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THE COSTS AND BENFITS OF INFORMALITY

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The Costs and Benefits of Informality *

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Abstract

We explore the costs and benefits of informality associated with the informal sector lying outside the tax regime in a two-sector New Keynesian model. The informal sector is more labour intensive, has a lower labour productivity, is untaxed and has a classical labour market. The formal sector bears all the taxation costs, produces all the government services and capital goods, and wages are determined by a real wage norm. We identify two welfare costs of informalization: (1) long-term costs restricting taxes to the formal sector and (2) short-term fluctuation costs of tax changes to finance fluctuations in government spending. The benefit of informality derives from its wage flexibility. We investigate whether taxing the informal sector and thereby reducing its size sees a net welfare improvement.

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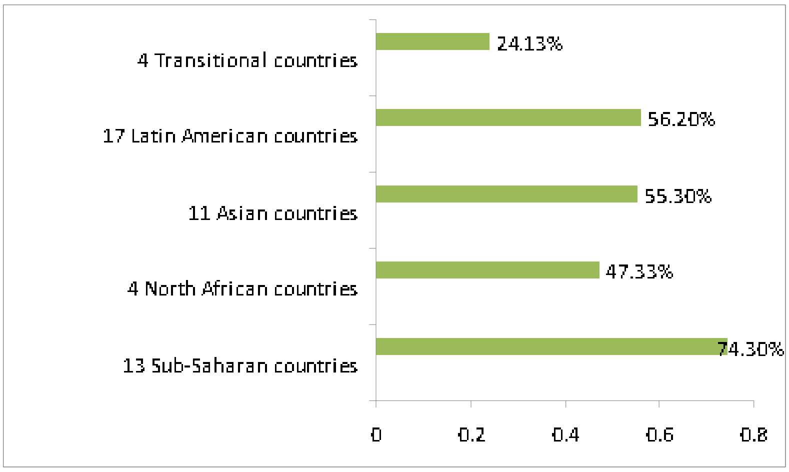
1 Introduction

With around 60% of workers employed informally, mainly in developing and emerging economies, and with a possible increase in the number of informally employed due to the recent economic crises, informality can be expected to stay for many years to come (Jutting and de Laiglesia (2009)). The OECD document suggests: *"Governments should face this reality and incorporate informal employment into their policy making"*. At present we do not know much about the efficacy of monetary and fiscal policies in economies with a large informal sector. We believe that the study of informality can shed new light on the impact of macroeconomic policies on the economic performance of developing and emerging economies. Below we show some figures describing the importance of the informal economy in developing and emerging economies in terms of employment and GDP shares. Informal employment, namely jobs and activities in the production and sales of legal goods which are not regulated or protected by the state, ranges from 25% to 75%.



Source: ILO, 2002

Figure 1: Informal Employment as % of Non-Agricultural Employment

The phenomenon, that we refer in our paper as 'informality' has been discussed using different terminology: unregistered, hidden, shadow, underground and, in a more restrictive sense black, economy. Chen (2007) describes the move from the 'old' concept of the informal sector to a more comprehensive view of the informal economy. The 'new' view of informality which focuses on the worker and informal employment, that is employment without any sort of protection, includes self-employment in unregistered firms and wage employment in unprotected jobs. [1] According to the definition used, the estimates of the

[1] In this paper we mainly look at the informal sector which can be defined as "all informal enterprises" so employment in the informal sector refers to all employment in enterprises classified as informal according

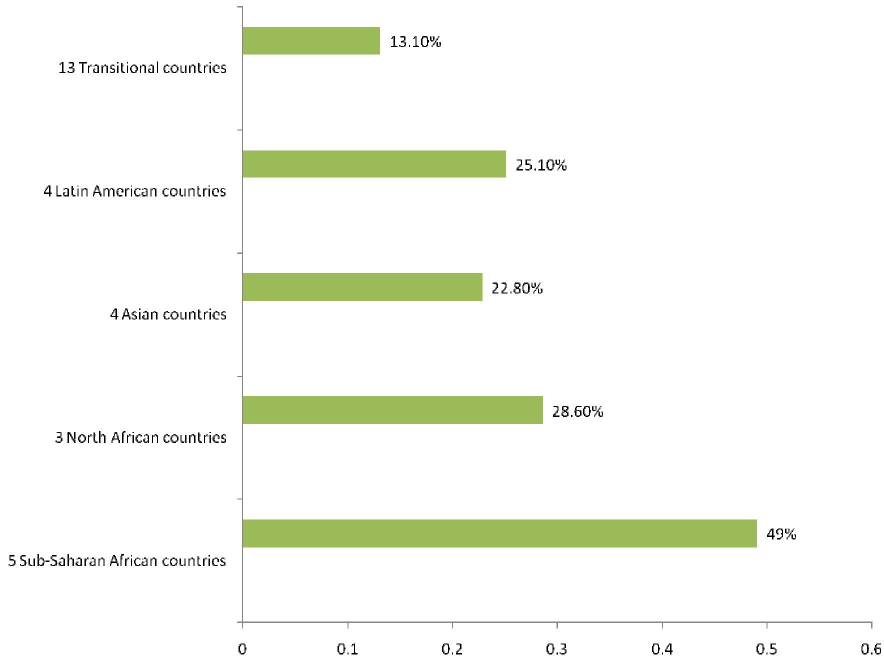


Figure 2: Informal Sector as % of GDP

size of the informal economy can be very different as figures 1 and 2 imply.

The common view is that a large informal sector is thought to be detrimental for the official economy, but here we do not take any particular position and we attempt to identify the impact of informality on the formal economy by weighting, both, costs and benefits of economies with a large informal sector. On the one hand, the informal enterprises can be seen as less productive due to the limited access to credit and/or public services. Similarly, the informal sector is often associated with inferior working conditions and low fiscal revenue. On the other hand, in a world with various kind of rigidities, the informal sector can benefit the formal economy by allowing more flexibility in the system[2]. Dell'Anno (2008) provides an interesting overview on the positive and negative impact of informality looking at the substitution or complementarity hypothesis between the two sectors. In the paper, the informal sector can act as a stabilizer, but the impact on GDP growth is ambiguous.[3] The assessment in terms of costs and benefits of informality is done by modelling what we believe are some of the most relevant positive and negative aspects of the informal economy.

to a common set of criteria (i.e. size and registration status). It follows that we do not distinguish informal workers in the informal and in the formal sectors. See Bosch (2006) and Bosch (2007) for a modeling of formal and informal contracts within the formal sector.

[2]See Batini *et al.* (2010).

[3]In the next section we provide more insights on this point.

Table 1: **Formal-Informal Sector Differences**

	Labour Market	**Productivity**	**Taxation**	**Labour Share**
F Sector	frictions	high	taxed	low
I Sector	no frictions	low	untaxed	high

Here we focus on the labour market aspects of informality, and study the costs and benefits of informality in an economy where the size of the informal sector depends on an employment tax. This model describes an economy with two sectors producing two different goods. In equilibrium, workers who do not find a job in the unionized formal labour market (i.e. the sector with a higher labour standard), move to the informal sector. In our model public goods are produced formally and the two sectors have different technologies, the informal sector being more labour intensive. A further distinction is that we introduce market friction in the labour market in the formal sector, whilst the informal sector is frictionless in this respect.

A further distinction is that taxes required to finance government spending are confined to the formal sector. Thus we capture some of the main characteristics of the informal sector: labour-intensiveness, low productivity and wage flexibility. These differences between the two sectors are summarized in Table 1.

Price stickiness is added to both sectors to give us a New Keynesian aspect and a model that can be used to investigate monetary policy. We study a balance budget fiscal policy where distortionary taxes adjust to exogenous government spending and optimal monetary policy. Our modelling approach then captures the *a priori* ambiguous impact of informality. On the one hand, the flexible and frictionless informal labour market reduces business cycle costs. On the other hand, informality brings about a cost owing to the realistic assumption that it lies outside the tax regime. Our experiment consists of allowing the taxes to be gathered from the informal sector (in which cases it looses a key characteristic of informality) resulting in a reduction in the size of this sector. There are then benefits from tax smoothing across the two sectors, but costs from a reduction in wage flexibility. Our objective is to quantify the net gain gain or loss from this change in the tax regime. To our knowledge, this is the first paper that try to quantify the costs and benefits of informality in a dynamic general equilibrium model with New Keynesian features.

The remainder of the paper is organized as follows. Section 2 shows how our general equilibrium economy relates to similar theoretical frameworks within the DSGE and the informal economy literatures. Section 3 sets out details of our model. Section 4 describes the calibration based on the steady state. Section 5 studies optimal monetary policy alongside the two tax regimes with a balanced budget constraint. Section 5 concludes.

2 Background Literature

Conesa *et al.* (2002) and Ihrig and Moe (2004) represents an attempt in introducing an informal sector within a dynamic general equilibrium RBC framework. Ihrig and Moe (2004) in a dynamic general equilibrium model describe the informal sector trade-off between taxes and productivity.[4] These papers introduce a second sector into a standard Real Business Cycle (RBC) model which is described as an “underground” economy that has a different technology, produces goods and services that could otherwise be produced in the formal sector, but is not registered in NI accounts.

