of such falls. This highlights the importance of the credibility of commitment.

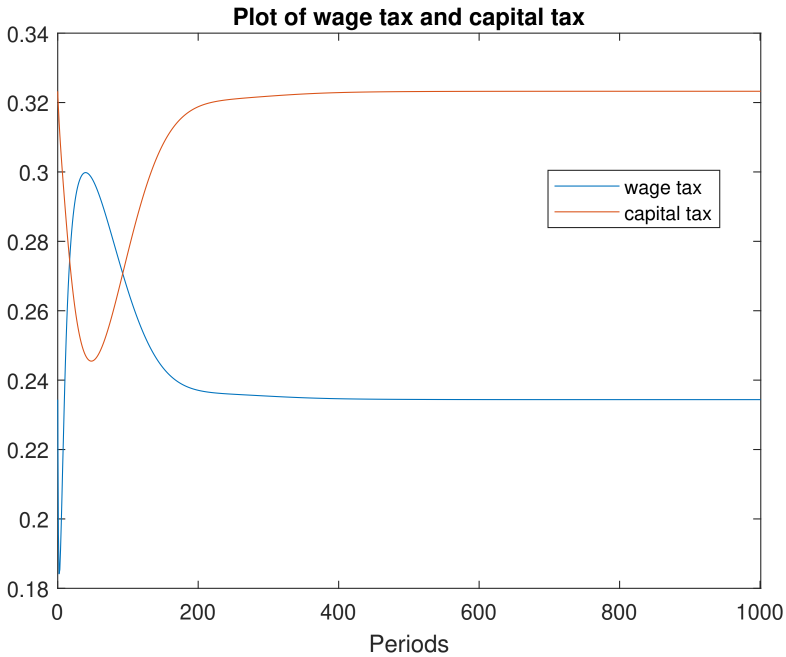

Figure 6: Short-Term Effect of Increases in Authorized Government Investment with the **CES production function**. Focus on the wage and capital tax rates in regime 2.

5.4 CD vs CES Production Function Effect

We now consider the choice of CD or CES production function in the model as discussed in Section 2.3. From the estimation results and Table 1, we found that $\rho = 1.2929$ indicating increasing returns to scale increasing the size of the output multiplier. We also found that $\vartheta = -1.1699$ which implies that the elasticity of substitution $\sigma = \frac{1}{1-\vartheta} < 1$ and public and private investment inputs into output are complements. Public capital then crowds in private investment, leading to high multipliers compared with the CD case with $\vartheta = 0$. Figure 7 confirms these effects.

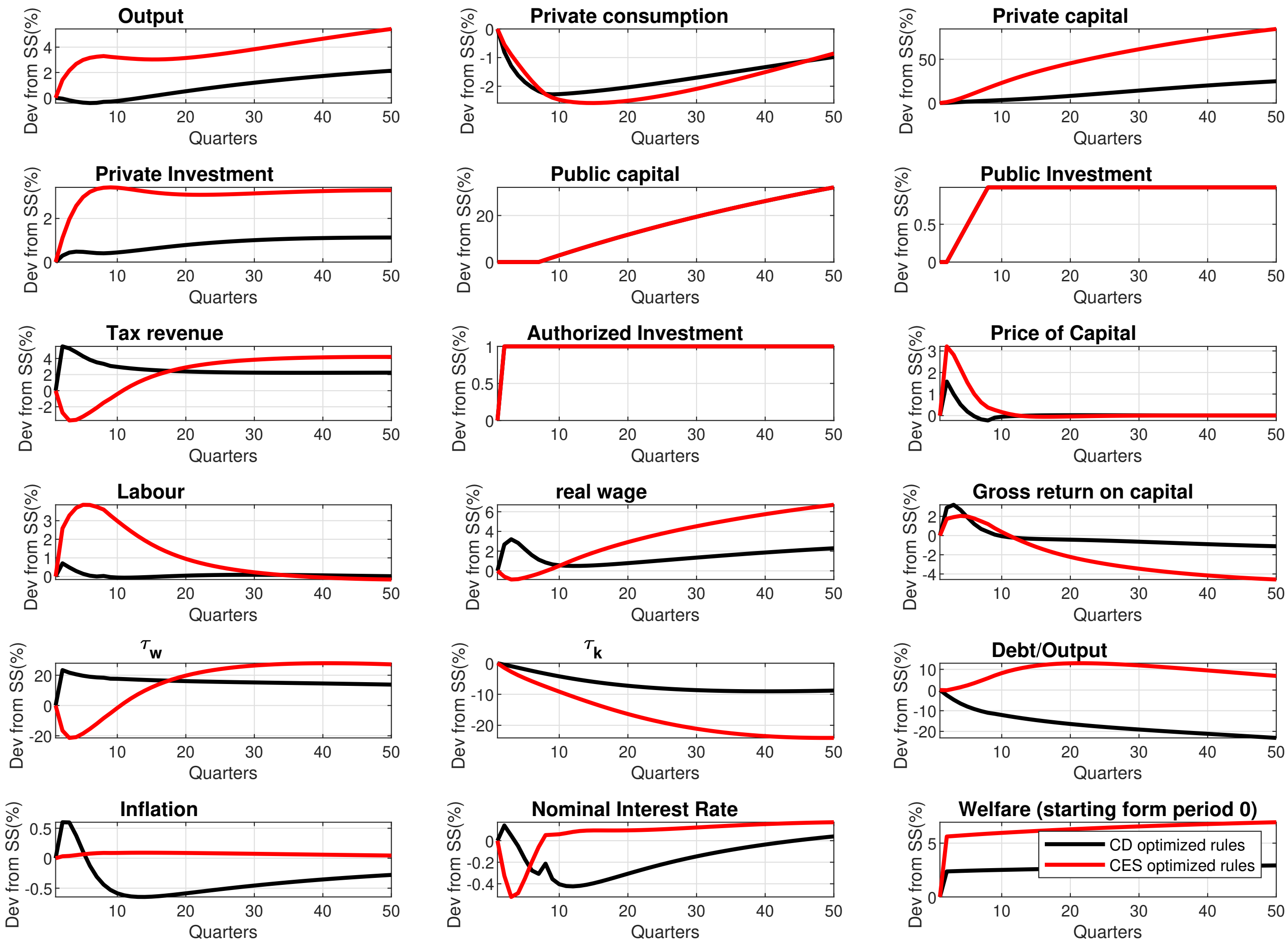

Figure 7: Effect of Increases in Government Investment. Comparison between the **CD and CES production function** for $\beta = 0.995$ and all rules are optimized. Rules are in form of equations (9) and (10). Monetary policy is in the implementable form. The initial value of public investment is equal, the permanent change to public investment is equal to one percent of the output. The **Output, Private consumption, Private capital, Private investment, Public capital, Public investment, Tax revenue, and Authorized Investment** are the deviations from their own steady state as a percentage of steady state output, other variables are the deviations from their own steady state as a percentage of their own steady state.

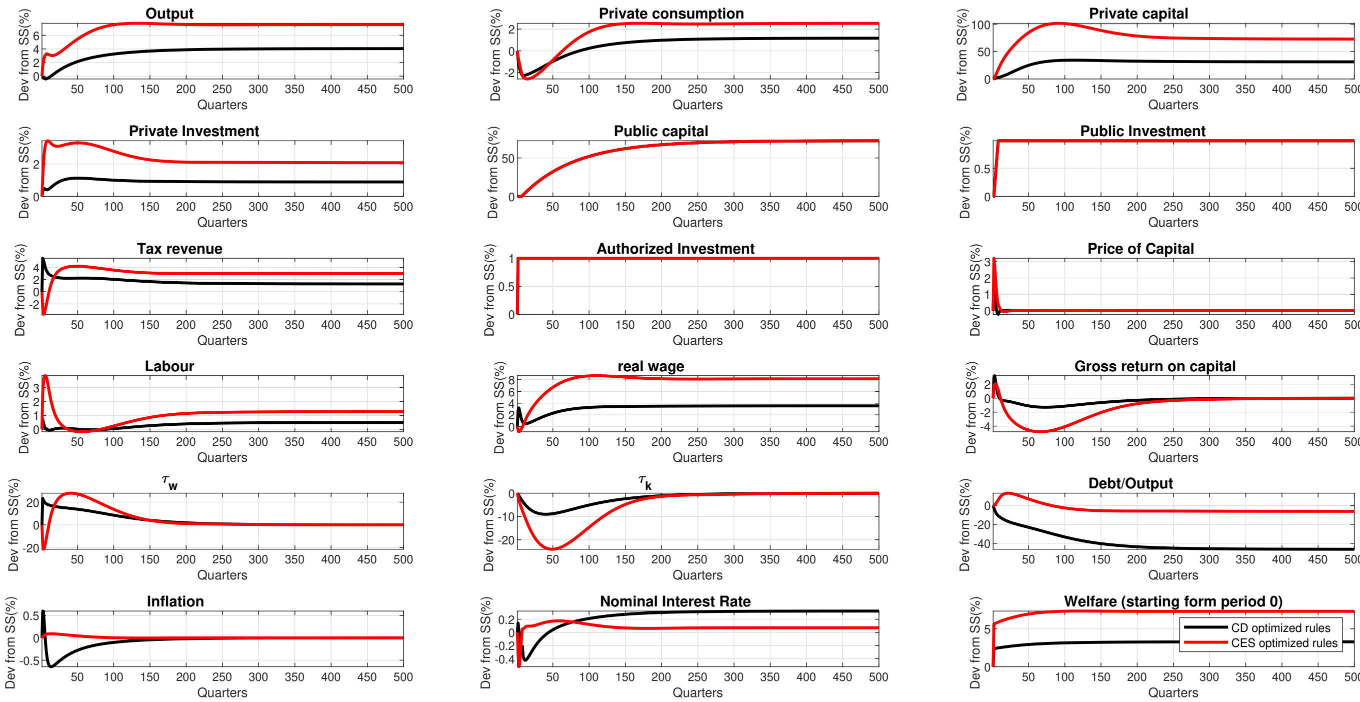

Figure 8: Effect of Increases in Government Investment. Comparison between the CD and CES production function for $\beta = 0.995$ and all rules are optimized. Rules are in form of equations (9) and (10). Monetary policy is in the implementable form. The initial value of public investment is equal, the permanent change to public investment is equal to one percent of the output. . The **Output, Private consumption, Private capital, Private investment, Public capital, Public investment, Tax revenue, and Authorized Investment** are the deviations from their own steady state as a percentage of steady state output, other variables are the deviations from their own steady state as a percentage of their own steady state.

Estimated FP	ρ_{τ_k}	$\theta_{\tau_{k,b}}$	$\theta_{\tau_{k,y}}$	$\theta_{\tau_{k,dy}}$	ρ_{τ_w}	$\theta_{\tau_{w,b}}$	$\theta_{\tau_{w,y}}$	$\theta_{\tau_{w,dy}}$
	0.9272	1.3219	0.1582	0.1422	0.8957	0.5310	0.1868	0.1530
Estimated MP	ρ_r	θ_π	θ_y	θ_{dy}				
	0.3667	2.2469	-0.0058	0.1995	Welfare	-449.85	CEV	0
Optimized FP	ρ_{τ_k}	$\theta_{\tau_{k,b}}$	$\theta_{\tau_{k,y}}$	$\theta_{\tau_{k,dy}}$	ρ_{τ_w}	$\theta_{\tau_{w,b}}$	$\theta_{\tau_{w,y}}$	$\theta_{\tau_{w,dy}}$
	0.9735	0.1000	6.0000	0.0027	0.0002	0.4050	0.9996	3.0602
Optimized MP	ρ_r	θ_π	θ_y	θ_{dy}				
	0.8512	3.3601	0.0215	20.1566	Welfare	-446.215	CEV_2	2.1941%

Table 3: OSR, * means the optimal parameters are at the lower or upper bounds. Estimated rule and OSR welfares for the CD production function. CE=1.53364

5.5 Lagged Fiscal Rules

Finally we examine whether our results still hold if a 2-period lag in the implementation of the fiscal rules is introduced. Table 4 reports the welfare-optimized FP rules for this case and shows that the CEV welfare loss from a lagged response compared to the instantaneous case is small (0.0439%). Figure 9 confirms that the strong multiplier effect of regime 3 still holds but is slightly weaker when there is a 2-period lag between the periods. It also indicates a slower fiscal consolidation of the debt-GDP ratio.

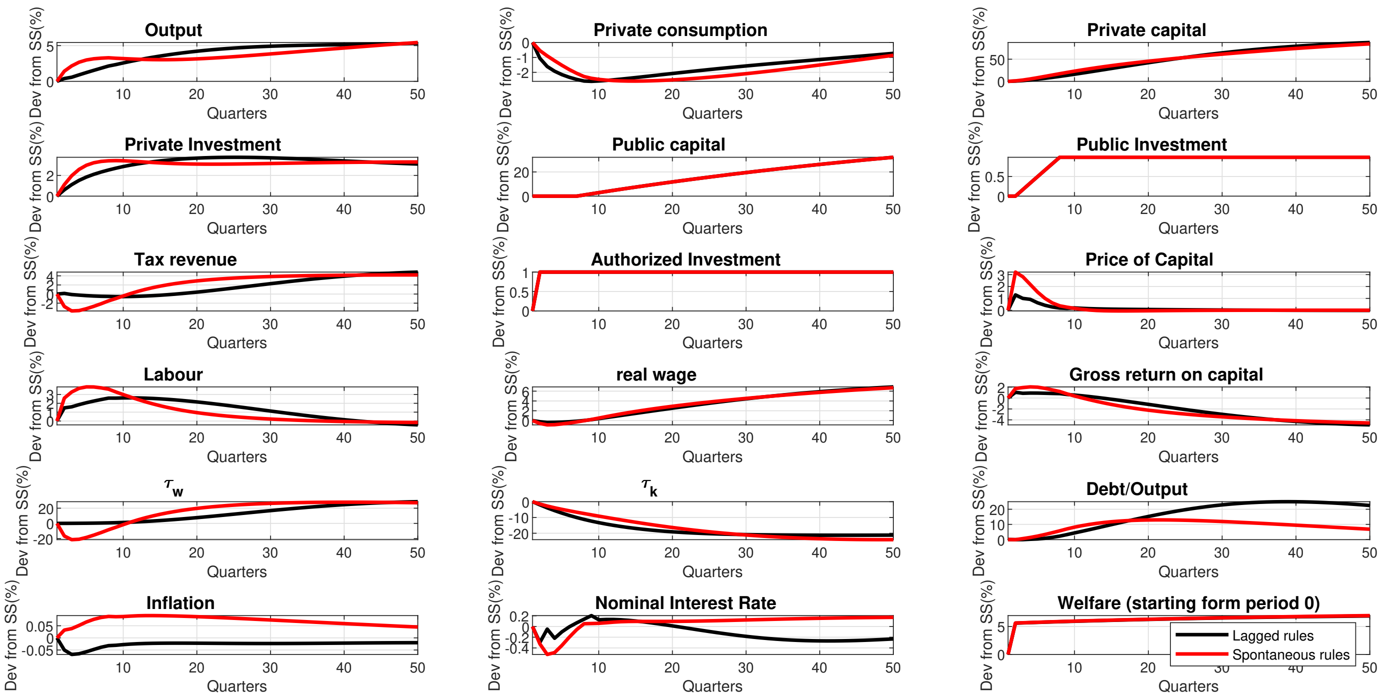

Figure 9: Effect of Increases in Government Investment. Comparison between the 2-lagged rules and the simultaneous rules for $\beta = 0.995$ and all rules are optimized. Rules are in form of equations (9) and (10). Monetary policy is in the implementable form. The initial value of public investment is equal, the permanent change to public investment is equal to one percent of the output. The **Output, Private consumption, Private capital, Private investment, Public capital, Public investment, Tax revenue, and Authorized Investment** are the deviations from their own steady state as a percentage of steady state output, other variables are the deviations from their own steady state as a percentage of their own steady state.

