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**FINANCIAL CRISIS, MONETARY BASE EXPANSION
AND RISK**

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Financial Crisis, Monetary Base Expansion and Risk

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Abstract

This paper examines the post-2008 European Central Bank's liquidity enhancing policies, namely 'Long Term Refinancing Operations', and the increase of banks' excess reserves that followed. To evaluate this in a quantitative environment, I build a dynamic, general equilibrium model that incorporates financial frictions in both the supply and demand for credit and allows banks to receive liquidity and hold reserves. Results suggest the existence of a risk-shifting channel of monetary policy in the recent ECB operations. Specifically, I show that when the central bank supplies liquidity during turbulent times, banks grant loans to riskier firms. This increases the firms' default on new credit and worsens the performance of the economy although the banks' health is improved. Additionally, I find that an increase in the riskiness of the non-financial corporations can explain the recent reserve accumulation by the banking system. Lastly, I evaluate the effects of negative interest rates on credit and assess the welfare implications of the recent policies.

JEL classification: E44, E58

Keywords: Excess reserves, LTRO, unconventional monetary policy, negative interest rates, DSGE

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1. Introduction

In Euro Area and the US since the onset of the Great Recession, the two central banks have used a number of non-standard monetary policy tools, most of them not previously implemented or analyzed in the macroeconomic policy literature. The extension of existing reverse operations under longer maturities and the asset purchase programs were the more popular among those regions. Although the key scope of these direct funding programs was the stabilization of economic activity through a credit expansion, especially in the Eurozone, a shrinkage in bank lending and output continues. Additionally, in both continents there has been a serious increase in the banks' reserves holdings which triggered many commentators, analysts and policy makers to criticize banks for hoarding reserves out of the emergency funds instead of lending them to the real sector.¹ Lastly, the ECB's decision to implement negative interest rates to its reserves accounts in a try to make reserve accumulation a non-profitable activity and stimulate bank lending, signals a new monetary policy tool which needs further exploration. This paper studies those recent macroeconomic developments and their effects in the Euro Area macroeconomy. The paper's main finding is that the ECB's liquidity provision was beneficial for the banking system but not for the macroeconomy due to the risk-shifting channel of monetary policy. Additional results show that an increase of riskiness in the credit demand side is the reason behind the banks' excess reserves accumulation. Lastly, in an attempt to asses the negative interest rates policy, it is shown that the central bank is able to induce banks to lower their reserves holdings and extend credit only when the interest rates on reserves become significantly negative.

This study introduces agency problems associated with financial intermediation in an otherwise standard business cycles model. Additionally, it introduces a modeling framework for the the banks' ability to receive and store emergency liquidity funds from the central bank into their reserve accounts. By combining Gertler and Kiyotaki (2010) with Bernanke, Gertler, and Gilchrist (1999) (henceforth GK and BGG respectively) a setting is developed where increased risk (in the sense of Christiano, Motto, and Rostagno (2014)), reduces firms net worth, increases their likelihood of default and makes banks reduce and shift credit to more risky firms and increase their reserves, the risk free asset, when the central bank provides liquidity.

¹Pisani-Ferry and Wolff (2012), The truth about all those excess reserves (*The Economist*), Central Bank reserve creation in the era of negative money multipliers (*Voxeu*), Draghi Unveils Historic Measures Against Deflation Threat (*Bloomberg*), ECB Doing Whatever It Takes Can't Make Euro-Area Banks Lend (*Bloomberg*) and many others. Philadelphia Fed President Charles Plosser expressed concern about what would occur "were all those excess reserves to start flowing out into the economy in the form of loans or purchases of other assets"

This setting makes it possible to simulate a crisis environment and evaluate the recent liquidity provision. The ECB proceeded in measures aiming to support banks' liquidity funding and therefore encouraging banks to provide credit.²The main tool used, the LTRO, is an open market operation that takes place as reverse transaction and is the basic liquidity provision tool of the ECB. Starting from October 2008 the ECB steadily increased the maturities of the LTRO from 3 months to 36 months.³Therefore, financial intermediaries could have unlimited access to short term funding. At the same time a significant increase of the banks excess reserves took place.⁴LTRO funding and the banks' accumulation of excess liquidity are depicted in Figure 1.