Our model is related to a series of works that incorporate labour market frictions into New Keynesin (NK) DSGE models to explain the cyclical behaviour of employment, job creation, job destruction and inflation rate in response to a monetary policy shock. Castillo and Montoro (2008) formally introduce an informal sector in a DSGE model developed from Blanchard and Gali (2007) and model a labour market economy with formal and informal labour contracts within a New Keynesian model with labour market frictions. Informality is a result of hiring costs, which are a function of the labour market tightness. In equilibrium , firms in the wholesale sectors balance the higher productivity of a formal production process with the lower hiring costs of the informal process. Marginal costs will then become a function also of the proportion of informal jobs in the economy. The interesting results of this theoretical framework is that during period of high aggregate demand the informal sector expands due to lower hiring costs associated with this technology. This creates a link between informality and the dynamics of inflation. In particular, the authors show that *“informal workers act as a buffer stock of labour that allows firms to expand output without putting pressure on wages”.* Castillo and Montoro (2008) allow for a voluntary decision where the marginal worker is indifferent between formal and informal sector. Labour market regulations may reduce labour demand without introducing segmentation per se. While we recognize this picture is realistic in many advanced economies and there is also evidence that shows the existence of a voluntary, small firms sector in some developing countries (see Perry *et al.* (2007)), we believe that in the majority of the developing world informality is a result of *segmentation* where workers turn to the informal labour market when they cannot find a job in the formal sector. For this reason

[4]The informal sector firms produce the same good of the formal sector paying lower taxes, but due to limited access to capital they are less productive than their formal counterpart. Due to the assumption of homogeneous good, the size of the informal sector is mainly driven by capital accumulation.

we depart from Castillo and Montoro (2008) and, as in Satchi and Temple (2009) and Marjit and Kar (2008) we model the idea that: *"Unemployment is a luxury"* and that *"informal sector activities provide an unofficial safety-net in the absence of state-provided unemployment insurance"*.

As in Zenou (2008), we allow for a frictionless informal labour market.[5] We do not model this idea explicitly, but our competitive informal labour market implies free-entry and an instantaneous hiring process. While Zenou's framework has no NK features and focuses on the evaluation of various labour market policies on the unemployment rate of an economy with an informal sector, we also introduce labour market frictions in the formal sector, but we do not explicitly model the matching process. Rather we follow another modelling option favoured in the literature by introducing a wage norm in the formal sector. While we explore the general equilibrium features of informality, our model is in line with the Harris and Todaro tradition (Harris and Todaro, 1970) in describing a very simple labour market structure where labour in the formal sector is fixed at a higher than the market clearing level. See also Marjit and Kar (2008) and Agenor and Montiel (1996) for a similar assumption. As discussed in Satchi and Temple (2009), a richer labour market structure implies a wage in the formal sector which is endogenously determined. While this can be a promising future development we believe the simplifying assumption allows us to obtain interesting conclusions without adding further complications to the already complex modelling framework. In this respect, we should also mention that, following the critics on the inability of the search matching model to generate the observed unemployment volatility as reported in Shimer (2005), a series of papers depart from the flexible wage assumption in order to generate enough volatility in the unemployment rate (see Blanchard and Gali (2007), Krause and Lubik (2007) and Christoffel and Linzert (2005)). The introduction of a real wage norm in New Keynesian models has been described as one of the possible way to reconcile the model with the data.[6]

Our paper contribution to this literature is as follows. First we compare the costs and benefits of *increasing* the size of the formal sector, by allowing a more equal distribution of taxes between the two regimes.[7] Second, we look at the efficacy of monetary policy and for this reason we require a more general framework with price rigidity. We introduce

[5] As clarified in Zenou's paper *".. in the informal sector, either people are self-employed or work with relatives or friends and thus do not apply formally for jobs posted in newspapers or employment agency"*.

[6] However the introduction of such real wage rigidity is not immune of critics. Thomas (2008) introduces staggered nominal wages and points to a series of advantages of his approach with respect to the real wage norm assumption while Hornstein *et al.* (2005) and Pissarides (2008) claim that wage rigidity needs to be accompanied by an unrealistic assumption on the labour share and points instead at the introduction of demand shocks as a possible solution to the unemployment volatility puzzle.

[7] Clearly, the exercise of increasing taxes in the informal sector is not costless given the difficulties of observing and taxing the informal sector. We comment on this point in the conclusions.

New Keynesian price rigidities in the usual way, as in Castillo and Montoro (2008), but then proceed to analyse the interaction of informal and formal sectors and the implications for monetary policy. Our analysis of both the steady-state (long-run) and business cycle (short-run) costs and benefits of an informal economy are particularly novel features.

3 The Model

Consider a two-sector "Formal" (F) and "Informal" (I) economy, producing different goods with different technologies which sell at different retail prices, $P_{F,t}$ and $P_{I,t}$, say. Labour and capital are the variable factor inputs and the informal sector is less capital intensive. Government spending is financed by an employment tax as in Zenou (2008). In the general set-up this can be shared by the formal and informal sectors giving us a framework in which the role of tax incidence can be studied as one of the drivers of informalization. The other driver in our model is the degree of real wage rigidity in the formal sector

To help the exposition, we first abstract from investment costs and government debt.

3.1 Households

A proportion $n_{F,t}$ of household members work in the formal sector. Hours $h_{F,t}$ and hours $h_{I,t}$ are supplied in the F and I sectors respectively. Members who work in sector $i = I, F$ derive utility $U(C_t, L_{i,t})$ where C_t is household shared consumption and leisure $L_{i,t} = 1 - h_{i,t}$ and we assume that[8]

$$U_C > 0,\ U_L > 0,\ U_{CC} \leq 0,\ U_{LL} \leq 0 \tag{1}$$

. The representative household single-period utility is

$$\Lambda_t = \Lambda(C_t, n_{F,t}, h_{F,t}, h_{I,t}) = n_{F,t} U(C_t, 1 - h_{F,t}) + (1 - n_{F,t}) U(C_t, 1 - h_{I,t}) \tag{2}$$

We construct Dixit-Stiglitz consumption and price aggregates

$$C_t = \left[\mathrm{w}^{\frac{1}{\mu}} C_{F,t}^{\frac{\mu-1}{\mu}} + (1-\mathrm{w})^{\frac{1}{\mu}} C_{I,t}^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \tag{3}$$

$$P_t = \left[\mathrm{w}(P_{F,t})^{1-\mu} + (1-\mathrm{w})(P_{I,t})^{1-\mu} \right]^{\frac{1}{1-\mu}} \tag{4}$$

[8] Our notation is $U_C \equiv \frac{\partial U}{\partial C}$, $U_L \equiv \frac{\partial U(C,L)}{\partial L}$, $U_{CC} \equiv \frac{\partial^2 U}{\partial C^2}$ etc.

Then standard inter-temporal and intra-temporal decisions lead to

$$\frac{\Lambda_{C,t}}{P_t} = \beta E_t \left[(1+R_{n,t}) \frac{\Lambda_{C,t+1}}{P_{t+1}} \right] \tag{5}$$

$$C_{F,t} = \mathrm{w} \left(\frac{P_{F,t}}{P_t} \right)^{-\mu} C_t \tag{6}$$

$$C_{I,t} = (1-\mathrm{w}) \left(\frac{P_{I,t}}{P_t} \right)^{-\mu} C_t \tag{7}$$

where $R_{n,t}$ is the nominal interest rate over the interval $[t, t+1]$ for riskless bonds set by the central bank at the beginning of the period. Note that substituting (6) and (7) into (3) gives (4) so that (4) or (3) are superfluous for the set-up. Total labour supply is found by equating the marginal rate of substitution between labour and leisure with the real wages for the two sectors:

$$\frac{U_{L_I,t}}{\Lambda_{C,t}} = \frac{W_{I,t}}{P_t} \tag{8}$$

$$\frac{U_{L_F,t}}{\Lambda_{C,t}} = \frac{W_{F,t}}{P_t} \tag{9}$$

We assume that the real wage in the formal sector is a combination of an exogenous real wage norm, RW_t and the market-clearing real wage in the informal sector:

$$\frac{W_{F,t}}{P_t} = RW_t > \frac{W_{I,t}}{P_t} \tag{10}$$

From $RW_t > \frac{W_{I,t}}{P_t}$, it follows from $U_{LL} < 0$ that the household will choose less leisure and more work effort in the formal sector; i.e., $h_{F,t} > h_{I,F}$.

3.2 Wholesale Firms

Wholesale output in the two sectors is given by a Cobb-Douglas production function

$$Y^W_{i,t} = F(A_{i,t}, N_{i,t}, K_{i,t}),\ i = I, F \tag{11}$$

where $A_{i,t}$ are a technology, total labour supply $N_{i,t} = n_{i,t} h_{i,t}$, $i = I, F$. Capital inputs are $K_{i,t}$, $i = I, F$ and we assume capital is accumulated from formal output only.

The first-order conditions are

$$P^W_{F,t}F_{N_F,t} = W_{F,t} + P_t\tau_{F,t} \tag{12}$$

$$P^W_{I,t}F_{N_I,t} = W_{I,t} + P_t\tau_{I,t} \tag{13}$$

$$P^W_{I,t}F_{K_I,t} = P^W_{F,t}F_{K_F,t} = P_t[R_t + \delta] \tag{14}$$

where $P^W_{F,t}$ and $P^W_{I,t}$ are wholesale prices, $\tau_{F,t}$, $\tau_{F,t}$ are the employment real tax rates in the formal sector and informal sectors respectively and $R_t + \delta$ is the real cost of capital (in consumption units), the ex post real interest rate over the interval $[t-1, t]$ plus the depreciation rate. R_t is defined by

$$1 + R_t = \left[(1 + R_{n,t-1})\frac{P_{t-1}}{P_t}\right] \tag{15}$$

where $R_{n,t}$ is the nominal interest charged on loans made in period t.