Optimized FP	ρ_{τ_k}	$\theta_{\tau_{k,b}}$	$\theta_{\tau_{k,y}}$	$\theta_{\tau_{k,dy}}$	ρ_{τ_w}	$\theta_{\tau_{w,b}}$	$\theta_{\tau_{w,y}}$	$\theta_{\tau_{w,dy}}$
	0.9766	0.1070	11.9273	0.4571	0.9251	1.0526	0.8493	0.7366
Optimized MP	ρ_r	θ_π	θ_y	θ_{dy}				
Regime 3	0.6061	2.3849	0.0448	1.3619	Welfare	- 287.4997	CEV	0.0439%

Table 4: OSR for the CES Production Function. Lagged Fiscal Rules. CE=1.2834. $CEV = \frac{\Omega(\text{regime 2}) - \Omega(\text{regime 3})}{CE}$

6 Conclusions

We use an estimated medium-scale New Keynesian model that replaces the Cobb–Douglas technology with an empirically supported CES production function to re-assess the strength of the Government investment multiplier. We find strong empirical support for a production function where public and private capital are complements and for increasing returns in the two inputs.

Public-investment expansions with time-to-build constraints are implemented under jointly welfare-maximizing simple monetary and tax rules. The analysis shows how the estimated low elasticity of substitution between private and public capital, increasing returns to scale and optimal fiscal-monetary policy interactions substantially increase both the short-run and long-run productive gains to the government investment fiscal multiplier.

References

Altig, D. et al. (2001). Simulating fundamental tax reform in the united states. *American Economic Review*, 91(3):574–595.

Antràs, P. (2004). Is the u.s. aggregate production function cobb–douglas? *American Economic Review*, 94:257–283.

Aschauer, D. A. (1989). Is public expenditure productive? *Journal of Monetary Economics*, 23(2):177–200.

Auerbach, A. J. and Gorodnichenko, Y. (2012). Measuring the output responses to fiscal policy. *American Economic Journal: Economic Policy*, 4(2):1–27.

Batini, N., Justiniano, A., Levine, P., and Pearlman, J. (2006). Robust Inflation-Forecast-Based Rules to Shield against Indeterminacy. *Journal of Economic Dynamics and Control*, 30:1491–1526.

Baxter, M. and King, R. G. (1993). Fiscal policy in general equilibrium. *American Economic Review*, 83(3):315–334.

Benigno, P. and Woodford, M. (2004). Optimal monetary and fiscal policy: A linear-quadratic approach. *NBER Macroeconomics Annual*, 19:271–333.

Blanchard, O. and Perotti, R. (2002). An empirical characterization of the dynamic effects of changes in government spending and taxes. *Quarterly Journal of Economics*, 117(4):1329–1368.

Bom, P. R. and Ligthart, J. E. (2014a). What have we learned from three decades of research on public capital productivity? *Journal of Economic Surveys*, 28(5):889–916.

Bom, P. R. and Ligthart, J. E. (2014b). What have we learned from three decades of research on the productivity of public capital? *Journal of Economic Surveys*, 28(5):889–916.

Bouakez, H., Guillard, M., and Roulleau-Pasdeloup, J. (2017). Public investment, time to build, and the zero lower bound. *Review of Economic Dynamics*, 23:60–79.

Calvo, G. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics*, 12(3):383–398.

Cantore, C., Levine, P., Pearlman, J., and Yang, B. (2015). CES Technology and Business Cycle Fluctuations. *Journal of Economic Dynamics and Control*, 65(2):133–151.

Chari, V. and Kehoe, P. (1999). Optimal fiscal and monetary policy. *Handbook of Macroeconomics*, 1:1671–1745.

Deak, S., Levine, P., Mirfatah, M., and Swarbrick., J. (2026). Kimball Preferences in the Smets-Wouters NK Model. Forthcoming Discussion Papers, University of Surrey, School of Economics.

Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimal product diversity. *American Economic Review*, 67(3):297–308.

Fernald, J. (1999). Roads to prosperity? assessing the link between public capital and productivity. *American Economic Review*, 89(3):619–638.

Ghaw, N. S. R., Obeng-Osei, R., and Wickstead, T. (2024). Public investment and potential output. Office for Budget Responsibilty, Discussion Paper no. 5.

Jones, J. B. (2002). Has fiscal policy helped stabilize the postwar U.S. economy? *Journal of Monetary Economics*, 49(4):709–746.

Khan, A. and Thomas, J. (2013). Credit shocks and aggregate fluctuations. *International Economic Review*, 54(4):737–755.

Kimball, M. S. (1995). The quantitative analytics of the basic neomonetarist model. *Journal of Money, Credit and Banking*, 27(4):1241–1277.

King, R., Plosser, C., and Rebelo, S. (1988). Production, Growth and Business Cycles I: The basic Neoclassical Model. *Journal of Monetary Economics*, 21:195–231.

Klenow, P. J. and Willis, J. L. (2016). Real Rigidities and Nominal Price Changes. *Economica*, 83(331):443–472.

Kliem, M. and Kriwoluzky, A. (2014). Toward a Taylor Rule for Fiscal Policy. *Review of Economic Dynamics*, 17:294–302.

Klump, R., McAdam, P., and Willman, A. (2007). Factor substitution and factor-augmenting technical change. *Review of Economics and Statistics*, 89(1):183–192.

Koeva, P. (2000). The facts about time-to-build. IMF Working Paper WP/00/138, International Monetary Fund.

Kydland, F. E. and Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6):1345–1370.

Leeper, E. M. (2017). Why central bank independence matters: Fiscal-monetary interactions and macroeconomic performance. *Journal of Economic Perspectives*, 31(4):205–224.

Leeper, E. M., Walker, T. B., and Yang, S.-C. S. (2010a). Government investment and fiscal stimulus. *Journal of Monetary Economics*, 57(8):1000–1012.

Leeper, E. M., Walker, T. B., and Yang, S.-C. S. (2010b). Government investment and fiscal stimulus. *Journal of Monetary Economics*, 57:1000–1012.

Levine, P. and Currie, D. A. (1987). The design of feedback rules in linear stochastic rational expectations models. *Journal of Economic Dynamics and Control*, 11:1–28.

León-Ledesma, M. A., McAdam, P., and Willman, A. (2010). Identifying the elasticity of substitution with biased technical change. *American Economic Review*, 100(4):1330–1357.

Li, J. and Li, R. (2018). Time-to-build, consumption complementarity, and fiscal stimulus. *Economics Letters*, 163:121–125.

Linde, J. and Trabandt, M. (2018). Should we use linearized models to calculate fiscal multipliers? *Journal of Applied Econometrics*, 33(7):937 – 965.

Mountford, A. and Uhlig, H. (2009). What are the effects of fiscal policy shocks? *Journal of Applied Econometrics*, 24(6):960–992.

Munnell, A. H. (1990). Why has productivity growth declined? productivity and public investment. *New England Economic Review*, pages 3–22.

Ramey, V. A. (2020). The macroeconomic consequences of infrastructure investment. *Journal of Economic Perspectives*, 34(2):89–110.

Ramey, V. A. and Zubairy, S. (2018). Government spending multipliers in good times and in bad. *Journal of Political Economy*, 126(2):850–901.

Schmitt-Grohe, S. and Uribe, M. (2007). Optimal Simple and Implementable Monetary and Fiscal Rules. *Journal of Monetary Economics*, 54(6):1702–1725.

Schmitt-Grohé, S. and Uribe, M. (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, 114(2):198–230.

Smets, F. and Wouters, R. (2007). Shocks and Frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review*, 97(3):586–606.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214.

Taylor, J. B. (1999). A Historical Analysis of Monetary Policy Rules. In Taylor, J. B., editor, *Monetary Policy Rules*, pages 319–41. Chicago: University of Chicago Press.

Turnovsky, S. J. (1997). Fiscal policy in a growing economy with public capital. *Journal of Public Economics*, 65(1):117–155.

Appendices

A From the CES to CD Production Function

We now show that as $\vartheta \to 0$, (2) becomes (1) with $\gamma_H = \alpha\rho\theta$, $\gamma_K = (1-\alpha)\rho\theta$ and $\gamma_G = \rho(1-\theta)$. Taking the limit of Y_t^W and using the fact that exp and ln are inverse functions we have

$$
\begin{aligned}
Y_t^W &= \lim_{\vartheta\to 0}\left[\theta\left((A_tH_t^d)^{\alpha}(u_tK_{t-1})^{1-\alpha}\right)^{\vartheta} + (1-\theta)\left(K_{t-1}^G\right)^{\vartheta}\right]^{\frac{\rho}{\vartheta}} \\
&= \exp\left\{\lim_{\vartheta\to 0}\frac{\rho}{\vartheta}\ln\left[\theta\left((A_tH_t^d)^{\alpha}(u_tK_{t-1})^{1-\alpha}\right)^{\vartheta} + (1-\theta)\left(K_{t-1}^G\right)^{\vartheta}\right]\right\} \\
&= \exp\left\{\rho\lim_{\vartheta\to 0}\frac{\theta\ln\left((A_tH_t^d)^{\alpha}(u_tK_{t-1})^{1-\alpha}\right)\left((A_tH_t^d)^{\alpha}(u_tK_{t-1})^{1-\alpha}\right)^{\vartheta} + (1-\theta)\ln\left(K_{t-1}^G\right)\left(K_{t-1}^G\right)^{\vartheta}}{\left[\theta\left((A_tH_t^d)^{\alpha}(u_tK_{t-1})^{1-\alpha}\right)^{\vartheta} + (1-\theta)\left(K_{t-1}^G\right)^{\vartheta}\right]}\right\} \\
&= \exp\left\{\rho\frac{\theta\ln\left((A_tH_t^d)^{\alpha}(u_tK_{t-1})^{1-\alpha}\right) + (1-\theta)\ln\left(K_{t-1}^G\right)}{[\theta + (1-\theta)]}\right\} \\
&= \exp\left\{\frac{\ln\left((A_tH_t^d)^{\alpha}(u_tK_{t-1})^{1-\alpha}\right)^{\rho\theta} + \ln\left(K_{t-1}^G\right)^{\rho(1-\theta)}}{[\theta + (1-\theta)]}\right\} \\
&= \exp\left\{\ln\left((A_tH_t^d)^{\alpha}(u_tK_{t-1})^{1-\alpha}\right)^{\rho\theta} + \ln\left(K_{t-1}^G\right)^{\rho(1-\theta)}\right\} \\
&= \left((A_tH_t^d)^{\alpha}(u_tK_{t-1})^{1-\alpha}\right)^{\rho\theta}\left(K_{t-1}^G\right)^{\rho(1-\theta)}
\end{aligned}
\tag{A.1}
$$

where from step 2 to step 3, we use the L'Hopital's rule for the $\frac{0}{0}$ limit.

B The Model

B.1 Households

At time $t = 0$, household i maximizes its expected lifetime utility

$$
\begin{aligned}
\Omega_0(i) &= \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[U_t(C_t(i), C_{t-1}(i), H_t^s(i) + \Gamma(G_t^B, K_t^B)\right] \\
&= \mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{[C_t(i) - \chi C_{t-1}(i)]^{1-\sigma_c}}{1-\sigma_c}\exp\left[(\sigma_c - 1)\frac{H_t^s(i)^{1+\psi}}{1+\psi}\right] + \Gamma(G_t^B, K_t^B)\right]\right\}
\end{aligned}
\tag{B.2}
$$

where $\mathbb{E}_t[\cdot]$ denotes rational expectations based on information available at time t, $C_t(i)$ is real consumption, $H_t^s(i)$ is hours supplied, β is the discount factor, χ controls for internal habit formation, σ_c is the inverse of the elasticity of inter-temporal substitution (for constant labour), and ψ is the inverse of the Frisch labour supply elasticity. Preferences chosen by SW in (B.2) are compatible with balanced growth (see King et al. (1988)).

We write the **nominal** household budget constraint in terms of the bond holdings, $B_t(i)$, the price P_t^B of a one-period bond issued at time t that pays one unit of currency in the next period as follows: The household's budget constraint in period t is given by

$$\begin{aligned} P_t(C_t(i)+I_t(i))+P_t^B B_t(i) &= B_{t-1}(i)+P_t\Big(\left((1-\tau_{k,t})r_t^K u_t(i)-a(u_t(i))\right)K_{t-1}(i) \\ &+ (1-\tau_{w,t})W_{h,t}H_t^s(i)+Z_t(i)+(1-\tau_{k,t})\Gamma_t\Big) \end{aligned} \quad \text{(B.3)}$$

where C_t is consumption, I_t is investment, K_t is end of period t capital stock, r_t^K is the real rental rate, $\tau_{k,t}$ is the tax rate on capital return, u_t is the utilization rate of capital, $a(u_t(i))$ is the physical cost of use of capital in consumption terms, $W_{h,t}$ is the real wage rate at which households supply labour that is homogeneous at this point to trade unions, $\tau_{w,t}$ is the tax rate on labour income, Z_t is a real tax transfer and Γ_t is the real profit of retail firms distributed to households[5].