Despite the fact that the ECB has more than doubled its balance sheet, creating a remarkable expansion of the Eurosystem's monetary base, bank lending has not shown any signs of expansion yet as Figure 2 shows. Monetary base expansion, although unprecedented in its size, has not worked as intended. Banks' credit growth remains low in the Eurozone and hence investment.

To simulate and evaluate these actions in a quantitative application, I employ the risk shock motivated by Christiano et al. (2014) as the main source of real business cycles fluctuations. The paper's main finding shows that an increase in the risk of the non-financial corporations leads to a reduction to the firms' net worth, a higher probability of default and a banks' aversion to expand new credit. To moderate the adverse effects, the central bank provides liquidity to the banking sector as occurred with the LTRO program. This improves the banks' net worth and at the same time increases their reserves holdings as we observe in the data. Nevertheless, the impact on the macroeconomy is negative. Banks supply new credit to riskier firms and a leverage effect is materializing. Firms react to the lower cost of borrowing by leveraging up their net worth and therefore the likelihood of default increases. This affects the firms' net worth and hence lowers investment and output. The existence

²ECB's response was in two phases with the use of non-standard monetary policies labeled as "enhanced credit support". Firstly at the onset of financial crisis and later when the Euro sovereign crisis took place. These included the maturity extension of Long Term Refinancing Operations (LTROs), the creation the Targeted Long Term Refinancing Operations (TLTROs), the reduction in banks' reserve requirements from 2% to 1%, an asset purchase program and numerous other non-standard measures described in detail by Cour-Thimann and Winkler (2012).

³Only for its second intervention, the ECB supplied to the banks 1 trillion Euro via the LTRO the scheme.

⁴In the Eurosystem framework, banks either hold their reserves as excess reserves where they get a zero remuneration or in the deposit facility, the account where banks make deposits with the central bank and earn an interest. Before 2008 both assets' level was insignificant and were only used for banking micro-management. Since I am not interested in the micro-management allocation of banks between the deposit facility and the current accounts, in the model I use the deposit facility account as the representative reserve account. The model can be extended easily to include also the current accounts (reserves outside of the deposit facility) as an asset that pays no interest.