3.3 Retail Firms

We now introduce a retail sector of monopolistic firms within each sector buying wholesale goods and differentiating the product at a proportional resource cost $c_iY^W_{i,t}$ in sectors $i = F, I$. In a free-entry equilibrium profits are driven to zero. Retail output for firm f in sector is then $Y_{i,t}(f) = (1 - c_i)Y^W_{i,t}(f)$ where $Y^W_{i,t}$ is produced according to the production technology (11) at prices $P^W_{i,t}$. Let the number of differentiated varieties produced in the informal and formal sectors be ν_F and ν_I respectively. Each is produced by a single retail firm and the numbers of these firms is fixed.[9] Let $C_{F,t}(f)$ and $C_{I,t}(f)$ denote the home consumption of the representative household of variety f produced in sectors F and I. Aggregate consumption of each category now become indices

$$C_{F,t} = \left[\left(\frac{1}{\nu_F}\right)^{\frac{1}{\zeta_F}} \sum_{f=1}^{\nu_F} C_{F,t}(f)^{(\zeta_F-1)/\zeta_F}\right]^{\zeta_F/(\zeta_F-1)} \tag{16}$$

$$C_{I,t} = \left[\left(\frac{1}{\nu_I}\right)^{\frac{1}{\zeta_I}} \left(\sum_{f=1}^{\nu_I} C_{I,t}(f)^{(\zeta_I-1)/\zeta_I}\right)\right]^{\zeta_I/(\zeta_I-1)} \tag{17}$$

[9]This model structure closely follows a model of two interacting economies in the New Open Economy Literature.

where $\zeta_F, \zeta_I > 1$ are the elasticities of substitution between varieties in the two sectors. Aggregate output is similarly defined:

$$Y_{F,t} = \left[\left(\frac{1}{\nu_F} \right)^{\frac{1}{\zeta_F}} \sum_{f=1}^{\nu_F} Y_{F,t}(f)^{(\zeta_F - 1)/\zeta_F} \right]^{\zeta_F/(\zeta_F - 1)} \tag{18}$$

$$Y_{I,t} = \left[\left(\frac{1}{\nu_I} \right)^{\frac{1}{\zeta_I}} \left(\sum_{f=1}^{\nu_I} Y_{I,t}(f)^{(\zeta_I - 1)/\zeta_I} \right) \right]^{\zeta_I/(\zeta_I - 1)} \tag{19}$$

Then the optimal intra-sectoral decisions are given by standard results:

$$C_{F,t}(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} C_{F,t} \tag{20}$$

$$C_{I,t}(f) = \left(\frac{P_{I,t}(f)}{P_{I,t}} \right)^{-\zeta_I} C_{I,t} \tag{21}$$

and inter-sector decisions are as before.

We introduce endogenous investment, I_t, and exogenous government spending G_t both assumed to consist entirely of formal output. Then maximizing the investment and government expenditure indices as for the consumer in (20) we have

$$I_t(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} I_t \tag{22}$$

$$G_t(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} G_t \tag{23}$$

Using (20)–(23) it follows that total demands for each differentiated product are given by

$$Y_{F,t}(f) = C_{F,t}(f) + I_t(f) + G_t(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} (C_{F,t} + I_t + G_t) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\zeta_F} Y_{F,t} \tag{24}$$

$$Y_{I,t}(f) = C_{I,t}(f) = \left(\frac{P_{I,t}(f)}{P_{I,t}} \right)^{-\zeta_I} C_{I,t} = \left(\frac{P_{I,t}(f)}{P_{I,t}} \right)^{-\zeta_I} Y_{I,t} \tag{25}$$

Retail firms follow Calvo pricing. In sector $i = F, I$, assume that there is a probability of $1 - \xi_i$ at each period that the price of each good f is set optimally to $\hat{P}_{i,t}(f)$. If the price is not re-optimized, then it is held constant.[10] For each producer f the objective is

[10] Thus we can interpret $\frac{1}{1-\xi_i}$ as the average duration for which prices are left unchanged in sector $i = F, I$.

at time t to choose $\hat{P}_{i,t}(f)$ to maximize discounted profits

$$E_t \sum_{k=0}^{\infty} \xi_i^k D_{t,t+k} Y_{i,t+k}(f) \left[\hat{P}_{i,t}(f) - P_{i,t+k}\mathrm{MC}_{i,t+k}\right]$$

where $D_{t,t+k}$ is the stochastic discount factor over the interval $[t, t+k]$, subject to a downward sloping demand from consumers of elasticity ζ_i given by (24) and (25), and $MC_{i,t} = \frac{P^W_{i,t}}{P_{i,t}}$ are real marginal costs. The solution to this is

$$E_t \sum_{k=0}^{\infty} \xi_i^k D_{t,t+k} Y_{i,t+k}(f) \left[\hat{P}_{i,t}(f) - \frac{\zeta_i}{(\zeta_i - 1)} P_{i,t+k}\mathrm{MC}_{i,t+k}\right] = 0 \tag{26}$$

and by the law of large numbers the evolution of the price index is given by

$$P_{i,t+1}^{1-\zeta_i} = \xi_i \left(P_{i,t}\right)^{1-\zeta_i} + (1-\xi_i)(\hat{P}_{i,t+1}(f))^{1-\zeta_i} \tag{27}$$

These summations can be expressed as difference equations as follows. First define for $i = I, F$, $\Pi_{i,t} \equiv \frac{P_{i,t}}{P_{i,t-1}} = \pi_{i,t} + 1$. Then from the Euler equation we have that $D_{t,t+k} = \beta^k \frac{U_{C,t+k}}{U_{C,t}}$. Using this result we can derive the aggregate price dynamics for $i = I, F$ as

$$\begin{aligned}
H_{i,t} - \xi_i \beta E_t[\Pi_{i,t+1}^{\zeta_i - 1} H_{i,t+1}] &= Y_{i,t} U_{C,t} && (28)\\
J_{i,t} - \xi_i \beta E_t[\Pi_{i,t+1}^{\zeta_i} J_{i,t+1}] &= \left(\frac{1}{1 - \frac{1}{\zeta_i}}\right) Y_{i,t} U_{C,t} MC_{i,t} && (29)\\
\frac{\hat{P}_{i,t}}{P_{i,t}} H_{i,t} &= J_{i,t} && (30)\\
1 &= \xi_i \Pi_t^{\zeta_i - 1} + (1-\xi_i)\left(\frac{\hat{P}_{i,t}}{P_{i,t}}\right)^{1-\zeta_i} && (31)
\end{aligned}$$

3.4 Equilibrium

Assuming Cobb-Douglas technology in the wholesale sectors (see all functional forms below) for each differentiated product in the F and I sectors we equate supply and demand in the retail sectors to give

$$\begin{aligned}
Y_{F,t}(f) &= (1-c_i)F(A_{F,t}, N_{F,t}(f), K_{F,t}(f)) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\zeta_F} Y_{F,t} && (32)\\
Y_{I,t}(f) &= (1-c_i)F(A_{I,t}, N_{I,t}(f), K_{I,t}(f)) = \left(\frac{P_{I,t}(f)}{P_{I,t}}\right)^{-\zeta_I} Y_{I,t} && (33)
\end{aligned}$$

using (24) and (25). Then solving for $N_{i,t}$, $i = F, I$ and defining aggregate employment-hours in each sector by $N_{i,t} = \sum_{j=1}^{\nu_i} N_{i,t}(j)$, $i = F, I$ we arrive to the aggregate production functions

$$Y_{i,t} = \frac{(1-c_i)A_{i,t}N_{i,t}^{\alpha_i}K_{i,t}^{1-\alpha_i}}{\Delta_{i,t}} \; ; \; i = F, I \tag{34}$$

where

$$\Delta_{i,t} = \sum_{j=1}^{\nu_i} \left(\frac{P_{i,t}(f)}{P_{i,t}} \right)^{-\frac{\zeta_i}{\alpha_i}} \tag{35}$$

is a measure of the price dispersion across firms in sector $i = F, I$. Then the aggregate equilibrium conditions in each retail sector are

$$Y_{F,t} = C_{F,t} + I_t + G_t \tag{36}$$

$$I_t = K_{t+1} - (1-\delta)K_t \tag{37}$$

$$K_t = K_{F,t} + K_{I,t} \tag{38}$$

$$Y_{I,t} = C_{I,t} \tag{39}$$

with aggregate production functions (34).

Given government spending G_t, technology $A_{i,t}$, the nominal interest rate $R_{n,t}$, the real wage norm RW_t and choice of numeraire, the above system defines a general equilibrium in C_t, P_t, $P_{i,t}$, $P_{i,t}^W$, $C_{i,t}$, $h_{F,t}$, $h_{I,t}$, $W_{F,t}$, $W_{I,t}$, $n_{i,t}$, $Y_{i,t} = (1-c_i)Y_{i,t}^W$ and $\hat{P}_{i,t}$ for $i = I, F$. The structure of the model is illustrated in Figure 3.

3.5 Monetary Policy and Government Budget Constraint

Monetary policy is conducted in terms of the nominal interest rate $R_{n,t}$ set at the beginning of period t. The expected real interest rate over the interval $[t, t+1]$ is given by

$$E_t[1 + R_{t+1}] = E_t \left[(1 + R_{n,t}) \frac{P_t}{P_{t+1}} \right] \tag{40}$$

In what follows we consider interest rate policy in the form of the optimal commitment policy.