Now put the price of the bond $P_t = \frac{1}{R_{n,t}}$ where $R_{n,t}$ is the nominal interest rate and define the real value of bond holdings by $D_t \equiv \frac{P_t^B B_t}{P_t}$. Then noting that $\frac{B_{t-1}}{P_t} = \frac{R_{n,t-1}D_{t-1}}{\Pi_t} = R_{t-1}D_{t-1}$, where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the inflation rate and $R_{t-1} = \frac{R_{n,t-1}}{\Pi_t}$ is the ex post real interest rate, the **real** household budget constraint can be written:

$$\begin{aligned} C_t(i)+I_t(i)+D_t(i) &= RP_{t-1}R_{t-1}D_{t-1}(i)+\left((1-\tau_{k,t})r_t^K u_t(i)-a(u_t(i))\right)K_{t-1}(i) \\ &+ (1-\tau_{w,t})W_{h,t}H_t^s(i)+Z_t(i)+(1-\tau_{k,t})\Gamma_t \end{aligned} \quad \text{(B.4)}$$

End of period capital stock, $K_t(i)$, accumulates according to

$$K_t(i) = (1-\delta)K_{t-1}(i)+(1-S(X_t(i)))I_t(i)\mathrm{IS}_t \quad \text{(B.5)}$$

where IS_t is an investment specific technological shock that follows an AR1 process, $X_t(i) = I_t(i)/I_{t-1}(i)$ is the growth rate of investment, and $S(\cdot)$ is an adjustment cost

[5] Γ_t is defined in Section B.5.

function such that $S(X) = 0$, $S'(X) = 0$, and $S''(\cdot) = 0$ where X is the steady state value of investment growth. For $S(X_t)$ in a symmetric equilibrium we choose the functional form: $S(X_t) = \phi_X(X_t - \bar{X}_t)^2$ where $\bar{X}_t$ is the balanced-growth steady-state trend. For $a(u_t)$ we choose the functional form: $a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2$ with $u_t = u = 1$ in the steady state.

Then the household first-order conditions consist of an Euler Consumption equation, an arbitrage condition, a first order condition equating the marginal rate of substitution between leisure and consumption with the real wage and first order conditions for investment, the price of capital Q_t (Tobin's Q) and capacity utilization:

$$\mathbb{E}_t[\Lambda_{t,t+1}(i)RP_{t+1}R_{t+1}] = 1 \tag{B.6}$$

$$\mathbb{E}_t[\Lambda_{t,t+1}(i)R^K_{t+1}] = 1 \tag{B.7}$$

$$-\frac{U_{H,t}(i)}{U_{C,t}(i)} = (1 - \tau_{w,t})W_{h,t} \tag{B.8}$$

$$Q_t = \mathbb{E}_t\left\{\Lambda_{t,t+1}(i)\left[(1 - \tau_{k,t+1})r^K_{t+1}u_{t+1}(i) - a(u_{t+1})(i) + Q_{t+1}(1 - \delta)\right]\right\} \tag{B.9}$$

$$\begin{aligned} 1 &= Q_t\left[1 - S(X_t(i)) - S'(X_t)(i)X_t(i)\right]IS_t \\ &+ \mathbb{E}_t\left[\Lambda_{t,t+1}Q_{t+1}S'(X_{t+1}(i))X_{t+1}(i)^2 IS_{t+1}\right] \end{aligned} \tag{B.10}$$

$$(1 - \tau_{k,t})r^K_t = a'(u_t(i)) \tag{B.11}$$

where $\Lambda_{t,t+1}(i) \equiv \beta\frac{U_{C,t+1}(i)}{U_{C,t}(i)}$ is the stochastic discount factor, $U_{C,t}(i) \equiv \frac{\partial U_t(i)}{\partial C_t(i)}$, $U_{H,t}(i) \equiv \frac{\partial U_t(i)}{\partial H_t(i)}$ are marginal utilities over two successive periods in the summation given by:

$$U_{C,t}(i) = (1 - \sigma_c)\left(\frac{U_t(i)}{C_t(i) - \chi C_{t-1}(i)} - \frac{\beta\chi U_{t+1}(i)}{C_{t+1}(i) - \chi C_t(i)}\right) \tag{B.12}$$

where $R^K_t = \frac{[(1-\tau_{k,t})r^K_t u_t - a(u_t) + (1-\delta)Q_t]}{Q_{t-1}}$ is the real gross returns on physical capital and Q_t is the price of capital (Tobin's Q). In a symmetric equilibrium of identical households $C_t(i) = C_t$, $H^s_t(i) = H^s_t$ etc.

B.2 The Labour Market

Households supply their homogeneous labour to trade unions that differentiate the labour services. A labour packer buys the differentiated labour from the trade unions and aggregate them into a composite labour using the Dixit-Stigliz aggregator[6] given aggregate demand H_t^d.

$$H_t^d = \left(\int_0^1 H_t(j)^{(\zeta_w-1)/\zeta_w} dj\right)^{\zeta_w/(\zeta_w-1)} dj \tag{B.13}$$

where ζ_w is the elasticity of substitution among different types of labour, and we index trade unions by j. The labour packer minimizes the cost $\int_0^1 W_{n,t}(j)H_t(j)dj$ of producing the composite labour service, where $W_{n,t}(j)$ denotes the nominal wage set by union j. This leads to the standard demand function

$$H_t(j) = \left(\frac{W_{n,t}(j)}{W_{n,t}}\right)^{-\zeta_w} H_t^d \tag{B.14}$$

where $W_{n,t}$ is the aggregate nominal wage given by the Dixit-Stigliz aggregator

$$W_{n,t} = \left[\int_0^1 W_{n,t}(j)^{1-\zeta_w} dj\right]^{\frac{1}{1-\zeta_w}}$$

Sticky wages are introduced through Calvo contracts supplemented with indexation. At each period there is a probability $1-\xi_w$ that trade union j can choose $W_{n,t}^O(j)$ to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} H_{t+k}(j) \left[\frac{W_{n,t}^O(j)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w} - W_{h,t+k}\right]$$

subject to the demand function (B.14), where $\gamma_w \in [0,1]$ is a wage indexation parameter.

The solution to the above problem is the first-order condition

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} H_{t+k}(j) \left[\frac{W_{n,t}^O(j)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w} - \frac{\text{MRSS}_{t+k}}{(1-1/\zeta_w)} W_{h,t+k}\right] = 0$$

where we have introduced a mark-up shock MRSS_t to the marginal rate of substitution that follows an AR1 process. This leads to

$$\frac{W_{n,t}^O(j)}{W_{n,t}} = \frac{\frac{1}{1-1/\zeta_w}\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} H_{t+k}(j) W_{h,t+k} \text{MRSS}_{t+k}}{W_{n,t}\mathbb{E}_t \sum_{k=0}^{\infty} \xi_w^k \Lambda_{t,t+k} H_{t+k}(j) \frac{1}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_w}}$$

where $\frac{W_{n,t}^O(j)}{W_{n,t}} = \frac{W_{n,t}^O}{W_{n,t}}$ in a symmetric equilibrium.

[6]See Dixit and Stiglitz (1977).Smets and Wouters (2007) generalize the aggregator to a Kimball form as in Kimball (1995) which introduces a variable mark-up even in the absence of wage stickiness. But as Klenow and Willis (2016) argues a significant difference between the two aggregators only emerges if one calibrates the model using an implausibly high price super-elasticity. See also Deak et al. (2026).

By the law of large numbers the evolution of the aggregate wage is given by

$$W_{n,t}^{1-\zeta_w} = \xi_w \left(W_{n,t-1}\Pi_{t-1}^{\gamma_w}\right)^{1-\zeta_w} + (1-\xi_w)(W_{n,t}^O)^{1-\zeta_w}$$

which can be written as

$$1 = \xi_w \left(\frac{\Pi_{t-1}^{\gamma_w}}{\Pi_t^w}\right)^{1-\zeta_w} + (1-\xi_w)\left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{1-\zeta_w} \tag{B.15}$$

Wage dispersion is defined as $\Delta_{w,t} = \int (W_{n,t}(j)/W_{n,t})^{-\zeta_w} dj$. Assuming that the number of trade unions is large, we obtain the following dynamic relationship:

$$\begin{aligned}\Delta_{w,t} &= \xi_w \int_{not\ optimize} \left(\frac{W_{n,t-1}^O(j)\Pi_{t-1}^{\gamma_w}}{W_{n,t}}\right)^{-\zeta_w} dj + (1-\xi_w)\int_{optimize}\left(\frac{W_{n,t}^O(j)}{W_{n,t}}\right)^{-\zeta_w} dj \\ &= \xi_w \frac{(\Pi_t^w)^{\zeta_w}}{\Pi_{t-1}^{\zeta_w\gamma_w}}\Delta_{w,t-1} + (1-\xi_w)\left(\frac{W_{n,t}^O(j)}{W_{n,t}}\right)^{-\zeta_w}\end{aligned}$$

B.3 Firms in the Retail Sector

The retail sector uses a homogeneous wholesale good to produce a basket of differentiated goods for aggregate consumption

$$C_t = \left(\int_0^1 C_t(m)^{(\zeta_p-1)/\zeta_p} dm\right)^{\zeta_p/(\zeta_p-1)} \tag{B.16}$$

where ζ_p is the elasticity of substitution. For each m, the consumer chooses $C_t(m)$ at a price $P_t(m)$ to maximize (B.16) given total expenditure $\int_0^1 P_t(m)C_t(m)dm$. This results in a set of consumption demand equations for each differentiated good m with price $P_t(m)$ of the form

$$C_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta_p} C_t \Rightarrow Y_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta_p} Y_t$$

where $P_t = \left[\int_0^1 P_t(m)^{1-\zeta_p} dm\right]^{\frac{1}{1-\zeta_p}}$. P_t is the aggregate price index. C_t and P_t are Dixit-Stigliz aggregates – see Dixit and Stiglitz (1977).

Following Calvo (1983), we now assume that there is a probability of $1-\xi_p$ at each period that the price of each retail good m is set optimally to $P_t^0(m)$. If the price is not re-optimized, then prices are indexed to last period's aggregate inflation, with indexation parameter γ_p. With indexation parameter $\gamma_p \geq 0$, this implies that successive prices with no re-optimization are given by $P_t^0(f)$, $P_t^0(f)\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_p}$, $P_t^0(f)\left(\frac{P_{t+1}}{P_{t-1}}\right)^{\gamma_p}$, ... For each retail producer m, given its real marginal cost (the inverse of the price mark-up)

$$MC_t = \frac{P_t^W}{P_t},$$

the objective is at time t to choose $\{P_t^O(m)\}$ to maximize discounted profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} Y_{t+k}(m) \left[\frac{P_t^O(m)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_p} - MC_{t+k} \right]$$

subject to

$$Y_{t+k}(m) = \left[\frac{P_t^O(m)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_p} \right]^{-\zeta_p} Y_t$$

The solution to this is

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} Y_{t+k}(m) \left[\frac{P_t^O(m)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\gamma_p} - \frac{1}{(1-1/\zeta_p)} MC_{t+k} \right] = 0$$

which leads to

$$\frac{P_t^O(m)}{P_t} = \frac{\frac{1}{1-1/\zeta_p} \mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} \frac{\left(\frac{P_{t+k}}{P_t}\right)^{\zeta_p}}{\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_p \zeta_p}} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi_p^k \Lambda_{t,t+k} \frac{\left(\frac{P_{t+k}}{P_t}\right)^{\zeta_p - 1}}{\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma_p(\zeta_p - 1)}} Y_{t+k}}$$

where $\frac{P_t^O(m)}{P_t} = \frac{P_t^O}{P_t}$ in a symmetric equilibrium.

By the law of large numbers the evolution of the price index is given by

$$P_t^{1-\zeta_p} = \xi_p \left(P_{t-1} \Pi_{t-1}^{\gamma_p} \right)^{1-\zeta_p} + (1-\xi_p)(P_t^O(m))^{1-\zeta_p}$$

which can be written as

$$1 = \xi_p \left(\frac{\Pi_{t-1}^{\gamma_p}}{\Pi_t} \right)^{1-\zeta_p} + (1-\xi_p) \left(\frac{P_t^O(m)}{P_t} \right)^{1-\zeta_p}$$

Price dispersion is defined as $\Delta_{p,t} = \int (P_t(m)/P_t)^{-\zeta_p} dm$. Assuming that the number of firms is large, we obtain the following dynamic relationship:

$$\begin{aligned}\Delta_{p,t} &= \xi_p \int_{not\ optimize} \left(\frac{P^O_{t-1}(m)\Pi^{\gamma_p}_{t-1}}{P_t}\right)^{-\zeta_p} dm + (1-\xi_w)\int_{optimize}\left(\frac{P^O_t(m)}{P_t}\right)^{-\zeta_p} dm \\ &= \xi_p \frac{\Pi_t^{\zeta_p}}{\Pi_{t-1}^{\zeta_p\gamma_p}}\Delta_{p,t-1} + (1-\xi_p)\left(\frac{P^O_t(m)}{P_t}\right)^{-\zeta_p}\end{aligned}$$

B.4 Firms in the Wholesale Sector

Wholesale firms employ a CES production function to produce a homogeneous output:

$$\begin{aligned}Y_t^W &= F\left(A_t, H_t^d, u_t K_{t-1}, K_{t-1}^G\right) \\ &= \left[\theta\left((A_t H_t^d)^\alpha (u_t K_{t-1})^{1-\alpha}\right)^\vartheta + (1-\theta)\left(K_{t-1}^G\right)^\vartheta\right]^{\frac{\rho}{\vartheta}} - F_t \end{aligned} \tag{B.17}$$

where F_t are exogenous fixed costs growing in a balanced-growth steady in line with the other real variables. θ is the share parameter of the private final output, ϑ is the degree of substitutability of the private finals and public capitals and ρ is the degree of homogeneity, with CRS if $\rho = 1$, IRS if $\rho > 1$ and DRS if $\rho < 1$.

Where we still assume the private sector output is a Cobb-Douglas production function which is empirically consistent with data. In the estimation, we shall fix the share on the effective labour input $\alpha = 2/3$, and estimating θ, ϑ and ρ.