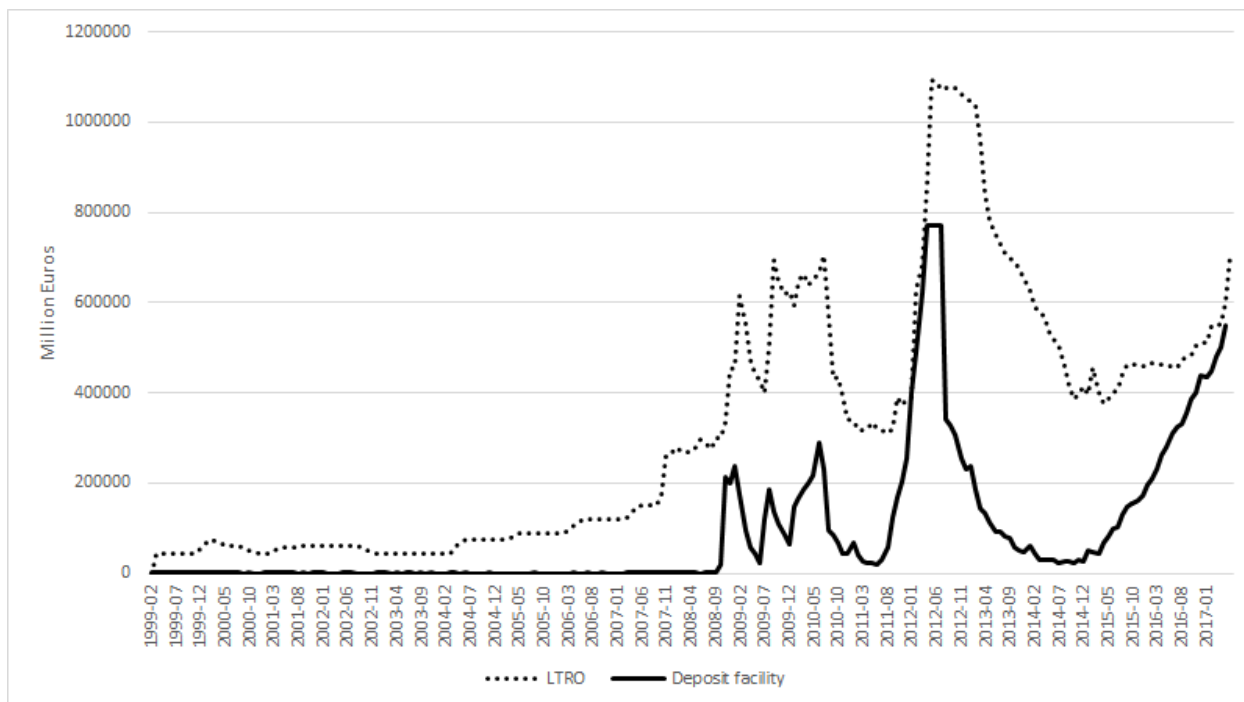


Fig. 1. LTRO and excess reserves in the Eurosystem. Data source: ECB

of risk-shifting in lending due to expansive monetary policy is also identified in the recent empirical literature on the risk taking channel by Jiménez, Ongena, Peydró, and Saurina (2014) and others.

This study also tries to shed some light on the effects of the newly introduced the negative interest rates or reserves. I proceed by introducing a penalty fee on reserves, in the same fashion as negative interest rates, to assess the recent policy practiced by the European Central Bank and other central banks⁵. After the introduction of the reserve penalty, a reduction of the banks' reserve position and an increase in credit follows which lead to an overall economic upturn. Lastly, using consumption equivalence measures based on conditional welfare as in Schmitt-Grohé and Uribe (2007), I find that the recent ECB's policies were welfare improving.

The modeling structure allows credit frictions to operate simultaneously originating from both the demand and the supply side of credit, an approach that has not yet been discussed in the literature. On the supply side, an agency problem between the depositors and the banks is introduced. The financial intermediaries can divert at any time a fraction of their

⁵Apart from the ECB, negative interest rates have been implemented also by Denmark's Nationalbank, Bank of Japan, Swiss National Bank and the Sveriges Riksbank.