Fiscal policy assumes a balanced budget constraint in which an employment tax on only formal firms, τ_t, finances government spending. This takes the form

$$P_{F,t}G_t = P_t(n_{F,t}h_{F,t}\tau_{F,t} + n_{I,t}h_{I,t}\tau_{I,t}) \tag{41}$$

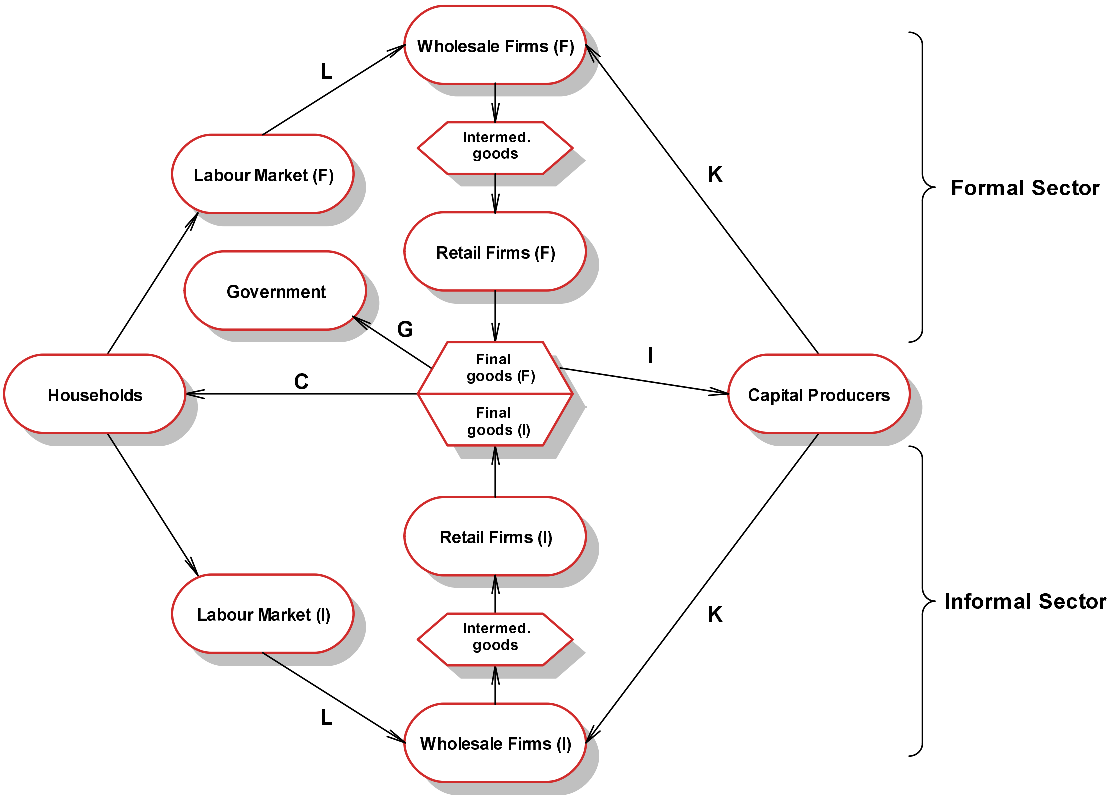


Figure 3: Model Structure

noting that government services are provided out of formal output. We assume a *tax rule*

$$\tau_{I,t} = k\tau_{F,t}\,;\;\; k \in [0,1] \tag{42}$$

allowing for the possibility that *some* tax can be collected in the informal economy.

3.6 Investment Costs

Now we generalize the model to allow for investment costs and government debt. It is convenient to introduce capital producing firms that at time t convert I_t of output into $(1 - S(X_t))I_t$ of new capital, where $X_t \equiv \frac{I_t}{I_{t-1}}$, sold at a real price Q_t. We then replace

(37) and (14) with

$$
\begin{aligned}
K_{t+1} &= (1-\delta)K_t + (1 - S(X_t))I_t\,;\ S',\, S'' \geq 0\,;\ S(1) = S'(1) = 0 \\
E_t[1 + R_{t+1}] &= \frac{E_t\left[(1-\alpha_F)\frac{P^W_{t+1}Y_{F,t+1}}{P_{t+1}K_{F,t+1}} + (1-\delta)Q_{t+1}\right]}{Q_t} \qquad (43) \\
Q_t(1 - S(X_t) - X_tS'(X_t)) &+ E_t\left[D_{t,t+1}Q_{t+1}S'(X_t)\frac{I^2_{t+1}}{I^2_t}\right] = 1 \qquad (44)
\end{aligned}
$$

Then as $S(X_t) \to 0$, $Q_t \to 1$ and (43) gets back to (14).

3.7 Functional Forms

We choose a Cobb-Douglas production function, AR(1) processes for government spending and labour-augmenting productivity, and a utility function consistent with balanced growth:

$$
\begin{aligned}
F(A_{i,t}, N_{i,t}) &= (A_{i,t}N_{i,t})^{\alpha_i}K_{i,t}^{1-\alpha_i} \qquad (45) \\
\log A_{i,t} - \log \bar{A}_{i,t} &= \rho_{A_i}(A_{i,t-1} - \bar{A}_{i,t-1}) + \epsilon_{A_i,t} \qquad (46) \\
\log G_t - \log \bar{G}_t &= \rho_G(G_{t-1} - \bar{G}_{t-1}) + \epsilon_{G,t} \qquad (47) \\
U_t(C_t, L_{i,t}) &= \frac{[C_t^{1-\varrho}L_{i,t}^{\varrho}]^{1-\sigma} - 1}{1-\sigma}\,; \quad \sigma > 1 \\
&= (1-\varrho)\log C_t + \varrho \log L_{i,t}\,; \quad \sigma = 1 \qquad (48) \\
\log\left[\frac{\bar{A}_{i,t}}{\bar{A}_{i,t-1}}\right] &= \log\left[\frac{\bar{G}_t}{\bar{G}_{t-1}}\right] = 1 + g \qquad (49)
\end{aligned}
$$

where $\epsilon_{A_i,t}, \epsilon_{G_i,t}, \sim ID$ with zero mean. The choice of utility function in (48) is chosen to be consistent with a steady state balanced growth path (henceforth BGP) where LAP $\bar{A}_t$ and $\bar{G}_t$ are time-varying. As pointed out in Barro and Sala-i-Martin (2004), chapter 9, this requires a careful choice of the form of the utility as a function of consumption and labour effort. It is achieved by a utility function which is non-separable in consumption and leisure unless $\sigma = 1$. A utility function of the form (48) achieves this. The marginal utilities are then then given by

$$
\begin{aligned}
\Lambda_{C,t} &= (1-\varrho)C_t^{(1-\varrho)(1-\sigma)-1}(n_{F,t}L_{F,t}^{\varrho(1-\sigma)} + (1-n_{I,t})L_{I,t}^{\varrho(1-\sigma)}) \qquad (50) \\
U_{L_F,t} &= \varrho C_t^{(1-\varrho)(1-\sigma)}L_{F,t}^{\varrho(1-\sigma)-1} \qquad (51) \\
U_{L_I,t} &= \varrho C_t^{(1-\varrho)(1-\sigma)}L_{I,t}^{\varrho(1-\sigma)-1} \qquad (52)
\end{aligned}
$$

4 Model Calibration and Steady State Analysis

The zero inflation balanced growth steady state of the model economy is given by

$$\frac{\bar{\Lambda}_{C,t+1}}{\bar{\Lambda}_{C,t}} \equiv 1 + g_{\Lambda_C} = \left[\frac{\bar{C}_{t+1}}{\bar{C}_t}\right]^{(1-\varrho)(1-\sigma)-1} = (1+g)^{((1-\varrho)(1-\sigma)-1)} \tag{53}$$

using (50). Thus from (5)

$$1 + R_n = 1 + R = \frac{(1+g)^{1+(\sigma-1)(1-\varrho)}}{\beta} \tag{54}$$

The rest of the steady state is given by

$$\begin{aligned} n_{I,t} + n_{F,t} &= 1 &(55)\\ P &= \left[\mathrm{w}(P_F)^{1-\mu} + (1-\mathrm{w})(P_I)^{1-\mu}\right]^{\frac{1}{1-\mu}} &(56) \end{aligned}$$

$$\begin{aligned} \bar{Y}_{i,t} &= (1-c_i)(n_i h_i \bar{A}_{i,t})^{\alpha_i} \bar{K}_{I,t}^{1-\alpha_i}\,;\; i = F, I &(57)\\ \frac{\varrho \bar{C}_t}{(1-\varrho)(1-h_i)} &= \bar{W}_{i,t}\,;\; i = F, I &(58)\\ \frac{\alpha_i P_i^W \bar{Y}_{i,t}^W}{P n_i h_i} &= \bar{W}_{i,t} + \bar{\tau}_{i,t}\,;\; i = F, I &(59)\\ \frac{\bar{W}_{F,t}}{P_t} &= \overline{RW}_t &(60)\\ \frac{P \bar{K}_{F,t}}{P^W \bar{Y}_{F,t}^W} &= \frac{1-\alpha_F}{R+\delta} &(61)\\ \frac{P \bar{K}_{I,t}}{P_I^W \bar{Y}_{I,t}^W} &= \frac{1-\alpha_I}{R+\delta} &(62)\\ \bar{I}_t &= (\delta + g)(\bar{K}_{I,t} + \bar{K}_{F,t}) &(63) \end{aligned}$$