Profit-maximizing demand for factors results in the first order conditions

$$\begin{aligned}W_t \equiv \frac{W_{n,t}}{P_t} &= \rho\alpha\frac{P_t^W}{P_t}\left[Y_t^W + F_t\right]^{\left(1-\frac{\vartheta}{\rho}\right)} \frac{\theta\left((A_t H_t^d)^\alpha (u_t K_{t-1})^{1-\alpha}\right)^\vartheta}{H_t^d} \\ r_t^K &= \rho(1-\alpha)\frac{P_t^W}{P_t}\left[Y_t^W + F_t\right]^{\left(1-\frac{\vartheta}{\rho}\right)} \frac{\theta\left((A_t H_t^d)^\alpha (u_t K_{t-1})^{1-\alpha}\right)^\vartheta}{u_t K_{t-1}}\end{aligned}$$

Note that firms take the public capital K_{t-1}^G as given. Public capital accumulation is as follows, first **without time to build (TTB)**:

$$K_t^G = (1-\delta_G)K_{t-1}^G + I_t^G \tag{B.18}$$

Where I_t^G is the government investment. **With TTB**, following Leeper et al. (2010b) (17) is replaced with

$$K_t^G = (1-\delta_G)K_{t-1}^G + AU_{t-n} \tag{B.19}$$

where AU_{t-n} denotes authorized appropriation at time $t-n$ that takes n periods to build and contribute to capital stock. Then implemented government investment at time t is given by

$$G_t^I = \sum_{i=0}^{n-1} \omega_i AU_{t-i} \tag{B.20}$$

where $\sum_{i=0}^{n-1} \omega_i = 1$.

B.5 Output Equilibrium

As with consumption goods, the demand equations for each differentiated good m with price $P_t(m)$ forming aggregate investment and public services takes the form

$$I_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} I_t\,; \quad I_t^G(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} I_t\,; \quad G_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta} G_t \tag{B.21}$$

Hence equilibrium for good m gives

$$Y_t^W(m) - F(m) = (C_t + I_t + I_t^G + G_t + a(u_t)K_{t-1}) \int_0^1 \left(\frac{P_t(m)}{P_t}\right)^{-\zeta_p} dm \tag{B.22}$$

where $F(m)$ is the fixed cost of converting wholesale good m into retail output. Integrating over m we arrive at the aggregate output equilbrium

$$Y_t^W - F = Y_t\,\Delta_{p,t} \tag{B.23}$$

We can now pin down fixed cost from imposing a steady state zero profit outcome in the retail sector arising from free entry. Nominal retail profits are given by

$$P_t\Gamma_t \equiv P_tY_t - P_t^W Y_t^W = P_t\left(\frac{Y_t^W - F_t}{\Delta_p}\right) - P_t^W Y_t^W \tag{B.24}$$

Defining $f \equiv \frac{F}{Y_t^W}$, this gives the free-entry zero profit condition in the steady state as

$$f = 1 - \Delta_p \frac{P_t^W}{P_t} = 1 - \Delta_p\left(1 - \frac{1}{\zeta_p}\right) \tag{B.25}$$

Noting that $\frac{P_t^W}{P_t}$ is the real marginal cost $MC_t = \left(1 - \frac{1}{\xi_p}\right)$. For a zero net inflation steady state, $\Delta_p = 1$ so with our prior $\zeta_p = 7$ this gives the markup=1/MC=1.1667 (about 17%) and $f = 0.17$.

C The Stationary Equilibrium

We now show how to obtain a stationary equilibrium around the trend $Z_t(\bar{Z}_t)$.

$$\frac{Y_t^W}{Z_t} = \left[\theta\left((A_t H_t^d)^\alpha \left(u_t \frac{K_{t-1}}{Z_{t-1}}\frac{Z_{t-1}}{Z_t}\right)^{1-\alpha}\right)^\vartheta \frac{1}{Z_t^{\frac{\vartheta}{\rho}-(1-\alpha)\vartheta}} + (1-\theta)\left(A_t^G \frac{K_{t-1}^G}{Z_{t-1}}\frac{Z_{t-1}}{Z_t}\right)^\vartheta \frac{1}{Z_t^{\frac{\vartheta}{\rho}-\vartheta}}\right]^{\frac{\rho}{\vartheta}} - \frac{F_t}{Z_t} \tag{C.1}$$

$$\frac{W_t}{Z_t} = \rho\alpha \frac{P_t^W}{P_t}\left[\frac{Y_t^W(1+F_t)}{Z_t}\right]^{\left(1-\frac{\vartheta}{\rho}\right)} \frac{\theta\left((A_t H_t^d)^\alpha (u_t \frac{K_{t-1}}{Z_{t-1}}\frac{Z_{t-1}}{Z_t})^{1-\alpha}\right)^\vartheta}{H_t^d}\frac{1}{Z_t^{\frac{\vartheta}{\rho}-\vartheta}} \tag{C.2}$$

$$r_t^K = \rho(1-\alpha)\frac{P_t^W}{P_t}\left[\frac{Y_t^W(1+F_t)}{Z_t}\right]^{\left(1-\frac{\vartheta}{\rho}\right)} \frac{\theta\left((A_t H_t^d)^\alpha (u_t \frac{K_{t-1}}{Z_{t-1}}\frac{Z_{t-1}}{Z_t})^{1-\alpha}\right)^\vartheta}{u_t \frac{K_{t-1}}{Z_{t-1}}\frac{Z_{t-1}}{Z_t}}\frac{1}{Z_t^{\frac{\vartheta}{\rho}-\vartheta}} \tag{C.3}$$

where

$$A_t = A_t^c \bar{A}_t \tag{C.4}$$

$$\bar{A}_t = (1+g_A)\bar{A}_{t-1} \tag{C.5}$$

$$Z_t = \bar{Z}_t = \bar{Z}_{t-1}(1+g_Z) \tag{C.6}$$

$$A_t^G = \bar{A}_t^G = \bar{A}_{t-1}^G(1+g_{A^G}) \tag{C.7}$$

When we have CRS or $\rho = 1$ then $Z_t = \bar{A}_t$. For the general value of $\rho \neq 1$, as we will see a balanced growth steady state needs some form of exogenous shock to the public capital in the production function, assuming it is denoted A_t^G. Estimating both A_t^G and A_t would create the singularity problem in the estimation. Therefore, we fix A_t^G at the steady state, and using the shock to Authorised Investment as in Ramey and Leeper. This is given by

$$\log AP_t - \log AP = \rho_{AU}(\log AP_{t-1} - \log AP) + \epsilon_{AU,t} \tag{C.8}$$

First putting the term $\frac{1}{Z_t^{\frac{\vartheta}{\rho}-(1-\alpha)\vartheta}}$ inside the private production function where A_t is positioned. Then in (C.1)- (C.3), we have

$$A_t = \bar{A}_t A_t^c \rightarrow \bar{A}_t A_t^c \frac{1}{Z_t^{\frac{1}{\alpha}\left(\frac{1}{\rho}-(1-\alpha)\right)}} = A_t^c \frac{\bar{A}_t}{\bar{A}_{t-1}} \left(\frac{Z_{t-1}}{Z_t}\right)^{\frac{1}{\alpha}\left(\frac{1}{\rho}-(1-\alpha)\right)} \bar{A}_{t-1} Z_{t-1}^{-\frac{1}{\alpha}\left(\frac{1}{\rho}-(1-\alpha)\right)} \quad \text{(C.9)}$$

Similarly, putting the term $\frac{1}{Z_t^{\frac{\vartheta}{\rho}-\vartheta}}$ inside the public capital product in the production function where A_t^G is positioned. Then in (C.1)- (C.3), we have

$$A_t^G = A_t^G = \bar{A}_t^G \rightarrow \bar{A}_t^G \frac{1}{Z_t^{\left(\frac{1}{\rho}-1\right)}} = \frac{\bar{A}_t^G}{\bar{A}_{t-1}^G} \bar{A}_{t-1}^G \frac{1}{Z_t^{\left(\frac{1}{\rho}-1\right)}} = \frac{\bar{A}_t^G}{\bar{A}_{t-1}^G} \frac{\bar{A}_{t-1}^G}{Z_{t-1}^{\left(\frac{1}{\rho}-1\right)}} \frac{Z_{t-1}^{\left(\frac{1}{\rho}-1\right)}}{Z_t^{\left(\frac{1}{\rho}-1\right)}} \quad \text{(C.10)}$$

The model is now in stationary form with Y_t^W, A_t, K_t and K_t^G replaced with stationary forms $\frac{Y_t^W}{Z_t}$, A_t^c, $\frac{K_t}{Z_t}$ and $\frac{K_t^G}{Z_t}$ and growing at a common balanced growth in the steady state if we impose

$$\frac{\bar{A}_{t-1}}{Z_{t-1}^{\frac{1}{\alpha}\left(\frac{1}{\rho}-(1-\alpha)\right)}} = \frac{\bar{A}_{t-1}^G}{Z_{t-1}^{\left(\frac{1}{\rho}-1\right)}} = 1 \quad \text{(C.11)}$$

Putting $A_{t-1}^G \left(Z_{t-1}\right)^{-\left(\frac{1}{\rho}-1\right)} = 1$ then the relationship between the growth rates to imply the balanced growth path is $g_{A^G} = \left(\frac{1}{\rho} - 1\right) g_Z$ which given g_Z, pins down g_{A^G}.

(C.1) in stationary form now becomes

$$\begin{aligned} \frac{Y_t^W}{Z_t} &= \left[\theta\left(\left(A_t^c \frac{\bar{A}_t}{\bar{A}_{t-1}}\left(\frac{Z_{t-1}}{Z_t}\right)^{\frac{1}{\alpha}\left(\frac{1}{\rho}-(1-\alpha)\right)} H_t^d\right)^{\alpha}\left(u_t \frac{K_{t-1}}{Z_{t-1}} \frac{Z_{t-1}}{Z_t}\right)^{1-\alpha}\right)^{\vartheta}\right. \\ &+ \left.(1-\theta)\left(\frac{A_t^G}{A_{t-1}^G} \frac{K_{t-1}^G}{Z_{t-1}}\left(\frac{Z_{t-1}}{Z_t}\right)^{\frac{1}{\rho}}\right)^{\vartheta}\right]^{\frac{\rho}{\vartheta}} - \frac{F_t}{Z_t} \end{aligned} \quad \text{(C.12)}$$

which we write as

$$\begin{aligned} \frac{Y_t^W}{Z_t} &= \left[\theta\left(\left(A_t^c(1+g_A)\left(\frac{1}{1+g_Z}\right)^{\frac{1}{\alpha}\left(\frac{1}{\rho}-(1-\alpha)\right)} H_t^d\right)^{\alpha}\left(u_t \frac{K_{t-1}^c}{1+g_Z}\right)^{1-\alpha}\right)^{\vartheta}\right. \\ &+ \left.(1-\theta)\left((1+g_{AG})\left(\frac{1}{1+g_Z}\right)^{\frac{1}{\rho}} K_{t-1}^{G^c}\right)^{\vartheta}\right]^{\frac{\rho}{\vartheta}} - \frac{F_t}{Z_t} \end{aligned} \quad \text{(C.13)}$$

where

$$\log A_t^c = \rho_A \log A_{t-1}^c + \epsilon_{A,t} \tag{C.14}$$

Finally we need to stationarize (B.20). We do this by writing

$$\frac{G_t^I}{Z_t} = \sum_{i=0}^{n-1} \omega_i \frac{AU_{t-i}}{Z_t} = \sum_{i=0}^{n-1} \omega_i \frac{AU_{t-i}}{Z_{t-i}} \frac{Z_{t-i}}{Z_i} = \sum_{i=0}^{n-1} \omega_i \frac{AU_{t-i}}{Z_{t-i}} \frac{1}{Z_{[t,t-i]}} \tag{C.15}$$

where we have defined

$$Z_{t,t} = 1 \tag{C.16}$$

$$Z_{t,t-1} = 1 + g_{Z,t} \tag{C.17}$$

$$Z_{t,t-2} = (1 + g_{Z,t})(1 + g_{Z,t-1}) \tag{C.18}$$

$$Z_{t,t-i} = (1 + g_{Z,t})(1 + g_{Z,t-1}) \cdots (1 + g_{Z,t-i+1})\,; i > 0 \tag{C.19}$$

Then we can define stationarized variables by

$$\begin{aligned}
\frac{\Omega_t}{\bar{Z}_t^{1-\sigma}} &= \frac{U_t}{\bar{Z}_t^{1-\sigma}} + \beta E_t \frac{\Omega_{t+1}}{\bar{Z}_{t+1}^{1-\sigma}} \left(\frac{\bar{Z}_{t+1}}{\bar{Z}_t} \right)^{1-\sigma} \\
\frac{U_t}{\bar{Z}_t^{1-\sigma}} &= \frac{\left[\frac{C_t}{\bar{Z}_t} - \chi \frac{C_{t-1}}{\bar{Z}_{t-1}} \frac{\bar{Z}_{t-1}}{\bar{Z}_t} \right]^{1-\sigma}}{1-\sigma} \exp \left[(\sigma - 1) \frac{H_t^{1+\psi}}{1+\psi} \right] \\
\Lambda_{t,t+1} &= \beta \frac{U_{C,t+1}}{U_{C,t}} = \beta (1 + g_Z)^{-\sigma)} \frac{U_{C,t+1}^c}{U_{C,t}^c} \equiv \frac{\beta_g}{1 + g_Z} \frac{U_{C,t+1}^c}{U_{C,t}^c}
\end{aligned}$$

where the growth-adjusted discount rate is defined as

$$\beta_g \equiv \beta(1 + g_Z b)^{1-\sigma},$$

the Euler equation is still

$$E_t \left[\Lambda_{t,t+1} R_{t+1} \right]$$

Now stationarize remaining variables by defining cyclical components:

$$\begin{aligned}
\frac{U_{C,t}}{\bar{Z}_t^{-\sigma}} &= \frac{(1-\sigma) \frac{U_t}{\bar{Z}_t^{1-\sigma}}}{\frac{C_t}{\bar{Z}_t} - \chi \frac{C_{t-1}}{\bar{Z}_{t-1}} \frac{\bar{Z}_{t-1}}{\bar{Z}_t}} - \beta \chi \left(\frac{\bar{Z}_{t+1}}{\bar{Z}_t} \right)^{-\sigma} \frac{(1-\sigma) \frac{U_{t+1}}{\bar{Z}_{t+1}^{1-\sigma}}}{\frac{C_{t+1}}{\bar{Z}_{t+1}} - \chi \frac{C_t}{\bar{Z}_t} \frac{\bar{Z}_t}{\bar{Z}_{t+1}}} \\
K_t^c &= (1-\delta) \frac{K_{t-1}^c}{1 + g_Z} + (1 - S(X_t^c)) I_t^c
\end{aligned}$$

$$
\begin{aligned}
X_t^c &= (1+g_Z)\frac{I_t^c}{I_{t-1}^c} \\
S(X_t^c) &= \phi_X(X_t^c - 1 - g_Z)^2 \\
S'(X_t^c) &= 2\phi_X(X_t^c - 1 - g_Z) \\
C_t^c &\equiv \frac{C_t}{\bar{Z}_t} \\
I_t^c &\equiv \frac{I_t}{\bar{Z}_t} \\
W_t^c &\equiv \frac{W_t}{\bar{Z}_t}
\end{aligned}
$$