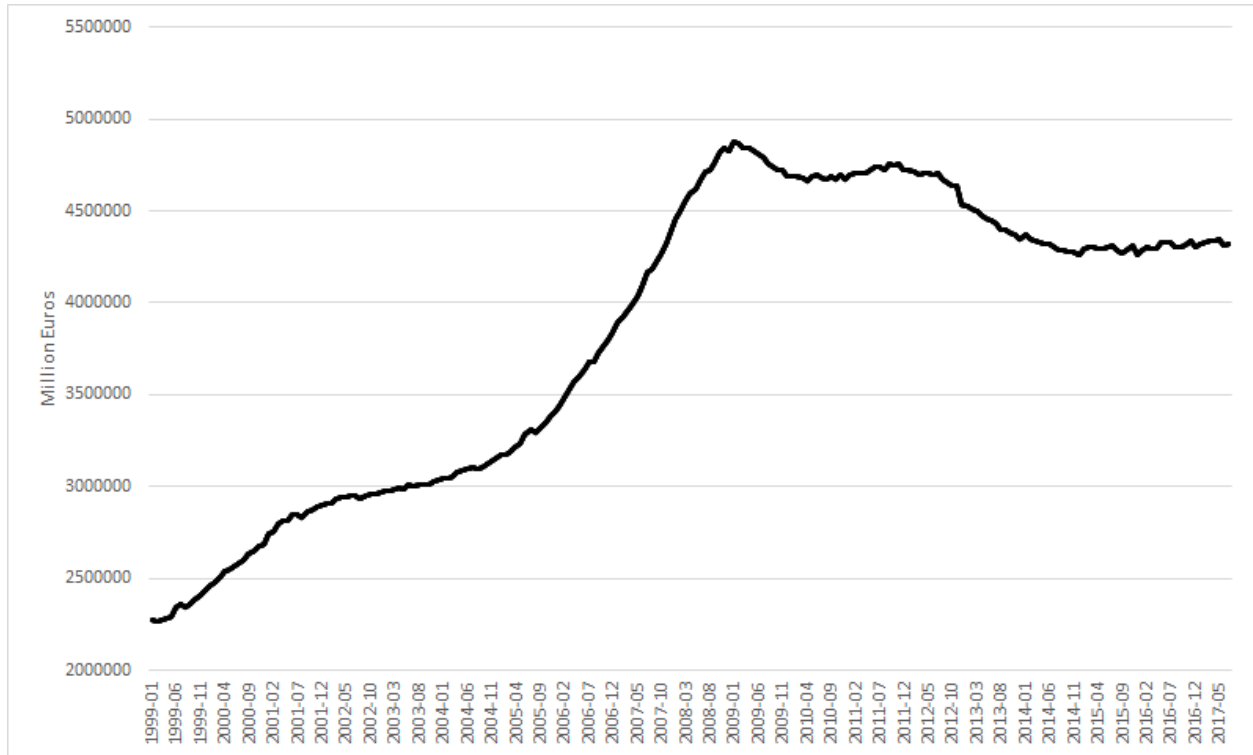


Fig. 2. Loans from Monetary Financial Institutions to Non-Financial Corporations in the Euro Area. Data source: ECB

assets and return it back to their families as in Hart and Moore (1998). This implies an endogenous constraint on the bank's ability to obtain funds that assures depositors' funds safety. A wedge between the interest rate on loans and the deposit interest rate is generated when the constraint is binding. As for the demand side friction, a costly state verification (CSV) problem as initially proposed by Townsend (1979) is introduced. Banks in order to monitor the defaulting entrepreneurs, must pay a cost to verify their state. A premium emerges between the interest rate on capital and the discount rate, the equivalent of the deposit rate in the model. An endogenously determined remain and exit probability of the entrepreneurs is introduced in this new framework. Entrepreneurs decide whether they exit taking as given the loan interest rate. They stay in life as long as the level of their leverage satisfies the minimum banks' profitability.

Several improvements arise in the new modeling framework introduced compared to the two original building blocks. In GK, firms cannot rely on their own net worth to finance investment. In other words, if there is no financial intermediation there is no production. In the same fashion, in BGG it is assumed that investors lend directly to borrowers, without

the intervention of financial intermediaries. In this model, both firms and banks have net worth. The net worth of the banking sector, as well as the net worth of entrepreneurs matters for the models' dynamics. Furthermore, BGG and GK consider financial frictions only on the borrowers and lenders side of credit markets respectively; credit-supply effects stemming from the lenders in BGG and borrowers in GK behavior were completely ignored. Yet, the global crisis showed the need to consider models that combine financial frictions on both the borrowers and the lenders side. This is achieved by the merge of both models. The external finance premium for the firm now depends also on the bank behavior. A banks' decision to reduce credit and increase the interest rate after a negative shock will affect the premium that the firm must pay. In the BGG model, the premium only depends on the firms' behavior and leverage. Lastly, now the bank leverage is procyclical as suggested by the data. This is not the case in GK, since bankers were the owners of the firms' capital. The net worth of the banks was affected by a change in value of capital. In this model, bank do not own firms' capital, but firms buy it directly from the capital goods producers.

Related Literature. An increased development of macroeconomic models which incorporate financial frictions in a general equilibrium framework with financial intermediation has taken place after the Great Recession (see Gertler and Kiyotaki (2010), Brunnermeier and Sannikov (2014) among many others).⁶ Most of the existing modern macroeconomic models do not take into account that monetary policy is implemented through the banking system, as it occurs in practice. Instead, most assume that central banks directly control interest rates or monetary aggregates and abstract from how the transmission of monetary policy may depend on the conditions of banks. Interactions between reserves, open market operations, banking and the macroeconomy introduced in this paper, aim to build a closer approach to the real world monetary policy implementation.