$$\begin{aligned} \bar{Y}_{I,t} &= \bar{C}_{I,t} = (1-\mathrm{w})\left(\frac{P_I}{P}\right)^{-\mu} \bar{C}_t &(64)\\ \bar{Y}_{F,t} &= \bar{C}_{F,t} + \bar{G}_t = \mathrm{w}\left(\frac{P_F}{P}\right)^{-\mu} \bar{C}_t + \bar{I}_t + \bar{G}_t &(65)\\ \frac{P_F}{P}\bar{G}_t &= (n_F h_F \bar{\tau}_{F,t} + n_I h_I \bar{\tau}_{I,t}) &(66)\\ \bar{\tau}_{i,t} &= \tau_i \bar{W}_{i,t}\,;\; i = F, I &(67)\\ \bar{\tau}_{I,t} &= k \bar{\tau}_{F,t} &(68)\\ P_i &= \frac{1}{1-\frac{1}{\zeta_i}} P_i^W &(69) \end{aligned}$$

where consumption, labour augmenting technical change, the real wage and tax rates, and government spending (all indicated by $\bar{X}_t$) are growing at a common growth rate. We impose a free entry condition on retail firms in this steady state which drives monopolistic profits to zero. This implies that costs of converting wholesale to retail goods are given by

$$c_i = 1/\zeta_i$$

which implies that:

$$P_i \bar{Y}_{i,t} = P_i^W \bar{Y}_{i,t}^W \; ; \, i = F, I$$

Given exogenous trends for $\bar{A}_{i,t}$ and $\bar{G}_t$, the tax rates and RW_t, these equations give 21 relationships in 22 variables R, P, P_F, P_I, P_F^W, P_I^W, $\bar{C}_t$, $\bar{C}_{F,t}$, $\bar{C}_{I,t}$, $\bar{Y}_{F,t}$, $\bar{Y}_{I,t}$, $\bar{W}_{I,t}$, $\bar{W}_{F,t}$, n_I, n_F, h_I, h_F, $\bar{I}$, $\bar{K}_F$ $\bar{K}_I$, $\bar{\tau}_{F,t}$, $\bar{\tau}_{I,t}$. One of the prices (it is convenient to choose P) can be chosen as the numeraire, so the system is determinate.

Imposed Parameters	Value
δ	0.025
σ	2.0
α_F	0.5
α_I	0.8
$\xi_F = \xi_I$	0.75
$\zeta_F = \zeta_I$	7.0
μ	1.5
$\rho_{aF} = \rho_{aI} = \rho_g$	0.7
$\rho_{uI} = \rho_{uF}$	0
sd(ε_{aF})=sd(ε_{aI})= sd(ε_g)=sd(ε_{uF})= sd(ε_{uI})	1.0
Observed Equilibrium	Value
g^{obs}	0.01
n_F^{obs}	0.5
h_F^{obs}	0.45
rw^{obs}	0.5
g_{yF}^{obs}	0.15
R^{obs}	0.03
Calibrated Parameters	Value
β	0.99
w	0.81
ϱ	0.58
A_F/A_I	0.58

Table 2. Calibration

Turning to calibration, the idea is to assume an observed baseline steady state equilibrium in the presence of some observed policy. We then use this observed equilibrium to solve for model parameters consistent with this observation The calibrated parameters are: $\frac{\bar{A}_{F,t}}{\bar{A}_{I,t}}$, ϱ, w, β given observations or measurements of n_F, h_F, $\frac{\bar{W}_F}{\bar{W}_I} \equiv 1 + rw$, R and $\frac{\bar{G}_t}{\bar{Y}_{F,t}} \equiv g_{yF}$. We also use estimates of δ, σ α_I and α_F from micro-econometric studies. Appendix A sets out the details of the calibration of the parameters of the model and this is summarized in Table 2.

In Table 3 the full steady-state benchmark equilibrium with no taxation in the informal sector used for the calibration is compared with a new steady state in which both sectors have the same tax rate. In this way, proceeding from $k = 1$ back to $k = 0$ we can show how the incentive to avoid taxation drives formalization. Thus we see in the tax-smoothing case a larger formal sector ($n_F = 0.58$ as opposed to $n_F = 0.5$ in the baseline case) and a lower relative price in the formal sector (because output is higher). The real wage falls slightly in both sectors and consequently less labour is supplied per household member. The rise (fall) in the relative price of informal goods sector brings about a higher (lower) capital-output ratio in the informal (formal) sector, but the overall investment ratio is almost unchanged.

Variable	$k = 0$	$k = 1$
$\frac{P_F}{P}$	1.00	0.96
$\frac{P_I}{P}$	1.00	1.24
$\frac{W_F}{P}$	1.37	1.35
$\frac{W_I}{P}$	0.91	0.90
n_F	0.50	0.58
h_F	0.45	0.43
h_I	0.18	0.15
rel	8.82	9.97
R	0.03	0.03
τ_F	0.43	0.32
τ_I	0.0	0.32
KY_I	3.12	3.86
KY_F	7.79	7.74
i_{yF}	0.35	0.34
c_{yF}	0.50	0.51
Λ	-1.642	-1.624
		$c_e = 3.10\%$

Table 3. Steady State Equilibrium Values: $k = 0, 1$[11]

All this is with the wage mark-up in the formal sector $rw = 0.5$, our measure of wage stickiness. Figure 4 shows this process of informalization for different degrees of wage rigidity and illustrates how an increase in this friction also drives down participation in the formal sector. For example, with $k = 0$ and no friction the size of the formal sector is close to $n_F = 0.82$. When $rw = 0.75$, this halves, falling to under $n_F = 0.4$.

Figure 5 shows the welfare effects on a representative household as the tax burden is smoothed over the two sectors. As k approaches unity the utility becomes very flat and

[11] $c_e = \Delta\Lambda/0.0065 = 4.38\%$.

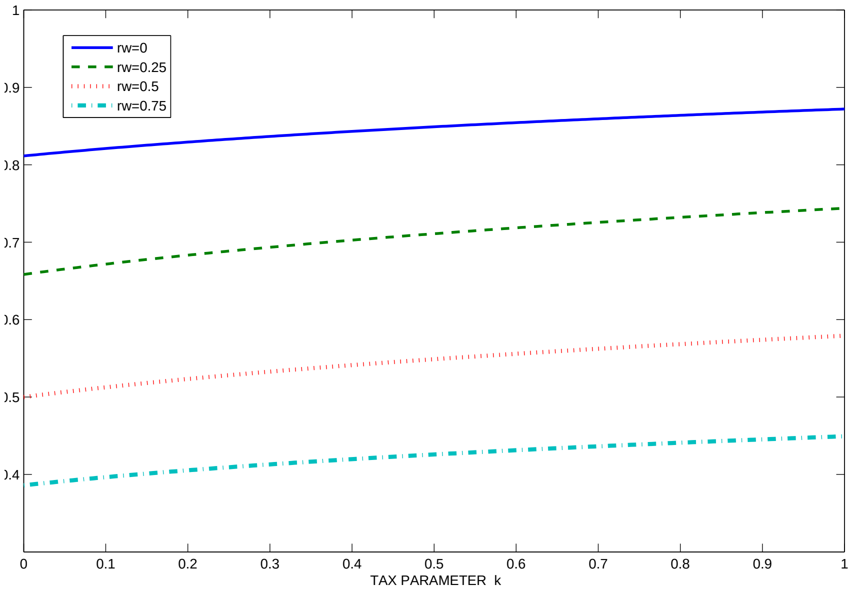


Figure 4: **The Size of Formal Sector and Tax Burden: k = Ratio of Informal-Formal Tax Rates. rw =wage mark-up in the formal sector.**

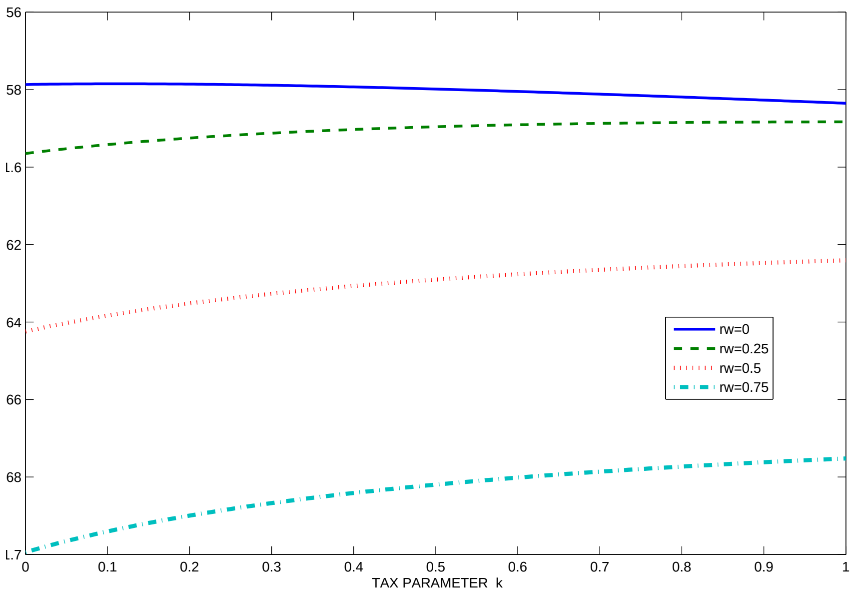


Figure 5: **Welfare and Tax Burden: k = Ratio of Informal-Formal Tax Rates. rw =wage mark-up in the formal sector.**

close to the optimum. We can work out the equivalent permanent increase in consumption implied by this optimum by first computing the increase from a 1% consumption change at any point on the balanced growth trend as $n_F U(1.01 \times \bar{C}_t, L_F) + (1 - n_F)U(\bar{C}_t, L_I)$ at some time $t = 0$ say. In our best steady state equilibrium for $rw = 0.5$ at $k = 1$, this works out as 0.0059, so any increase in welfare $D\Lambda$ implies a consumption equivalent $c_e = \frac{D\Lambda}{0.0059}\%$ as calculated in Table 3.