Rewrite the equilibrium conditions as

Household:

$$
\begin{aligned}
\frac{\Omega_t}{\bar{Z}_t^{1-\sigma}} &= \frac{U_t}{\bar{Z}_t^{1-\sigma}} + \beta E_t \frac{\Omega_{t+1}}{\bar{Z}_{t+1}^{1-\sigma}}\left(\frac{\bar{Z}_{t+1}}{\bar{Z}_t}\right)^{1-\sigma} \\
\frac{U_t}{\bar{Z}_t^{1-\sigma}} &= \frac{\left[\frac{C_t}{\bar{Z}_t} - \chi\frac{C_{t-1}}{\bar{Z}_{t-1}}\frac{\bar{Z}_{t-1}}{\bar{Z}_t}\right]^{1-\sigma}}{1-\sigma}\exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] \\
\frac{K_t}{\bar{Z}_t} &= (1-\delta)\frac{K_{t-1}}{\bar{Z}_{t-1}}\frac{\bar{Z}_{t-1}}{\bar{Z}_t} + (1-S(X_t))\frac{I_t}{\bar{Z}_t} IS_t \\
X_t &= \frac{\frac{I_t}{\bar{Z}_t}\bar{Z}_t}{\frac{I_{t-1}}{\bar{Z}_{t-1}}\bar{Z}_{t-1}} \\
S(X_t) &= \phi_X(X_t - 1 - g_Z)^2 \\
S'(X_t) &= 2\phi_X(X_t - 1 - g_Z) \\
\frac{\lambda_t}{\bar{Z}_t^{-\sigma}} &= \frac{(1-\sigma)\frac{U_t}{\bar{Z}_t^{1-\sigma}}}{\frac{C_t}{\bar{Z}_t} - \chi\frac{C_{t-1}}{\bar{Z}_{t-1}}\frac{\bar{Z}_{t-1}}{\bar{Z}_t}} - \beta\chi\left(\frac{\bar{Z}_{t+1}}{\bar{Z}_t}\right)^{-\sigma}\frac{(1-\sigma)\frac{U_{t+1}}{\bar{Z}_{t+1}^{1-\sigma}}}{\frac{C_{t+1}}{\bar{Z}_{t+1}} - \chi\frac{C_t}{\bar{Z}_t}\frac{\bar{Z}_t}{\bar{Z}_{t+1}}} \\
\frac{W_{h,t}}{\bar{Z}_t} &= \frac{\left[\frac{C_t}{\bar{Z}_t} - \chi\frac{C_{t-1}}{\bar{Z}_{t-1}}\frac{\bar{Z}_{t-1}}{\bar{Z}_t}\right]H_t^{\psi}}{1 - \beta\chi\frac{U_{t+1}/\bar{Z}_{t+1}^{1-\sigma}}{U_t/\bar{Z}_t^{1-\sigma}}\left(\frac{\bar{Z}_{t+1}}{\bar{Z}_t}\right)^{-\sigma}\frac{\frac{C_t}{\bar{Z}_t} - \chi\frac{C_{t-1}}{\bar{Z}_{t-1}}\frac{\bar{Z}_{t-1}}{\bar{Z}_t}}{\frac{C_{t+1}}{\bar{Z}_{t+1}} - \chi\frac{C_t}{\bar{Z}_t}\frac{\bar{Z}_t}{\bar{Z}_{t+1}}}} \\
r_t^K &= a'(u_t) \\
1 &= RPS_t\mathbb{E}_t\left[\Lambda_{t,t+1}R_{t+1}\right] \\
Q_t &= \mathbb{E}_t\left\{\Lambda_{t,t+1}\left[r_{t+1}^K u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta)\right]\right\} \\
1 &= Q_t\left[1 - S(X_t) - S'(X_t)X_t\right] IS_t
\end{aligned}
$$

$$+\mathbb{E}_t\left[\Lambda_{t,t+1}Q_{t+1}S'(X_{t+1})X_{t+1}^2 IS_{t+1}\right]$$

$$\Lambda_{t,t+1}=\beta\frac{\frac{\lambda_{t+1}}{\bar{Z}_{t+1}^{-\sigma}}\bar{Z}_{t+1}^{-\sigma}}{\frac{\lambda_t}{\bar{Z}_t^{-\sigma}}\bar{Z}_t^{-\sigma}}$$

$$R_t=\left[\frac{R_{n,t-1}}{\Pi_t}\right]$$

$$a(u_t)=\gamma_1(u_t-1)+\frac{\gamma_2}{1-\gamma_2}\frac{\gamma_1}{2}(u_t-1)^2$$

$$a'(u_t)=\gamma_1+\frac{\gamma_2}{1-\gamma_2}\gamma_1(u_t-1)$$

Wage setting:

$$\Pi_t^w=\frac{\frac{W_t}{\bar{Z}_t}}{\frac{W_{t-1}}{\bar{Z}_{t-1}}\frac{\bar{Z}_{t-1}}{\bar{Z}_t}}\Pi_t$$

$$\frac{J_t^w}{\bar{Z}_t}=\frac{1}{1-\frac{1}{\zeta_w}}\frac{W_{h,t}}{\bar{Z}_t}H_t^d MRSS_t$$

$$+\xi_w\mathbb{E}_t\Lambda_{t,t+1}\frac{\left(\Pi_{t,t+1}^w\right)^{\zeta_w}}{(\Pi_{t-1,t})^{\gamma_w\zeta_w}}\frac{J_{t+1}^w}{\bar{A}_{t+1}}\frac{\bar{Z}_{t+1}}{\bar{Z}_t}$$

$$JJ_t^w=H_t^d+\xi_w\mathbb{E}_t\Lambda_{t,t+1}\frac{\left(\Pi_{t,t+1}^w\right)^{\zeta_w}}{(\Pi_{t-1,t})^{\gamma_w(\zeta_w-1)}\Pi_{t,t+1}}JJ_{t+1}^w$$

$$\frac{W_{n,t}^O}{W_{n,t}}=\frac{\frac{J_t^w}{\bar{Z}_t}}{\frac{W_t}{\bar{Z}_t}JJ_t^w}$$

$$1=\xi_w\left(\frac{\Pi_{t-1}^{\gamma_w}}{\Pi_t^w}\right)^{1-\zeta_w}+(1-\xi_w)\left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{1-\zeta_w}$$

$$\Delta_{w,t}=\xi_w\frac{(\Pi_t^w)^{\zeta_w}}{\Pi_{t-1}^{\zeta_w\gamma_w}}\Delta_{w,t-1}+(1-\xi_w)\left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{-\zeta_w}$$

Retail firms:

$$\frac{Y_t^W}{Z_t}=\Bigg[\theta\left((A_tH_t^d)^\alpha\left(u_t\frac{K_{t-1}}{Z_{t-1}}\frac{Z_{t-1}}{Z_t}\right)^{1-\alpha}\right)^\vartheta\frac{1}{Z_t^{\frac{\vartheta}{\rho}-(1-\alpha)\vartheta}}$$

$$+(1-\theta)\left(A_t^G\frac{K_{t-1}^G}{Z_{t-1}}\frac{Z_{t-1}}{Z_t}\right)^\vartheta\frac{1}{Z_t^{\frac{\vartheta}{\rho}-\vartheta}}\Bigg]^{\frac{\rho}{\vartheta}}-\frac{F_t}{Z_t}\qquad \text{dotte}$$

$$\frac{W_t}{Z_t} = \rho\alpha \frac{P_t^W}{P_t} \left[\frac{Y_t^W(1+F_t)}{Z_t}\right]^{\left(1-\frac{\vartheta}{\rho}\right)} \frac{\theta\left((A_t H_t^d)^\alpha (u_t \frac{K_{t-1}}{Z_{t-1}} \frac{Z_{t-1}}{Z_t})^{1-\alpha}\right)^\vartheta}{H_t^d} \frac{1}{Z_t^{\frac{\vartheta}{\rho}-\vartheta}}$$

$$r_t^K = \rho(1-\alpha) \frac{P_t^W}{P_t} \left[\frac{Y_t^W(1+F_t)}{Z_t}\right]^{\left(1-\frac{\vartheta}{\rho}\right)} \frac{\theta\left((A_t H_t^d)^\alpha (u_t \frac{K_{t-1}}{Z_{t-1}} \frac{Z_{t-1}}{Z_t})^{1-\alpha}\right)^\vartheta}{u_t \frac{K_{t-1}}{Z_{t-1}} \frac{Z_{t-1}}{Z_t}} \frac{1}{Z_t^{\frac{\vartheta}{\rho}-\vartheta}}$$

Price setting:

$$\begin{aligned}
MC_t &= \frac{P_t^W}{P_t} \\
\frac{J_t^p}{\bar{Z}_t} &= \frac{1}{1-\frac{1}{\zeta_p}} \frac{Y_t}{\bar{Z}_t} MC_t MCS_t \\
&\quad + \xi_p \mathbb{E}_t \Lambda_{t,t+1} \frac{(\Pi_{t,t+1})^{\zeta_p}}{(\Pi_{t-1,t})^{\gamma_p \zeta_p}} \frac{J_{t+1}^p}{\bar{Z}_{t+1}} \frac{\bar{Z}_{t+1}}{\bar{Z}_t} \\
\frac{JJ_t^p}{\bar{Z}_t} &= \frac{Y_t}{\bar{Z}_t} + \xi_p \mathbb{E}_t \Lambda_{t,t+1} \frac{(\Pi_{t,t+1})^{\zeta_p - 1}}{(\Pi_{t-1,t})^{\gamma_p(\zeta_p - 1)}} \frac{JJ_{t+1}^p}{\bar{Z}_{t+1}} \frac{\bar{Z}_{t+1}}{\bar{Z}_t} \\
\frac{P_t^0}{P_t} &= \frac{\frac{J_t^p}{\bar{Z}_t}}{\frac{JJ_t^p}{\bar{Z}_t}} \\
1 &= \xi_p \left(\frac{\Pi_{t-1}^{\gamma_p}}{\Pi_t}\right)^{1-\zeta_p} + (1-\xi_p)\left(\frac{P_t^0}{P_t}\right)^{1-\zeta_p} \\
\Delta_{p,t} &= \xi_p \frac{\Pi_t^{\zeta_p}}{\Pi_{t-1}^{\zeta_p \gamma_p}} \Delta_{p,t-1} + (1-\xi_p)\left(\frac{P_t^0}{P_t}\right)^{-\zeta_p}
\end{aligned}$$

Monetary policy:

$$\begin{aligned}
\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) \\
&\quad + (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\right) \\
&\quad + \log MPS_t
\end{aligned}$$

Aggregation:

$$\begin{aligned}
\frac{Y_t}{\bar{Z}_t} &= \frac{C_t}{\bar{Z}_t} + \frac{G_t}{\bar{Z}_t} + \frac{I_t}{\bar{Z}_t} + \frac{I_t^G}{\bar{Z}_t} + \frac{a(u_t)}{IS_t} \frac{K_{t-1}}{\bar{Z}_{t-1}} \frac{\bar{Z}_{t-1}}{\bar{Z}_t} \\
H_t &= \Delta_{w,t} H_t^d
\end{aligned}$$

$$\frac{Y_t^W}{\bar{Z}_t} = \Delta_{p,t}\frac{Y_t}{\bar{Z}_t}$$
$$R_t^K = \frac{r_t^K u_t - a(u_t) + Q_t(1-\delta)}{Q_{t-1}}$$

Shock processes:

$$\log A_t - \log A = \rho_A(\log A_{t-1} - \log A) + \epsilon_{A,t}$$
$$\log G_t - \log G = \rho_G(\log G_{t-1} - \log G) + \epsilon_{G,t}$$
$$\log MCS_t - \log MCS = \rho_{MCS}(\log MCS_{t-1} - \log MCS) + \epsilon_{MCS,t}$$
$$\log MRSS_t - \log MRSS = \rho_{MRSS}(\log MRSS_{t-1} - \log MRSS) + \epsilon_{MRSS,t}$$
$$\log IS_t - \log IS = \rho_{IS}(\log IS_{t-1} - \log IS) + \epsilon_{IS,t}$$
$$\log AP_t - \log AP = \rho_{IS}(\log AP_{t-1} - \log AP) + \epsilon_{AU,t}$$
$$\log MPS_t - \log MPS = \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t}$$
$$\log RPS_t - \log RPS = \rho_{RPS}(\log RPS_{t-1} - \log RPS) + \epsilon_{RPS,t}$$

D Summary of the Dynamic Equilibrium

Use this change of variables and dropping the superscript c in trended variables such Ω_t^c, U_t^c, C_t^c etc to arrive to the following stationarized equilibrium conditions:

Household:

$$\Omega_t = U_t + \beta(1+g_Z)^{1-\sigma}E_t\Omega_{t+1} \tag{D.1}$$
$$U_t = \frac{\left[C_t - \chi\frac{C_{t-1}}{1+g_Z}\right]^{1-\sigma}}{1-\sigma}\exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] \tag{D.2}$$
$$K_t = (1-\delta)\frac{K_{t-1}}{1+g_Z} + (1-S(X_t))I_t IS_t \tag{D.3}$$
$$X_t = \frac{I_t}{I_{t-1}}(1+g_Z) \tag{D.4}$$
$$S(X_t) = \phi_X(X_t - 1 - g_Z)^2 \tag{D.5}$$

$$S'(X_t) = 2\phi_X(X_t - 1 - g_Z) \tag{D.6}$$

$$\lambda_t = \frac{(1-\sigma)U_t}{C_t - \chi\frac{C_{t-1}}{1+g_Z}} - \beta\chi(1+g_Z)^{-\sigma}\frac{(1-\sigma)U_{t+1}}{C_{t+1} - \chi\frac{C_t}{1+g_Z}} \tag{D.7}$$

$$W_{h,t} = \frac{\left[C_t - \chi\frac{C_{t-1}}{1+g}\right]H_t^{\psi}}{1 - \beta\chi(1+g_Z)^{-\sigma}\frac{U_{t+1}}{U_t}\frac{C_t - \chi\frac{C_{t-1}}{1+g_Z}}{C_{t+1} - \chi\frac{C_t}{1+g_Z}}} \tag{D.8}$$

$$r_t^K = a'(u_t) \tag{D.9}$$

$$1 = RPS_t\mathbb{E}_t\left[\Lambda_{t,t+1}R_{t+1}\right] \tag{D.10}$$

$$Q_t = \mathbb{E}_t\left\{\Lambda_{t,t+1}\left[r_{t+1}^K u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta)\right]\right\} \tag{D.11}$$

$$\begin{aligned} 1 &= Q_t\left[1 - S(X_t) - S'(X_t)X_t\right]IS_t \\ &\quad + \mathbb{E}_t\left[\Lambda_{t,t+1}Q_{t+1}S'(X_{t+1})X_{t+1}^2 IS_{t+1}\right] \end{aligned} \tag{D.12}$$

$$\Lambda_{t,t+1} = \beta(1+g_Z)^{-\sigma}\frac{\lambda_{t+1}}{\lambda_t} \tag{D.13}$$

$$R_t = \left[\frac{R_{n,t-1}}{\Pi_t}\right] \tag{D.14}$$

$$a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{1-\gamma_2}\frac{\gamma_1}{2}(u_t - 1)^2 \tag{D.15}$$

$$a'(u_t) = \gamma_1 + \frac{\gamma_2}{1-\gamma_2}\gamma_1(u_t - 1) \tag{D.16}$$

Wage setting:

$$\Pi_t^w = (1+g_Z)\frac{W_t}{W_{t-1}}\Pi_t \tag{D.17}$$

$$\begin{aligned} J_t^w &= \frac{1}{1-\frac{1}{\zeta_w}}W_{h,t}H_t^d MRSS_t \\ &\quad + \xi_w(1+g)\mathbb{E}_t\Lambda_{t,t+1}\frac{\left(\Pi_{t,t+1}^w\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w\zeta_w}}J_{t+1}^w \end{aligned} \tag{D.18}$$

$$JJ_t^w = H_t^d + \xi_w\mathbb{E}_t\Lambda_{t,t+1}\frac{\left(\Pi_{t,t+1}^w\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w(\zeta_w-1)}\Pi_{t,t+1}}JJ_{t+1}^w \tag{D.19}$$

$$\frac{W_{n,t}^O}{W_{n,t}} = \frac{J_t^w}{W_t JJ_t^w} \tag{D.20}$$

$$1 = \xi_w\left(\frac{\Pi_{t-1}^{\gamma_w}}{\Pi_t^w}\right)^{1-\zeta_w} + (1-\xi_w)\left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{1-\zeta_w} \tag{D.21}$$

$$\Delta_{w,t} = \xi_w\frac{(\Pi_t^w)^{\zeta_w}}{\Pi_{t-1}^{\zeta_w\gamma_w}}\Delta_{w,t-1} + (1-\xi_w)\left(\frac{W_{n,t}^O}{W_{n,t}}\right)^{-\zeta_w} \tag{D.22}$$

Retail firm:

$$Y_t^{W,c} = \Bigg[\theta\left(\left(\left(A_t^c(1+g_A)\left(\frac{1}{1+g_Z}\right)^{\frac{1}{\alpha}\left(\frac{1}{\rho}-(1-\alpha)\right)}H_t^d\right)^{\alpha}\left(u_t\frac{K_{t-1}^c}{1+g_Z}\right)^{1-\alpha}\right)^{\vartheta}$$

$$+(1-\theta)\left((1+g_{AG})\left(\frac{1}{1+g_Z}\right)^{\frac{1}{\rho}}K_{t-1}^{G^c}\right)^{\vartheta}\Bigg]^{\frac{\rho}{\vartheta}} - F_t \tag{D.23}$$

$$W_t^c = \rho\alpha\frac{P_t^W}{P_t}\left[Y_t^{W,c}(1+F_t)\right]^{\left(1-\frac{\vartheta}{\rho}\right)}$$

$$\frac{\theta\left((A_t(1+g_A)\left(\frac{1}{1+g_Z}\right)^{\frac{1}{\alpha}\left(\frac{1}{\rho}-1\right)+1}H_t^d)^{\alpha}(u_t\frac{K_{t-1}}{1+g_Z})^{1-\alpha}\right)^{\vartheta}}{H_t^d} \tag{D.24}$$

$$r_t^K = \rho(1-\alpha)\frac{P_t^W}{P_t}\left[Y_t^{W,c}(1+F_t)\right]^{\left(1-\frac{\vartheta}{\rho}\right)}$$

$$\frac{\theta\left((A_t(1+g_A)\left(\frac{1}{1+g_Z}\right)^{\frac{1}{\alpha}\left(\frac{1}{\rho}-1\right)+1}H_t^d)^{\alpha}(u_t\frac{K_{t-1}}{1+g_Z})^{1-\alpha}\right)^{\vartheta}}{u_t\frac{K_{t-1}}{1+g_Z}} \tag{D.25}$$

Price setting:

$$MC_t = \frac{P_t^W}{P_t} \tag{D.26}$$

$$J_t^p = \frac{1}{1-\frac{1}{\zeta_p}}Y_tMC_tMCS_t$$

$$+\xi_p(1+g_Z)\mathbb{E}_t\Lambda_{t,t+1}\frac{(\Pi_{t,t+1})^{\zeta_p}}{(\Pi_{t-1,t})^{\gamma_p\zeta_p}}J_{t+1}^p \tag{D.27}$$

$$JJ_t^p = Y_t + \xi_p(1+g_Z)\mathbb{E}_t\Lambda_{t,t+1}\frac{(\Pi_{t,t+1})^{\zeta_p-1}}{(\Pi_{t-1,t})^{\gamma_p(\zeta_p-1)}}JJ_{t+1}^p \tag{D.28}$$

$$\frac{P_t^0}{P_t} = \frac{J_t^p}{JJ_t^p} \tag{D.29}$$

$$1 = \xi_p\left(\frac{\Pi_{t-1}^{\gamma_p}}{\Pi_t}\right)^{1-\zeta_p} + (1-\xi_p)\left(\frac{P_t^0}{P_t}\right)^{1-\zeta_p} \tag{D.30}$$

$$\Delta_{p,t} = \xi_p\frac{\Pi_t^{\zeta_p}}{\Pi_{t-1}^{\zeta_p\gamma_p}}\Delta_{p,t-1} + (1-\xi_p)\left(\frac{P_t^0}{P_t}\right)^{-\zeta_p} \tag{D.31}$$

Monetary policy:

$$\log\left(\frac{R_{n,t}}{R_n}\right) = \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) + (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy} \log\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \log MPS_t \quad \text{(D.32)}$$

Fiscal policy:

$$\log\left(\frac{\tau_{w,t}}{\tau_{w,t}}\right) = \rho_{\tau_w} \log\left(\frac{\tau_{w,t-1}}{\tau_{w,t}}\right) + (1-\rho_{\tau_w})\left(\theta_{\tau_w} \log\left(\frac{b_{t-1}}{b}\right) + \theta_{y,\tau_w} \log\left(\frac{Y_{t-1}}{Y}\right) + \theta_{dy,\tau_w} \log\left(\frac{Y_{t-1}}{Y_{t-2}}\right)\right) + \epsilon_{\tau_w,t} \quad \text{(D.33)}$$

$$\log\left(\frac{\tau_{k,t}}{\tau_{k,t}}\right) = \rho_{\tau_k} \log\left(\frac{\tau_{k,t-1}}{\tau_{k,t}}\right) + (1-\rho_{\tau_k})\left(\theta_{\tau_k} \log\left(\frac{b_{t-1}}{b}\right) + \theta_{y,\tau_k} \log\left(\frac{Y_{t-1}}{Y}\right) + \theta_{dy,\tau_k} \log\left(\frac{Y_{t-1}}{Y_{t-2}}\right)\right) + \epsilon_{\tau_k,t} \quad \text{(D.34)}$$

Aggregation:

$$Y_t = C_t + G_t + I_t + I_t^G + a(u_t)\frac{K_{t-1}}{1+g} \quad \text{(D.35)}$$

$$H_t = \Delta_{w,t} H_t^d \quad \text{(D.36)}$$

$$Y_t^W = \Delta_{p,t} Y_t \quad \text{(D.37)}$$

$$R_t^K = \frac{r_t^K u_t - a(u_t) + Q_t(1-\delta)}{Q_{t-1}} \quad \text{(D.38)}$$

Shock processes:

$$\log A_t - \log A = \rho_A(\log A_{t-1} - \log A) + \epsilon_{A,t} \quad \text{(D.39)}$$

$$\log G_t - \log G = \rho_G(\log G_{t-1} - \log G) + \epsilon_{G,t} \quad \text{(D.40)}$$

$$\log MCS_t - \log MCS = \rho_{MCS}(\log MCS_{t-1} - \log MCS) + \epsilon_{MCS,t} \quad \text{(D.41)}$$

$$\log MRSS_t - \log MRSS = \rho_{MRSS}(\log MRSS_{t-1} - \log MRSS) + \epsilon_{MRSS,t} \quad \text{(D.42)}$$

$$\log IS_t - \log IS = \rho_{IS}(\log IS_{t-1} - \log IS) + \epsilon_{IS,t} \tag{D.43}$$

$$\log AP_t - \log AP = \rho_{AU}(\log AP_{t-1} - \log AP) + \epsilon_{AU,t} \tag{D.44}$$

$$\log MPS_t - \log MPS = \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \tag{D.45}$$

$$\log RPS_t - \log RPS = \rho_{RPS}(\log RPS_{t-1} - \log RPS) + \epsilon_{RPS,t} \tag{D.46}$$

This is a system of 50 equation in the following 50 macroeconomic variables (in order of appearance): V, U, C, H, K, K^G, $S(X)$, X, I, I^G, $AU(AP)$, IS, $S'(X)$, λ, W_h, r^K, $a'(u)$, RPS, Λ, R, Q, u, $a(u)$, τ_w, τ_k, RPS_t, $b^{A,r}$, R_n, Π, Π^w, W, J^w, H^d, $MRSS$, JJ^w, $\frac{W_n^O}{W_n}$, Δ_w, Y^W, A, $\frac{P^W}{P}$, MC, J^p, Y, MCS, JJ^p, $\frac{P^0}{P}$, Δ_p, MPS, G, R^K plus 10 AR1 Shock Processes.

Finally we define a consumption equivalent welfare measure CEV_t as the inter-temporal increase in welfare resulting from a permanent 1% increase in the equilibrium path of consumption as

$$\begin{aligned}
CEV_t &= \mathbb{E}_t\left[\sum_{t=s}^{\infty}\beta^s U(1.01C_{t+s}, 1.01C_{t-1+s}, H_{t+s})\right] \\
&- \mathbb{E}_t\left[\sum_{t=s}^{\infty}\beta^s U(C_{t+s}, C_{t-1+s}, H_{t+s})\right] \\
&= \frac{[1.01C_t - \chi 1.01C_{t-1}]^{1-\sigma}}{1-\sigma}\exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] - U(C_t, C_{t-1}, H_t) \\
&+ \beta\mathbb{E}_t CEV_{t+1} \\
&= (1.01^{1-\sigma} - 1)U_t + \beta\mathbb{E}_t CEV_{t+1}
\end{aligned} \tag{D.47}$$

The stationary version is then

$$CEV_t = (1.01^{1-\sigma} - 1)U_t + \beta(1+g)^{1-\sigma}\mathbb{E}_t CEV_{t+1} \tag{D.48}$$

In our results we compute consumption equivalent differences using the stationary steady state CEV.

E The Balanced-Growth Deterministic Steady State

Having stationarized the model we now drop the superscript c. The exogenous variables have steady states $A = MCS = MRSS = IS = MPS = RPS = 1$, $G = g_y Y$. Moreover, $u = 1$ in steady state. Given the steady state inflation rate Π and hours H, the steady state values of the other variables can be computed in stationary form as

$$S(X) = 0 \tag{E.1}$$

$$S'(X) = 0 \tag{E.2}$$

$$\Pi^w = (1+g)\Pi \tag{E.3}$$

$$Q = 1 \tag{E.4}$$

$$\Lambda = \beta(1+g)^{-\sigma} \tag{E.5}$$

$$r^K = \frac{1}{\Lambda} - (1-\delta) \tag{E.6}$$

$$a(u) = 0 \tag{E.7}$$

$$a'(u) = \gamma_1 \tag{E.8}$$

$$r^K = \gamma_1 \Rightarrow \gamma_1 = \frac{1}{\beta(1+g)^{-\sigma}} - (1-\delta) \tag{E.9}$$

$$\frac{P^0}{P} = \left(\frac{1-\xi_p \Pi^{(1-\gamma_p)(\zeta_p-1)}}{1-\xi_p}\right)^{\frac{1}{1-\zeta_p}} \tag{E.10}$$

$$\Delta_p = \frac{1-\xi_p}{1-\xi_p \Pi^{\zeta_p(1-\gamma_p)}} \left(\frac{P^0}{P}\right)^{-\zeta_p} \tag{E.11}$$

$$MC = \left(1 - \frac{1}{\zeta_p}\right) \frac{1-\xi_p(1+g)\Lambda\Pi^{\zeta_p(1-\gamma_p)}}{1-\xi_p(1+g)\Lambda\Pi^{(\zeta_p-1)(1-\gamma_p)}} \frac{P^0}{P} \tag{E.12}$$

$$\frac{P^W}{P} = MC \tag{E.13}$$

$$\frac{W_n^O}{W_n} = \left(\frac{1-\xi_w \Pi^{\gamma_w(1-\zeta_w)}(\Pi^w)^{\zeta_w-1}}{1-\xi_w}\right)^{\frac{1}{1-\zeta_w}} \tag{E.14}$$

$$\Delta_w = \frac{1-\xi_w}{1-\xi_w \frac{(\Pi^w)^{\zeta_w}}{\Pi^{\zeta_w\gamma_w}}} \left(\frac{W_n^O}{W_n}\right)^{-\zeta_w} \tag{E.15}$$

$$H^d = \frac{H}{\Delta_w} \tag{E.16}$$

$$\frac{K}{Y^W} = \frac{(1-\alpha)(1+g)(1+\tilde{F})}{ur^K} \frac{P^W}{P} \tag{E.17}$$

$$Y^W = \frac{H^d}{(1+\tilde{F})^{\frac{1}{\alpha}}} \left(\frac{\frac{K}{Y^W}}{1+g}\right)^{\frac{1-\alpha}{\alpha}} \tag{E.18}$$

$$K = Y^W \frac{K}{Y^W} \tag{E.19}$$

$$Y = \frac{Y^W}{\Delta_p} \tag{E.20}$$

$$I = \frac{K}{1}\frac{g+\delta}{1+g} \tag{E.21}$$

$$G = g_y Y \tag{E.22}$$

$$C = Y - G - I \tag{E.23}$$

$$JJ^w = \frac{H^d}{1 - \xi_w \Lambda \left(\Pi^w\right)^{\zeta_w} \Pi^{\gamma_w(1-\zeta_w)-1}} \tag{E.24}$$

$$W = \alpha \frac{P^W}{P} \frac{Y^W + F}{H^d} \tag{E.25}$$

$$J^w = \frac{W_n^O}{W_n} W JJ^w \tag{E.26}$$

$$\begin{aligned} \frac{W_h}{W} &= \frac{\left(1 - \xi_w(1+g)\Lambda \frac{(\Pi^w)^{\zeta_w}}{\Pi^{\gamma_w \zeta_w}}\right)\left(1 - \frac{1}{\zeta_w}\right) J^w}{W H^d} \\ &= \frac{\left(1 - \xi_w(1+g)\Lambda \frac{(\Pi^w)^{\zeta_w}}{\Pi^{\gamma_w \zeta_w}}\right)\left(1 - \frac{1}{\zeta_w}\right) \frac{W_n^O}{W_n}}{1 - \xi_w \Lambda \left(\Pi^w\right)^{\zeta_w} \Pi^{\gamma_w(1-\zeta_w)-1}} \end{aligned} \tag{E.27}$$