Studies closer to the ECB's unconventional LTROs, (Cahn, Matheron, and Sahuc (2017), Joyce, Miles, Scott, and Vayanos (2012), Bocola (2016), van der Kwaak (2017)), assume a direct relationship between the non-standard measures and bank lending. They omit the reserves that are being created from these operations. Thus, it is assumed that all the emergency funding from the central bank transforms directly to credit, which is an strong assumption.

In the recent excess reserves literature, Bianchi and Bigio (2014) develop a new framework to study the implementation of monetary policy through the banking system. Their results are in line with this paper. They find that the unprecedented increase in reserves is due to a substantial and persistent contraction in loan demand since the benefits of holding reserves relative to loans are increased. Primus (2017) designs a DSGE model where banks

⁶Also Eggertsson and Woodford (2003), Curdia and Woodford (2011), Gertler and Karadi (2011).

hold reserves but mainly focuses on the effects that reserve requirements can have in the middle-income countries.

Finally, the risk-taking channel literature as developed by Allen and Gale (2000), Diamond and Rajan (2012) and others concede with this paper’s main outcome of the monetary policy risk-shifting channel existence. In an empirical framework Jiménez et al. (2014) and Ioannidou, Ongena, and Peydró (2014) find that monetary expansion induces banks to grant loans to more risky firms which increases the likelihood of default. Adrian and Shin (2010) build a theoretical model and show that expansionary monetary policy increases the risk taking of the banking sector by relaxing the bank capital constraint due to moral hazard problems. In my knowledge this is the first study that introduces the channel of risk-shifting in lending after liquidity operations in a quantitative framework.

Layout. The paper is organized as follows. Section 2 presents the model and section 3 describes the important economic mechanisms. Section 4 the data and the calibration takes place and section 5 presents the quantitatively analysis. Section 6 concludes.

2. The Model

The model is built on and extends two leading approaches in the literature of credit market frictions in general equilibrium. The seminal work of Bernanke et al. (1999) that introduced the “financial accelerator” in a general equilibrium framework and Gertler and Kiyotaki (2010), one of the first attempts to incorporate an active banking sector in the business cycles literature. Section 2.1 describes the standard part of the model, employed in the most Real Business Cycles literature. Section 2.2 describes the financial frictions components.

All variables are in real terms abstracting from the notion of money. There are five types of agents. Households, financial intermediaries, entrepreneurs, capital goods producers and retailers, and a government that conducts both fiscal and monetary policy.

2.1. Standard Part of the Model

Households.— There is a continuum of households with identical preferences. Within each household there are three different member types: ϖ workers, ς bankers and $(1 - \varpi - \varsigma)$ entrepreneurs. Household members differ in the way they obtain earnings. Workers supply labor, bankers manage the financial intermediaries and entrepreneurs manage the non-financial firms. All return their earnings back to their families.⁷ Within the family

⁷This approach follows GK and allows for within-household heterogeneity but also sticks to the representative approach representation. Abstracting from consumption for the bankers and entrepreneurs makes the

there is perfect consumption insurance.

The preferences of the representative household take the following form:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \zeta_{c,t} [\ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\epsilon} N_{t+i}^{1+\epsilon}], \quad (1)$$

C_t denotes the per capita consumption of the household members and N_t the supply of labor. $\beta \in [0, 1]$ is the discount factor, $\gamma \in [0, 1]$ is the habit parameter, ϵ is the inverse Frisch elasticity of labor supply, $\chi > 0$ is the relative utility weight of labor and $t + i$ is the time subscript. Finally, $\zeta_{c,t}$ is preference shock that follows an AR(1) process. Because of the stochastic setting, households make expectations for the future based on what they know in time t and \mathbb{E}_t is the expectation operator at time t . I allow for habit formation of consumption as in Boldrin, Christiano, and Fisher (2001) where the utility of agents depends on current consumption but also to past consumption. As Fuhrer (2000) shows, model performance in monetary policy shocks is significantly improving with this adjustment.