5 Optimal Stabilization Policy

We adopt a linear-quadratic framework for the optimization problem facing the monetary authority. This is particularly convenient as we can then summarize outcomes in terms of unconditional (asymptotic) variances of macroeconomic variables and the local stability and determinacy of particular rules. The framework also proves useful for addressing the issue of the zero lower bound on the nominal interest rate.

In our model there are three distortions that result in the steady state output being below the social optimum: namely, from monopolistic competition, from distortionary taxes and from the non-market clearing wage norm. We assume that these distortions are small in the steady state and following Woodford (2003), we can adopt a 'small distortions' quadratic approximation to the household's single period utility which is accurate in the vicinity of our zero-inflation steady state. Details of this quadratic approximation are provided in Appendix C. The loss function is given by

$$\Omega_0 = \frac{1}{2} E_t \left[\sum_{t=0}^{\infty} \beta^t [w_c c_t^2 + w_{hI} \hat{h}_I^2 + w_{hF} \hat{h}_F^2 + w_{\pi F} \pi_{F,t}^2 + w_{\pi I} \pi_{I,t}^2] \right] \tag{70}$$

where coefficients w_c, w_{hI}, w_{hF}, $w_{\pi F}$ and $w_{\pi I}$ are defined in that Appendix.

To work out the welfare in terms of a consumption equivalent percentage increase, expanding $U(C, L)$ as a Taylor series, a 1% permanent increase in consumption of 1 per cent yields a first-order welfare increase $U_C C \times 0.01$. Since standard deviations are expressed in terms of percentages, the welfare loss terms which are proportional to the covariance matrix (and pre-multiplied by 1/2) are of order 10^{-4}. The losses reported in the paper are scaled by a factor $1 - \beta$. Letting $\Delta\Omega$ be these losses relative to the optimal policy, then $c_e = \Delta\Omega \times 0.01\%$.

We can modify welfare criterion so as to approximately impose an interest rate zero lower bound (ZLB) so that this event hardly ever occurs. Our quadratic approximation to the single-period loss function can be written as $L_t = \mathsf{y}_t' Q \mathsf{y}_t$ where $\mathsf{y}_t' = [\mathsf{z}_t', \mathsf{x}_t']'$ and Q is a symmetric matrix. As in Woodford (2003), chapter 6, the ZLB constraint is implemented by modifying the single period welfare loss to $L_t + w_r r_{n,t}^2$. Then following Levine *et al.* (2008), the policymaker's optimization problem is to choose w_r and the unconditional distribution for $r_{n,t}$ (characterized by the steady state variance) shifted to the right about a new non-zero steady state inflation rate and a higher nominal interest rate, such that the probability, p, of the interest rate hitting the lower bound is very low. This is implemented by calibrating the weight w_r *for each of our policy rules* so that $z_0(p)\sigma_r < R_n$ where $z_0(p)$ is the critical value of a standard normally distributed variable Z such that prob $(Z \leq z_0) = p$, $R_n = \frac{1}{\beta(1+g_{u_c})} - 1 + \pi^* \equiv R_n(\pi^*)$ is the steady state nominal interest rate, $\sigma_r^2 = \mathrm{var}(r_n)$ is the unconditional variance and π^* is the new steady state inflation rate. Given σ_r the steady state positive inflation rate that will ensure $r_{n,t} \geq 0$ with probability $1 - p$ is given by[12]

$$\pi^* = \max[z_0(p)\sigma_r - R_n(0) \times 100, 0] \tag{71}$$

n_F	Tax Distortion	Ω_0	σ_r^2	Pr(ZLB)	π^*	c_e
0.50	No	104	0.38	0.000	0	0
0.59	No	110	0.11	0.000	0	0.06
0.50	Yes	133	0.07	0.000	0	0.29
0.59	Yes	127	0.11	0.000	0	0.23

Table 4. Optimal Rules with Commitment

Table 4 sets out results for optimal stabilization monetary rule in the face of exogenous stochastic shocks as calibrated in Table 2. We compute outcomes with and without tax distortions so that we can distinguish the consequences of a loss of wage flexibility from the gains of tax smoothing in stabilization policy as we move from a smaller to a larger formal sector. The last column in table 4 gives the consumption equivalent welfare loss from fluctuations relative to the lowest loss which occurs in the first row where tax distortions are eliminated and the size of the informal sector is highest.

Examining the first four rows without tax distortions we see that proceeding from a high ($n_I = 0.5$) to a low size of the informal sector ($n_I = 1 - 0.59 = 0.41$) results in an *increase* in welfare costs of $c_e = 0.06\%$. In fact owing to the high calibrated steady state nominal interest rate and the low volatilities reported in the Table ZLB considerations are irrelevant in this exercise. [13] The final four rows of the table incorporate tax distortions in

[12] If the inefficiency of the steady-state output is negligible, then $\pi^* \geq 0$ is a credible new steady state inflation rate. Note that in our LQ framework, the zero interest rate bound is very occasionally hit in which case the interest rate is allowed to become negative.

[13] But this is not generally the case. Levine *et al.* (2008) show how a choice of w_r and π^* can be chosen

stabilization policy and now we observe that proceeding from a larger to a smaller informal sector results in a net *decrease* in welfare costs of $c_e = 0.29 - 0.23 = 06\%$. It follows that the tax distortion effect sees a benefit of 0.12% from reducing the size of the informal sector. This is offset by losses of 0.06% from the wage flexibility effect. Table 5 summarizes this cost-benefit analysis bringing the earlier steady state and stabilization results together.

Source of Cost	c_e (%)
No Tax Smoothing at Steady State	3.10
Stabilization Cost: Wage Flexibility	-0.06
Stabilization Cost: No Tax Smoothing	0.12
Net Stabilization Cost	0.06

Table 5. Summary of The Costs of Informality.

These stabilization effects it should be noted are very small compared with the tax distortion effect on the steady state which sees a benefit $c_e = 3.10\%$ from reducing the size of the informal sector. We have performed some sensitivity analysis allowing the calibrated values to change within the limits discussed in our evidence set out for emerging economies in Appendix A and this qualitative conclusion remains intact. Of course the assumption that tax revenues can be equalized in the two sectors ($k = 1$) is an extreme one so this figure is an upper bound. Moreover it is important to stress that stabilization depend on the calibrated volatilities of the shocks. We assumed a standard deviation of 1% for all shocks, which is a plausible figure for developing economies and in line with DSGE Bayesian estimation, but on the small size for emerging economies. So let the standard deviation be scaled by factor $\kappa \geq 1$. Then the net stabilization cost in Table 5 rises to $0.06\kappa^2$. Even with $\kappa = 5$, an implausibly high value, the net stabilization effect is still dominated by the steady state effect. We must conclude that in our model and with our calibration, the steady state gains from reducing the size of the informal sector by tax equalization far outweigh the benefits from stabilization.

6 Conclusions

We have examined the possible welfare benefits of reducing the size of the informal sector by eliminating the tax incentive to be informal. The main conclusions of our paper are that there are considerable welfare benefits from tax smoothing and net benefits from

optimally to satisfy the ZLB and that the gains from commitment taking into account of this constraint rise considerably.

stabilization with tax smoothing benefits outweighing the costs in terms of less wage flexibility. We also find that the reduction in long-term costs are much more relevant in size than the short-term costs and benefits. This is in part due to the assumption of a standard deviation of 1% for all shocks. By increasing the volatility in line with evidence of emerging economies, we still see the long-term impact dominating the business cycle costs and benefits.

A couple of caveats should be mentioned. First, it would be desirable to estimate the model by Bayesian methods as is now commonplace in the literature. For advanced economies the informal sector would become the hidden economy leading to the need to properly take into account the lack of observability of this sector in solving for the rational expectations equilibrium and the estimation. This is not done in this paper, nor indeed in the DSGE literature as a whole[14] and would be an important future direction for research. Second, we have alluded to the fact that no assessment has been made of the costs of tax collection in the informal sector. However our findings indicate that the case for tolerating a large informal sector may rest entirely on these being very substantial, rather than the benefits from an increase in wage flexibility.

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[14] An exception is Levine *et al.* (2007).

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A Calibration

The idea of calibration is to assume an observed baseline steady state equilibrium in the presence of some observed policy. We then use this observed equilibrium to solve for model parameters consistent with this observation For this baseline and for the purpose of calibration only, it is convenient to choose units of wholesale output such that their prices are unitary; i.e., $P_F^W = P_I^W = 1$. Then from (69)

$$P = \left[\frac{\mathrm{w}}{\left(1 - \frac{1}{\zeta_F}\right)^{1-\mu}} + \frac{(1-\mathrm{w})}{\left(1 - \frac{1}{\zeta_I}\right)^{1-\mu}} \right]^{\frac{1}{1-\mu}} \tag{A.1}$$

We assume $\zeta_F = \zeta_I = \zeta$ in which case $P = P_F = P_I = \frac{1}{1-\frac{1}{\zeta}}$. Similarly, we can choose units of labour supply h_I, h_F so that $A_I = 1$.

We now calibrate the parameters $\frac{\bar{A}_{F,t}}{\bar{A}_{I,t}}$, ϱ, w, β given observations or measurements of n_F, h_F, $\frac{\bar{W}_F}{\bar{W}_I} \equiv 1 + rw$, R and $\frac{\bar{G}_t}{\bar{Y}_{F,t}} \equiv g_{yF}$. We also use estimates of δ, σ α_I and α_F from micro-econometric studies. For the latter Cobb-Douglas production function parameter values we draw upon a range of values estimated in the literature using the lower and upper bounds for the formal and informal sectors respectively. As is standard in the literature we assume on average one-year price contracts in both sectors so that in our quarterly model $\xi_I = \xi_F = 0.25$, and a 15% mark-up of monopolistic prices giving $\zeta_I = \zeta_F \approx 7$.