To examine the impact of trend inflation Π on the steady state further we consider the zero growth case $g = 0$ for which wage and price inflation are equal ($\Pi^w = \Pi$). Then we have for price-setting:

$$\begin{aligned} \frac{P^0}{P} &= \left(\frac{1 - \xi_p \Pi^{(1-\gamma_p)(\zeta_p - 1)}}{1 - \xi_p}\right)^{\frac{1}{1-\zeta_p}} \\ \Delta_p &= \frac{1 - \xi_p}{1 - \xi_p \Pi^{\zeta_p(1-\gamma_p)}} \left(\frac{P^0}{P}\right)^{-\zeta_p} \\ MC &= \left(1 - \frac{1}{\zeta_p}\right) \frac{1 - \xi_p \Lambda \Pi^{\zeta_p(1-\gamma_p)}}{1 - \xi_p(1+g)\Lambda \Pi^{(\zeta_p - 1)(1-\gamma_p)}} \frac{P^0}{P} \end{aligned}$$

and for wage-setting:

$$\begin{aligned} \frac{W_n^O}{W_n} &= \left(\frac{1 - \xi_w \Pi^{(1-\gamma_w)(\zeta_w - 1)}}{1 - \xi_w}\right)^{\frac{1}{1-\zeta_w}} \\ \Delta_w &= \frac{1 - \xi_w}{1 - \xi_w \Pi^{(1-\gamma_w)\zeta_w}} \left(\frac{W_n^O}{W_n}\right)^{-\zeta_w} \\ \frac{W_h}{W} &= \frac{\left(1 - \xi_w \Lambda \Pi^{(1-\gamma_w)\zeta_w}\right)\left(1 - \frac{1}{\zeta_w}\right) \frac{W_n^O}{W_n}}{1 - \xi_w \Lambda \Pi^{(1-\gamma_w)(\zeta_w - 1)}}. \end{aligned}$$

Thus for $\zeta_p > 1$, both the optimized price $\frac{P^0}{P}$ and price dispersion Δ_p *increase* with the trend inflation rate Π. However noting that the price mark-up is the inverse of the real marginal cost, i.e., equal to $= 1/MC$, we can see that the price response to the re-optimized price *decreases* with Π. Analogous results for $\zeta_w > 1$ hold for the optimized nominal wage, wage dispersion and the wage mark-up which is the inverse of $\frac{W_h}{W}$.

E.1 Solution of the Deterministic Steady State

We solve for the steady state as follows:

1. We guess the value of H.

2. We solve for the steady state of the model given our guess.

3. We use the foc on hours

$$W_{h,t} = \frac{\left[C_t - \chi \frac{C_{t-1}}{1+g}\right] H_t^{\psi}}{1 - \beta\chi(1+g)^{-\sigma} \frac{U_{t+1}}{U_t} \frac{C_t - \chi \frac{C_{t-1}}{1+g}}{C_{t+1} - \chi \frac{C_t}{1+g}}}$$

   to evaluate our guess. Note that the above equation in steady state simplifies to

$$W_h = \frac{\left[C - \chi \frac{C}{1+g}\right] H^{\psi}}{1 - \beta\chi(1+g)^{-\sigma}}$$

   which eliminates the need to compute the steady state value for utility.

The rest of the variables can be computed as

$$U = \frac{\left[C - \chi \frac{C}{1+g}\right]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma - 1)\frac{H^{1+\psi}}{1+\psi}\right]$$

$$V = \frac{U}{1 - \beta(1+g)^{1-\sigma}} labelVss$$

$$X = 1 + g$$

$$\lambda = \frac{(1-\sigma)U}{C - \chi \frac{C}{1+g}} - \beta\chi(1+g)^{-\sigma} \frac{(1-\sigma)U}{C - \chi \frac{C}{1+g}}$$

$$R = \frac{1}{\Lambda}$$

$$R_n = R\Pi$$

$$J^p = \frac{YMCMCS}{\left(1 - \frac{1}{\zeta_p}\right)\left(1 - \xi_p(1+g)\Lambda\Pi^{\zeta_p(1-\gamma_p)}\right)}$$

$$JJ^p = \frac{J^p}{\frac{P^0}{P}}$$

$$R^K = r^K + 1 - \delta$$

$$CEV = \frac{(1.01^{1-\sigma} - 1)U}{1 - \beta(1+g)^{1-\sigma}}$$

E.2 Calibrated and Estimated Parameters

From our non-zero-inflation-growth steady state we impose the restrictions

$$R_n = \frac{\Pi}{\beta(1+g)^{-\sigma}} \tag{E.28}$$

on β. This implies that β **can** be calibrated as

$$\beta = \frac{\Pi}{R_n(1+g)^{-\sigma}} \tag{E.29}$$

However, in order to evaluate welfare ranking with a consistent form of the objective function, we set β given (E.29) with $\bar{\Pi}$ and g both estimated directly as the trend of the data with σ imposed at the prior given by 1.5. For our US data and estimation period, this gives $\beta = 0.9995$ which is then imposed on the rest of the estimation and used for the optimized rules.

The first-order condition for capital utilisation is

$$r_t^K = a'(u_t) \tag{E.30}$$

which has the linear approximation

$$\hat{r}_t^K = \frac{\gamma_2}{\gamma_1}\hat{u}_t \tag{E.31}$$

Smets and Wouters write the above equation as (see equation (6) in their paper)

$$z_t = z_1 r_t^k \tag{E.32}$$

where $z_1 = \frac{1-\psi}{\psi}$ and they estimate ψ. Consequently, $z_1 = \frac{\gamma_1}{\gamma_2}$.

Recall that the capital utilisation adjustment function is

$$a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2 \tag{E.33}$$

which can be rewritten as

$$a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{\gamma_1}\frac{\gamma_1}{2}(u_t - 1)^2$$
$$= \gamma_1(u_t - 1) + \frac{1}{z_1}\frac{\gamma_1}{2}(u_t - 1)^2$$
$$= \gamma_1(u_t - 1) + \frac{\psi}{1-\psi}\frac{\gamma_1}{2}(u_t - 1)^2 \tag{E.34}$$

Its derivative is

$$a'(u_t) = \gamma_1 + \frac{\psi}{1-\psi}\gamma_1(u_t - 1) \tag{E.35}$$

The production function (equation (5) in the paper) is given by

$$y_t = \phi_p(\alpha k_t^s + (1-\alpha)l_t + \varepsilon_t^a) \tag{E.36}$$

where $\phi_p = \frac{y_* + \Phi}{y_*}$ is one plus the share of fixed costs in production.[7] They use the prior $\phi_p \sim \mathcal{N}(1.25, 0.25)$ for the parameter (may be missing from the paper altogether), which implies that $\frac{\Phi}{y^*} \sim \mathcal{N}(0.25, 0.25)$. Hence we need to rewrite the equilibrium condition (**??**) as

$$Y_t^W = \left(A_t H_t^d\right)^\alpha (u_t K_{t-1})^{1-\alpha} - \tilde{F} Y^W \tag{E.39}$$

and define the prior on $\tilde{F} = \frac{F}{\frac{Y^W}{A_t}}$.

In the steady state we empirically calibrate $\tau_k = 0.323237$ and $\tau_w = 0.234539$ (Dataset covers from 1979Q1 to 2019Q4).

The government bond component equation is:

$$B_t^{A,r} = \frac{RPS_{t-1}R_{n,t-1}}{\Pi_t(1+g)}B_{t-1}^{A,r} + G^A + Z_t^A + \frac{\tau_{k,t} r_t^K u_t K_{t-1}^A}{(1+g)} + \tau_{w,t} W_t H_t^A \tag{E.40}$$

Where superscript A denoting the detrended variables with positive growth rate g. The

[7] In the technical appendix the production function is given by

$$y_t(i) = Z_t k_t(i)^\alpha L_t(i)^{1-\alpha} - \Phi \tag{E.37}$$

which becomes

$$\hat{y}_t = \alpha\frac{y_* + \Phi}{y_*}\hat{k}_t + (1-\alpha)\frac{y_* + \Phi}{y_*}\hat{L}_t + \frac{y_* + \Phi}{y_*}\hat{Z}_t \tag{E.38}$$

when loglinearized.

steady state version of the equation above is given as follows:

$$b^{A,r} = \frac{RPS \quad R_n}{\Pi(1+g)} b^{A,r} + gy + zy - [\frac{r^K u K^A/(1+g)}{Y}\tau_k + \frac{WH^A}{Y}\tau_w] \tag{E.41}$$

Equivalently,

$$b^{A,r} = \left(1 - \frac{RPS \quad R_n}{\Pi(1+g)}\right)^{-1} \left(gy + zy - \left[\frac{r^K u K^A/(1+g)}{Y}\tau_k + \frac{WH^A}{Y}\tau_w\right]\right) \tag{E.42}$$

$$b^{A,r} = \left(\frac{(1+g)}{(1+g) - RPS \quad R_n/\Pi}\right) \left(gy + zy - \left[\frac{r^K u K^A/(1+g)}{Y}\tau_k + \frac{WH^A}{Y}\tau_w\right]\right) \tag{E.43}$$

Where $\frac{G}{Y} = gy = 0.164$ as in the data and $\frac{B^{A,r}}{Y} = b^{A,r} = 2.6$. The transfer-to-income ratio is calibrated such that the steady state real debt-output ratio is equal to 65% quarterly.

F The Data and Measurement Equations

Our observables used in the estimation are: GDP per capita growth (dyobs), consumption expenditure per capita growth (dcobs), investment per capita growth (dinvobs), real wage growth (dwobs), percentage deviation of hours worked per capita from mean (labobs), monetary policy rate (robs), inflation rate (pinfobs), income tax rate (tauwobs), capital tax rate (taukobs) and public investment growth (dginvobs). The corresponding measurement equations expressed in terms of stationarized variables[8] are:

$$\begin{aligned}
\text{dyobs} &= \log\left((1+g)\frac{Y_t}{Y_{t-1}}\right) \\
\text{dcobs} &= \log\left((1+g)\frac{C_t}{C_{t-1}}\right) \\
\text{dinvobs} &= \log\left((1+g)\frac{I_t}{I_{t-1}}\right) \\
\text{dginvobs} &= \log\left((1+g)\frac{I_t^G}{I_{t-1}^G}\right) \\
\text{dwobs} &= \log\left((1+g)\frac{W_t}{W_{t-1}}\right)
\end{aligned}$$

[8]See Online Appendix C.

$$
\begin{aligned}
\text{labobs} &= \frac{H_t^d - H^d}{H^d} \\
\text{robs} &= R_{n,t} - 1 \\
\text{pinfobs} &= \log\left(\Pi_t\right) \\
\text{tauwobs} &= \tau_{w,t} \\
\text{taukobs} &= \tau_{k,t}
\end{aligned}
$$

The steady state values of the observables are dyobs=dinvobs=dginvobs=dcobs=dwobs=log(1+ g), labobs=0, robs=$R_n - 1$, pinfobs=log(Π), tauwobs $= \tau_w$ and taukobs $= \tau_k$.

The original data are taken from the FED Database available through the Federal Reserve Bank of St.Louis for the US economy. The data consists of 10 quarterly time series, namely log output growth (*dyobs*), log consumption growth (*dcobs*), log investment growth (*dinvobs*), log wage growth (*dwobs*), labour hours supply (*labobs*), the net inflation (*pinfobs*), and finally the policy rate measurement (*robs*). The sample period is 1979Q1 to 2019Q4. There is a pre-sample period of 4 quarters so the observations actually used for the estimation go from 1979Q1 to 2019Q4, 160 observations.

Table 5: St. Louis FED original data

Original FED data	Description	Model to data
GDPC96 LFU800000000	Real GDP Population level - 16 Years and Older	$yobs = Ln(GDPC96/LFU800000000)$
PCEC index LNS10000000	Personal consumption expenditure LNS10000000(1992:3)=1 Labour Force Status - 16 Years and older	$cobs = Ln((PCEC/GDPDEF)/index)$
FPI	Fixed private investment	$invobs = Ln((FPI/GDPDEF)/index)$
A782RC1Q027SBEA	Gross public investment	$ginvobs = Ln((GGI/GDPDEF)/index)$
PRS85006103	Hourly wage	$wobs = Ln(PRS85006103/GDPDEF)$
PRS85006023 index_ce16ov	Average weekly hours Employment - 16 Years and older	$labobs = Ln\left(\frac{(PRS85006023*index\_ce16ov/100)}{LFU800000000}\right)$
GDPDEF	GDP-Implicit Price Deflator-1996=100	$pinfobs = Ln(GDPDEF/GDPDEF(-1))$
FEDFUNDS	Federal Funds Effective Rate	$robs = FEDFUNDS$,

Then we calculate the log growth rates of output (*dyobs*), consumption (*dcobs*), investment (*dinvobs*), public investment (*dginvobs*) and wage (*dwobs*) by taking the first difference of *yobs*, *cobs*, *invobs*, *ginvobs* and *wobs*, respectively.