Denote observations by n_F^{obs} etc. With these observations and the steady state of the model we can deduce the unobserved variables in the steady state and the parameter values as follows:

From (58) we have

$$\frac{1 - h_I}{1 - h_F^{obs}} = \frac{\bar{W}_F}{\bar{W}_I} = 1 + rw^{obs} \tag{A.2}$$

which determines h_I.

From the government budget constraint (66) in our baseline where only the formal sector is taxed ($k = 0$) we have

$$g_{yF} = \tau_F w s_F = \frac{\tau_F}{1 + \tau_F} \alpha_F \tag{A.3}$$

which determines τ_F and $\tau_I = k\tau_F$ when both sectors are taxed. Then from (59)

$$\alpha_F = w s_F (1 + \tau_F) \tag{A.4}$$

$$\alpha_I = w s_I \tag{A.5}$$

determining ws_i, $i = I, F$. Hence from the definitions $ws_i \equiv \frac{\bar{W}_i n_i h_i}{P_i^W \bar{Y_{i,t}}^W}$ we obtain

$$ws_I = \frac{ws_F(1-n_F^{obs})h_I\,rel}{(1+rw^{obs})n_F^{obs}h_F^{obs}} \tag{A.6}$$

from which $rel \equiv \frac{P_F^W \bar{Y}_{F,t}^W}{P_I^W \bar{Y}_{I,t}^I}$ is obtained.

Now write the production functions (57) as

$$\bar{Y}_{i,t}^W = A_{i,t} n_i h_i \left(KY_i\right)^{\frac{1-\alpha_i}{\alpha_i}} \quad i = I, F \tag{A.7}$$

where KY_i is the capital-labour ratio in sector i. From (61) and (62) and using $P_F^W = P_I^W = 1$ in the baseline steady state we have

$$KY_i = \frac{1-\alpha_i}{R^{obs}+\delta}\,;\ i = I, F \tag{A.8}$$

and (A.7) we have

$$\frac{\bar{Y}_{F,t}}{\bar{Y}_{I,t}} = rel = \frac{\bar{A}_{F,t}}{\bar{A}_{I,t}} \frac{n_F^{obs}}{1-n_F^{obs}} \frac{h_F^{obs}}{h_I} \frac{KY_F^{\frac{1-\alpha_F}{\alpha_F}}}{KY_I^{\frac{1-\alpha_I}{\alpha_I}}} \tag{A.9}$$

from which $\frac{\bar{A}_{F,t}}{\bar{A}_{I,t}}$ is obtained.

To obtain w use (64) and (65) to give

$$\frac{w}{1-w} = \frac{\bar{Y}_{F,t}(1-i_{yF}-g_{yF})}{\bar{Y}_{I,t}} = rel\ c_{yF} \tag{A.10}$$

where

$$i_{yF} \equiv \frac{\bar{I}_t}{\bar{Y}_t} = \frac{(\delta+g)(\bar{K}_{I,t}+\bar{K}_{F,t})}{(1-c_F)\bar{Y}_{F,t}^W} = \frac{(\delta+g)\left(\frac{KY_I}{\hat{rel}} + KY_F\right)}{(1-c_F)} \tag{A.11}$$

$$c_{yF} = 1 - i_{yF} - g_{yF}^{obs} \tag{A.12}$$

From (A.10), (A.11) and (A.12) we now can determine w.

Finally from (58) and (64) we have

$$\frac{\varrho}{(1-\varrho)(1-h_I)} = (1-w)\frac{\bar{W}_{I,t}}{\bar{Y}_{I,t}} = \frac{(1-w)ws_I}{(1-n_F^{obs})h_I} \tag{A.13}$$

from which ϱ is obtained. Data on emerging economies can be obtained from IMF, World Bank and ILO statistics. As discussed in Neumeyer and Perri (2004) real interest rates in emerging economies are very volatile and difficult to calculate. Though nominal interest rate statistics are usually reported by local Central Banks, due to the high variability of inflation in emerging economies, the calculation of the real interest rate in EMEs countries is often cumbersome.

Uribe and Yue (2006) report quarterly data on equilibrium real interest rate for various emerging economies over the period 1994:1-2001:4 for seven developing countries. [15] We follow them and choose a quarterly real interest rate for emerging economies of 3.00%.

For GDP growth rates we assume an annual percentage change of 4%. Reinhart and Rogoff (2003) report an average GDP growth rate for a wide selection of emerging economies (annual % change) around 4% over the period 1990-2009.

We refer to LABSTAT (ILO) (ILO (2002) for the calculation of hours of work in emerging economies and choose h=45/100. Data on government shares can be obtained from different sources such as IMF and World Bank. We choose World Bank and calculate an average for selected EMEs countries to obtain a value equal to 15%. [16] For values of wage mark-up in the formal sector, we refer to Perry *et al.* (2007) where Latin American data are reported. Table 3.1 shows that, on average, informal salaried workers earn between 40 to 66 percent less than formal salaried workers. Looking at this figures, we choose a mark-up of 50%. Finally, data on the formal sector employment as reported in various ILO's documents range from 60 percent to 35% in selected EMEs countries with a particular low level of 15% in India. We choose a value of 50% which is also consistent with Spatz (2003) for Bolivia (see table 4 of their working paper).

This completes the calibration of the parameters describing the endogenous component of the model. There are currently two exogenous shocks in the model to labour productivity in both sectors and government spending. In the linearized model of Appendix B these are denoted respectively by $a_{i,t}$ and $g_{i,t}$, $i = I, F$. We also add mark-up shocks to the linearized Phillips Curves $u_{i,t}$, $i = I, F$. Again following the literature we assume AR(1) processes with calibrated persistence parameters 0.7 for the technology and demand shocks. Mark-up shocks are assumed to be transient. The standard deviations of the innovation processes are taken to be unity, but later we examine more volatile economies with a standard deviation $k > 1$. This completes the calibration; observations, imposed and calibrated parameters are summarized in Table 2 of the main text.

B Linearization

Define lower case variables $x_t = \log \frac{X_t}{X_t}$ if X_t has a long-run trend or $x_t = \log \frac{X_t}{X}$ otherwise where X is the steady state value of a non-trended variable. For variables $n_{F,t}$, $n_{I,t}$ and

[15] Argentina, Brazil, Ecuador, Mexico, Peru', Philippines and South Africa.

[16] In general, government spending in emerging economies is lower than the one in developed economies.

h_t define $\hat{x}_t = \log \frac{x_t}{x}$; $r_{n,t} \equiv \log \left(\frac{1+R_{n,t}}{1+R_n} \right)$; $\pi_{i,t} \equiv \log \left(\frac{1+\Pi_{i,t}}{1+\Pi_i} \right)$, $i = I, F$ are log-linear *gross* interest and inflation rates.

Our linearized model about the BGP zero-inflation steady state then takes the state-space form

$$
\begin{aligned}
a_{F,t+1} &= \rho_{aF} a_{F,t} + \varepsilon_{aF,t+1} && \text{(B.1)} \\
a_{I,t+1} &= \rho_{aI} a_{I,t} + \varepsilon_{aI,t+1} && \text{(B.2)} \\
g_{t+1} &= \rho_g g_t + \varepsilon_{g,t+1} && \text{(B.3)} \\
u_{F,t+1} &= \rho_{uF} u_{F,t} + \varepsilon_{uF,t+1} && \text{(B.4)} \\
u_{I,t+1} &= \rho_{uI} u_{I,t} + \varepsilon_{uI,t+1} && \text{(B.5)} \\
\tau_t &= \tau_{t-1} + \pi_{I,t} - \pi_{F,t} && \text{(B.6)} \\
k_t &= \frac{1-\delta}{1+g} + \frac{\delta+g}{1+g} i_t && \text{(B.7)} \\
E_t[\lambda_{C,t+1}] &= \lambda_{C,t} - E_t[r_{t+1}] && \text{(B.8)} \\
\beta E_t[\pi_{F,t+1}] &= \pi_{F,t} - \lambda_F (mc_{F,t} + u_{F,t}) && \text{(B.9)} \\
\beta E_t[\pi_{I,t+1}] &= \pi_{I,t} - \lambda_I (mc_{I,t} + u_{I,t}) && \text{(B.10)}
\end{aligned}
$$