Regarding the detail of calculating the tax rates, we follow Jones (2002) to calculate the **average tax rates**. We first begin by finding τ, the average personal income tax rate:

$$\tau = \frac{FIT + SIT}{PI + PRI + CP + NI} = \frac{FIT + SIT}{PI + PRI/2 + CI} \tag{F.1}$$

Where FIT denotes federal income taxes; SIT is state and local income taxes; PI is Personal income that persons receive in return for their provision of labor, land, and capital used in current production and the net current transfer payments that they receive from business and from government; PRI denotes the proprietor's income; CP denotes corporate profits; and NI denotes the net interest. And $CI = PRI/2 + CP + NI$ is the capital income.

The labor tax rate, τ_w, is then calculated as

$$\tau_w = \frac{\tau[PI + PRI/2] + CSI}{EC + PRI/2}, \tag{F.2}$$

where

- CSI = Total contributions to social insurance;
- EC = Total employee compensation.

In addition to wages and salaries, employee compensation includes contributions to social insurance and untaxed benefits.

The capital tax rate, τ_k, is calculated as

$$\tau_k = \frac{\tau CI + CT + PT}{CI + PT}, \tag{F.3}$$

where

- CT is the Corporate taxes
- PT Property taxes

We add property taxes to the denominator as they are deducted from profits. On the other hand, we exclude returns (net of depreciation) to durable goods.

For the public investment we use the data on the **Gross government investment (GGI)**

The time series are constructed for the period from 1979Q1 to 2019Q4.

G Bayesian Estimation

Table 6: Estimation results - Production Parameters and the Loglikelihood.

Parameters		Prior		Post. CD		Post. CES	
	pdf	Mean	Std	Mean	s.d	Mean	s.d
(ϵ_A)	**IG**	0.001	0.02	0.0070	0.0004	0.0070	0.0005
(ϵ_G)	**IG**	0.001	0.02	0.0308	0.0018	0.0305	0.0018
(ϵ_{AU})	**IG**	0.001	0.02	0.0179	0.0011	0.0180	0.0011
(ϵ_{MCS})	**IG**	0.001	0.02	0.0171	0.0031	0.0176	0.0025
(ϵ_{MRSS})	**IG**	0.001	0.02	0.0297	0.0033	0.0297	0.0033
(ϵ_{MPS})	**IG**	0.001	0.02	0.0038	0.0003	0.0038	0.0003
(ϵ_{RPS})	**IG**	0.001	0.02	0.0067	0.0020	0.0075	0.0018
(ϵ_{IS})	**IG**	0.001	0.02	0.0208	0.0023	0.0203	0.0023
(ϵ_{τ_w})	**IG**	0.001	0.02	0.0198	0.0012	0.0198	0.0012
(ϵ_{τ_k})	**IG**	0.001	0.02	0.0224	0.0013	0.0223	0.0014
(ρ_A)	**B**	0.50	0.20	0.9840	0.0041	0.9839	0.0041
(ρ_G)	**B**	0.50	0.20	0.9576	0.0109	0.9581	0.0110
(ρ_{AU})	**B**	0.50	0.20	0.9735	0.0127	0.9741	0.0126
(ρ_{MCS})	**B**	0.50	0.20	0.9326	0.0242	0.9309	0.0265
(ρ_{MRSS})	**B**	0.50	0.20	0.9478	0.0091	0.9479	0.0091
(ρ_{MPS})	**B**	0.50	0.20	0.5250	0.0540	0.5825	0.0579
(ρ_{RPS})	**B**	0.50	0.20	0.4861	0.1009	0.4307	0.1038
(ρ_{IS})	**B**	0.50	0.20	0.8699	0.0337	0.8687	0.0355
(σ_c)	**N**	1.50	0.275	1.3213	0.0870	1.3448	0.0903
(ψ)	**N**	2	0.75	2.2051	0.3542	2.1658	0.3550
(χ)	**B**	0.50	0.10	0.3970	0.0402	0.3991	0.0401
(ϕ_X)	**N**	2.0	0.75	1.0774	0.2533	1.0646	0.2309
(ξ_p)	**B**	0.50	0.10	0.7366	0.0288	0.7429	0.0303
(ξ_w)	**B**	0.50	0.10	0.4377	0.0485	0.4417	0.0504
(γ_p)	**B**	0.50	0.10	0.2797	0.0588	0.2675	0.0656
(γ_w)	**B**	0.50	0.10	0.5360	0.1039	0.5409	0.1040
(γ_2)	**B**	0.50	0.15	0.9146	0.0293	0.9138	0.0302
(ϕ_b)	**N**	0.0055	0.10	0.0003	0.0001	0.0004	0.0001
(θ)	**B**	0.50	0.20	0.9150	0.0359	0.6198	0.3760
(ρ)	**N**	1.0	0.75	1.3393	0.0784	1.3267	0.0734
(ϑ)	**N**	0.50	0.75			-1.1021	0.3830

Table 7: Estimation results - Parameters (the rules)

Parameters	Prior			Post. CD		Post. CES	
	pdf	Mean	Std	Mean	s.d	Mean	s.d
(ρ_r)	**B**	0.75	0.10	0.3667	0.0588	0.3296	0.0602
(θ_π)	**N**	1.50	0.25	2.2469	0.1765	2.2626	0.1711
(θ_y)	**N**	0.12	0.05	-0.0058	0.0148	-0.0087	0.0152
(θ_{dy})	**N**	0.12	0.05	0.1995	0.0376	0.2011	0.0378
(ρ_{τ_k})	**B**	0.75	0.10	0.9272	0.0165	0.9264	0.0162
(θ_{b,τ_k})	**N**	1.50	0.25	1.3219	0.2216	1.3278	0.2253
(θ_{y,τ_k})	**N**	0.12	0.05	0.1582	0.0236	0.1420	0.0988
(θ_{dy,τ_k})	**N**	0.12	0.05	0.1422	0.0348	0.1528	0.1000
(ρ_{τ_w})	**B**	0.75	0.10	0.8957	0.0323	0.8781	0.0326
(θ_{b,τ_w})	**N**	1.50	0.25	0.5310	0.1278	0.3944	0.1246
(θ_{y,τ_w})	**N**	0.12	0.05	0.1868	0.0910	0.2027	0.0920
(θ_{dy,τ_w})	**N**	0.12	0.05	0.1530	0.0998	0.1469	0.0998

H Reproduction of Ramey (2020)

This section reproduces Ramey (2020) which uses a CD production function with time-to-build but replaces the 1% permanent increase in authorized investment with an AR(1) process with persistence coefficient ρ_{AU} at the estimated value (table (6)) as in (C.8).

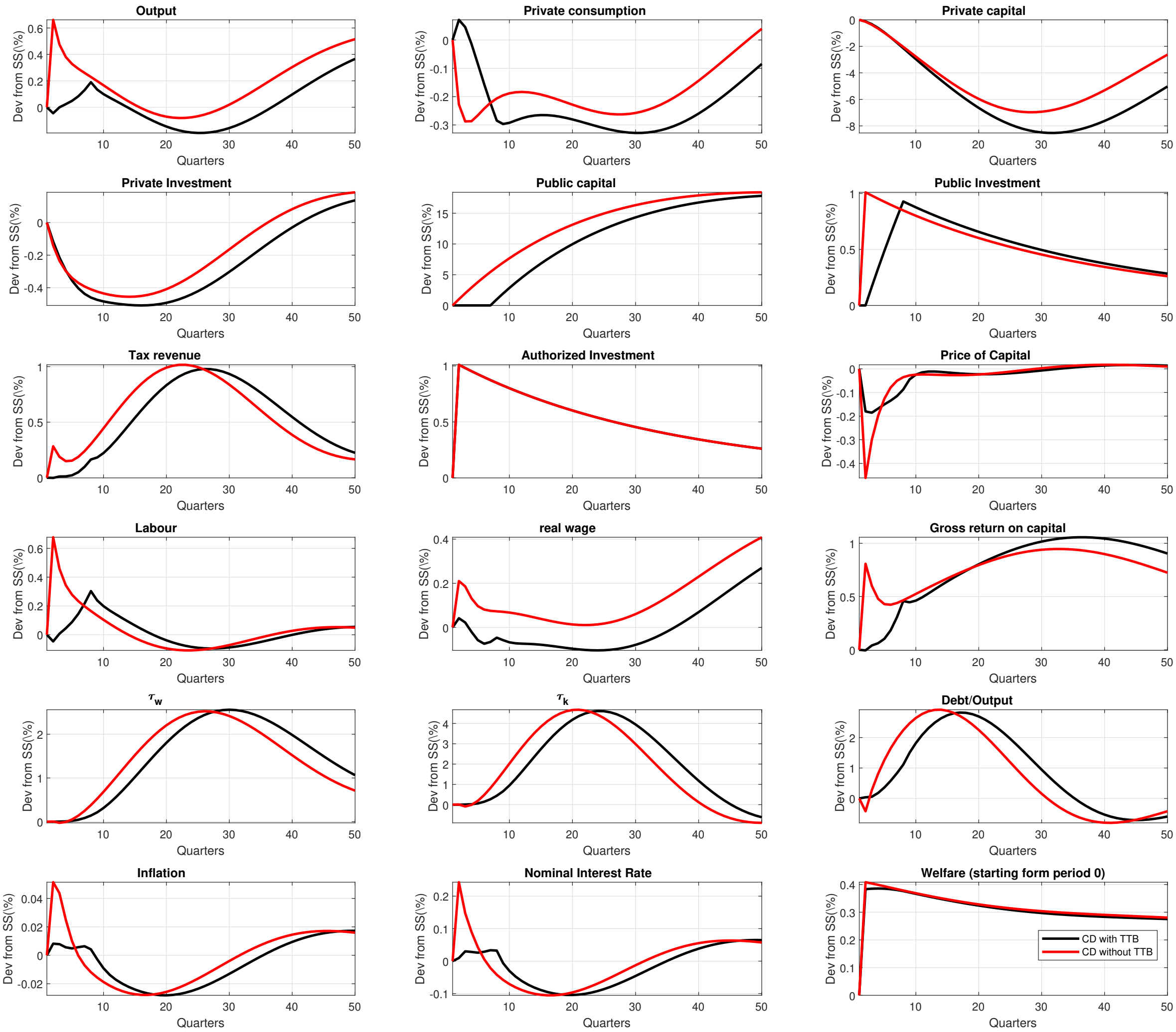

Figure 10: CD production with and without TTB. The **Output, Private consumption, Private capital, Private investment, Public capital, Public investment, Tax revenue, and Authorized Investment** are the deviations from their own steady state as a percentage of steady state output, other variables are the deviations from their own steady state as a percentage of their own steady state.

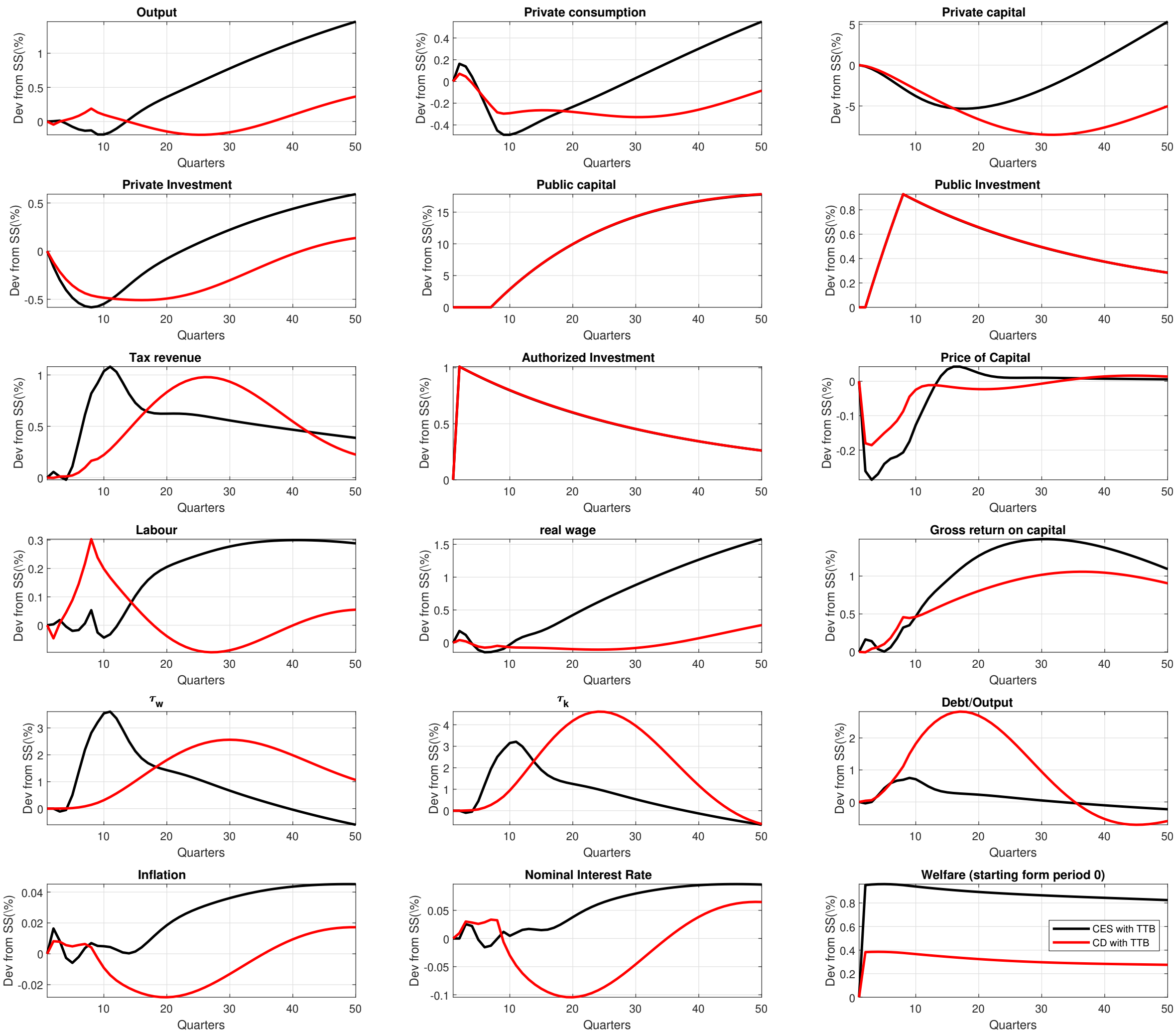

Figure 11: CES vs CD production with TTB. The **Output, Private consumption, Private capital, Private investment, Public capital, Public investment, Tax revenue, and Authorized Investment** are the deviations from their own steady state as a percentage of steady state output, other variables are the deviations from their own steady state as a percentage of their own steady state.