with outputs defined by

$$
\begin{aligned}
E_t[r_{t+1}] &= r_{n,t} - E_t[\pi_{t+1}] && \text{(B.11)} \\
E_t[\pi_{t+1}] &= \mathrm{w} E_t[\pi_{F,t+1}] + (1-\mathrm{w}) E_t[\pi_{I,t+1}] && \text{(B.12)} \\
\pi_t &= \mathrm{w} \pi_{F,t} + (1-\mathrm{w}) \pi_{I,t} && \text{(B.13)} \\
c_t: \ \lambda_{C,t} &= -(1 + (\sigma-1)(1-\varrho) c_t \\
&+ \frac{n_F (L_F^{\varrho(1-\sigma)} - L_I^{\varrho(1-\sigma)}) \hat{n}_{F,t} + \varrho(\sigma-1)(n_F L_F^{\varrho(1-\sigma)} \ell_{F,t} + (1-n_F) L_I^{\varrho(1-\sigma)} \ell_{I,t})}{n_F L_F^{\varrho(1-\sigma)} + (1-n_F) L_I^{\varrho(1-\sigma)}} \\
& && \text{(B.14)} \\
u_{L_I,t} &= u_{C,t} + c_t + \frac{h_I}{1-h_I} \hat{h}_{I,t} && \text{(B.15)} \\
u_{L_F,t} &= u_{C,t} + c_t + \frac{h_F}{1-h_F} \hat{h}_{F,t} && \text{(B.16)} \\
w_{I,t} - p_t &= u_{L_I,t} - \lambda_{C,t} && \text{(B.17)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{h}_{F,t}:\ w_{F,t}-p_t &= u_{L_F,t}-\lambda_{C,t} && \text{(B.18)}\\
w_{F,t}-p_t &= \omega(w_{I,t}-p_t) && \text{(B.19)}\\
c_{F,t} &= c_t+\mu(1-\mathrm{w})\tau_t && \text{(B.20)}\\
c_{I,t} &= c_t-\mu\mathrm{w}\tau_t && \text{(B.21)}\\
\hat{n}_{F,t}:\ y_{F,t} &= a_{F,t}+\alpha_F(\hat{n}_{F,t}+\hat{h}_{F,t})-(1-\alpha_F)k_{F,t} && \text{(B.22)}\\
\hat{h}_{I,t}:\ y_{I,t} &= a_{I,t}+\alpha_I(\hat{n}_{I,t}+\hat{h}_{I,t})-(1-\alpha_I)k_{I,t} && \text{(B.23)}\\
\hat{n}_{I,t} &= -\frac{n_F}{n_I}\hat{n}_{F,t} && \text{(B.24)}\\
mc_{F,t} &= \frac{1}{1+\tau_F}(w_{F,t}-p_t)+\frac{\tau_F}{1+\tau_F}\hat{\tau}_{F,t}+(1-\mathrm{w})\tau_t-a_{F,t}\\
&+ (1-\alpha_F)(\hat{n}_{F,t}+\hat{h}_{F,t}-k_{F,t}) && \text{(B.25)}\\
mc_{I,t} &= \frac{1}{1+\tau_I}(w_{I,t}-p_t)+\frac{\tau_I}{1+\tau_I}\hat{\tau}_{I,t}-\mathrm{w}\tau_t-a_{I,t}\\
&+ (1-\alpha_I)(\hat{n}_{I,t}+\hat{h}_{I,t}-k_{I,t}) && \text{(B.26)}\\
y_{I,t} &= c_{I,t} && \text{(B.27)}\\
i_t:\ y_{F,t} &= c_{yF}c_{F,t}+i_{yF}i_t+g_{yF}\,g_t && \text{(B.28)}\\
g_t &= (1-\mathrm{w})\tau_t+\frac{n_F\tau_F}{n_F\tau_t+n_I\tau_I}(\hat{n}_{F,t}+\hat{h}_{F,t}+\hat{\tau}_{F,t})+\frac{n_I\tau_I}{n_F\tau_t+n_I\tau_I}(\hat{n}_{I,t}+\hat{h}_{I,t}+\hat{\tau}_{I,t})\\
\hat{\tau}_{I,t} &= \hat{\tau}_{F,t} && \text{(B.29)}
\end{aligned}
$$

$$
\begin{aligned}
k_{I,t}:\ k_t &= \frac{\bar{K}_{F,t}}{\bar{K}_t}k_{F,t}+\frac{\bar{K}_{I,t}}{\bar{K}_t}k_{I,t} && \text{(B.30)}\\
y_{F,t}:\ y_{F,t}-k_{F,t} &= (1-w)\tau_t-mc_{F,t}+\frac{1+R}{R+\delta}r_t && \text{(B.31)}\\
k_{F,t}:\ mc_{F,t} &= mc_{I,t}+\tau_t+y_{I,t}-y_{F,t}+k_{F,t}-k_{I,t} && \text{(B.32)}
\end{aligned}
$$

where $\lambda_i \equiv \frac{(1-\beta\xi_i)(1-\xi_i)}{\xi_i}$, and $\tau_i \equiv \frac{\bar{\tau}_i}{W_t/P}$ $i = I, F$. Note that (B.14) defines c_t, (B.22) defines $\hat{n}_{F,t}$ and (B.23) defines $\hat{h}_t$. Let $\tau_I = (1-k)\tau_F$ where $k \in [0,1]$ to allow taxation to be enforced in the informal sector. Also (B.20) and (B.21) implies $c_t = \mathrm{w}c_{F,t} + (1-\mathrm{w})c_{I,t}$

The flexi-price 'natural rate' economy is found by putting $mc_{F,t} = mc_{I,t} = 0$ and making taxes non-distortionary.

C Quadratic Approximation of Welfare

To formulate this quadratic approximation first consider the simpler case without capital and with leisure constrained to be the same in both informal and formal sectors. Then we simply approximate the utility function $U_t = U(C_t, L_t)$ in consumption, C_t and leisure $L_t = 1 - h_t$ we start with the Taylor Series expansion about the BGP steady state[17]

$$U_t = U + U_C C c_t + \frac{1}{2} U_{CC} C^2 c_t^2 + U_L L l_t + \frac{1}{2} U_{LL} L^2 l_t^2 + \text{higher order terms} \tag{C.1}$$

Next we write $c_t = \mathrm{w} c_{F,t} + (1-\mathrm{w}) c_{I,t}$, $l_t = -\frac{h}{1-h}\hat{h}_t$ and use the linearized resource constraints

$$\begin{aligned} y_{F,t} &= a_{F,t} + \alpha_F(\hat{n}_{F,t} + \hat{h}_t) - d_{F,t} = (1 - g_{Fy}) c_{F,t} + g_{Fy} g_t && \text{(C.2)} \\ y_{I,t} &= a_{I,t} + \alpha_F(\hat{n}_{I,t} + \hat{h}_t) - d_{I,t} = c_{I,t} && \text{(C.3)} \end{aligned}$$

where

$$d_{i,t} = \log\left[\frac{\Delta_{i,t}}{\Delta_i}\right] \; ; i = I, F \tag{C.4}$$

and $\Delta_{i,t}$ is the price dispersion effect given by (35). By standard results (see, for example, Gali (2008), p88) $d_{i,t}$ is a *second order* term given by

$$d_{i,t} = \frac{\zeta_i(\alpha_i + (1-\alpha_i)\zeta_i)}{2\alpha_i} \mathrm{var}(p_{i,t}(j)) \; ; i = I, F \tag{C.5}$$

and

$$\sum_{t=0}^{\infty} \beta^t \mathrm{var}(p_{i,t}(j)) = \frac{\xi_i}{(1-\beta\xi_i)(1-\xi_i)} \sum_{t=0}^{\infty} \beta^t \pi_{i,t}^2 \; ; i = I, F \tag{C.6}$$

Then using the linearized resource constraints and the properties of efficiency in the steady state: $\frac{U_L}{U_C} = F_{N_F} = F_{N_I}$ the first order terms in c_t and l_t disappear in (C.1) and we are left the quadratic approximation to the utility function

$$U_t = U + U_C C\left[-\frac{\mathrm{w}}{(1-g_{Fy})} d_{F,t} - (1-\mathrm{w}) d_{I,t}\right] + \frac{1}{2} U_{CC} C^2 c_t^2 + U_L L l_t + \frac{1}{2} U_{LL} L^2 l_t^2 + \text{higher order terms} \tag{C.7}$$

Finally using the results (C.4)–(C.7) we can write the quadratic form of the intertemporal expected welfare loss at time $t = 0$ as

$$\Omega_0 = \frac{1}{2} E_t \left[\sum_{t=0}^{\infty} \beta^t [w_c c_t^2 + w_h \hat{h}_t^2 + w_{\pi F} \pi_{F,t}^2 + w_{\pi I} \pi_{I,t}^2]\right] \tag{C.8}$$

[17] The BGP is time-varying but here we drop the bar and time-script in $\bar{U}_t, \bar{C}_t$ etc.

where for our choice of utility function (48)

$$\begin{aligned}
w_c &= -\frac{U_{CC}C}{U_C} = 1+(\sigma-1)(1-\varrho) \\
w_h &= -\frac{U_{LL}h^2}{U_C C} = \frac{(1+\varrho(\sigma-1))h^2}{(1-\varrho)(1-h)^2} \\
w_{\pi F} &= \mathrm{w}\frac{\zeta_F(\alpha_F+(1-\alpha_F)\zeta_F)}{c_{Fy}\alpha_F\lambda_F} \\
w_{\pi I} &= \frac{(1-\mathrm{w})\zeta_I(\alpha_I+(1-\alpha_I)\zeta_I)}{\alpha_I\lambda_I} \\
\lambda_i &= \frac{\xi_i}{(1-\beta\xi_i)(1-\xi_i)}\ ;\ i=F,I
\end{aligned}$$

For the actual model with capital and different choices of work effort in the two sectors we use a modified version of this approximation:

$$\Omega_0 = \frac{1}{2}E_t\left[\sum_{t=0}^{\infty}\beta^t[w_c c_t^2 + w_{hI}\hat{h}_I^2 + w_{hF}\hat{h}_F^2 + w_{\pi F}\pi_{F,t}^2 + w_{\pi I}\pi_{I,t}^2]\right] \tag{C.9}$$

where now

$$w_h = -\frac{U_{LL}h^2}{U_C C} = \frac{(1+\varrho(\sigma-1))h^2}{(1-\varrho)(1-h)^2}\ ;\ h=h_I,h_F$$