The budget constraint of the representative household is

$$C_t + T_t + D_{h,t+1} = W_t N_t + \Pi_t + R_t D_{h,t}, \quad (2)$$

where

$$D_{h,t+1} = D_{t+1} + D_{g,t+1}. \quad (3)$$

Household allocates funds to consumption, taxes T_t and two types of savings: lending deposits D_{t+1} to banks and one period government bonds $D_{g,t+1}$. Both assets have no risk and are perfect substitutes of each other. R_t is the gross return for the bonds and the deposit holdings respectively (the interest factor) in period t . The household's financial resources are from labor income, W_t is the real wage, bond and deposits returns and the net payouts to the household from ownership of both non-financial firms and financial intermediaries Π_t .

The problem of the representative household is to choose $C_t, N_t, D_t, D_{h,t}$ in order to maximize its expected utility (1) subject to the budget constraint (2) at every period. Solution of the household's problem is shown in Appendix A. Finally, there is a turnover between workers, bankers and the entrepreneurs. This will be explained in detail in the next subsection.⁸

Capital and Consumption Goods Production.— The non-financial firms are sep-

model presentation simpler.

⁸This follows Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Gertler, Kiyotaki, and Queralto (2012). It ensures that bankers and entrepreneurs will never accumulate enough own funds to finance their activities.

arated into two types: goods producers and capital producers. Capital evolves according to the law of motion of capital

$$K_{t+1} = k_{t+1}^q [I_t + (1 - \delta)K_t]. \quad (4)$$

k_t^q denotes a capital quality shock that follows a first order autoregressive process. This is a simple way to introduce an exogenous source of variation in the value of capital.⁹

Goods Producers.— Goods producers are owned by the entrepreneurs. They combine capital that is taken from the entrepreneurs and labor to produce goods under a constant returns to scale production function of the following form subject to a total factor productivity shock A_t that follows an AR(1) process:

$$Y_t = A_t K_t^\alpha N_t^{1-\alpha}.$$

The decision problem of the goods producers is to choose K_t and N_t in order to maximize their profits. The firms choose labor in order to satisfy $W_t = (1 - \alpha)\left(\frac{K_t}{N_t}\right)^\alpha$ and capital to satisfy $Z_t = \alpha\left(\frac{N_t}{K_t}\right)^{1-\alpha}$.

Capital Goods Producers.— Capital goods producers are the investors in the model. They produce new capital and sell it to the entrepreneurs at a price Q_t . Investment is subject to adjustment costs. The objective of a capital producer is to choose $\{I_t\}_{t=0}^\infty$ to solve:

$$\max_{I_\tau} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau I_\tau - \left[1 + \tilde{f}\left(\frac{I_\tau}{I_{\tau-1}}\right) I_\tau \right] \right\}.$$

where the adjustment cost function \tilde{f} captures the cost of investors to increase their capital stock as follows:

$$\tilde{f}\left(\frac{I_\tau}{I_{\tau-1}}\right) = \frac{\eta}{2} \left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2 I_\tau$$

and η is the inverse elasticity of net investment to the price of capital. The solution to the decision problem of the investors who want to maximize I_τ yields the competitive price of capital.

$$Q_t = 1 + \left(\eta \frac{I_\tau}{I_{\tau-1}} \left(\frac{I_\tau}{I_{\tau-1}} - 1\right) + \frac{\eta}{2} \left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2 - \eta \Lambda_{t,\tau} \frac{I_{\tau+1}^2}{I_\tau^2} \left(\frac{I_\tau}{I_{\tau-1}} - 1\right) \right).$$

⁹Many recent papers have also used this exogenous disturbance in the capital value has also been used by Gertler and Karadi (2011), Brunnermeier and Sannikov (2014) among others.

2.2. Financial Frictions

Entrepreneurs.— Each entrepreneur i purchases raw capital $k_{i,t+1}$ from the capital goods producers at price Q_t in a competitive market and fund this purchase with their equity $n_{i,t+1}^E$ and credit $l_{i,t+1}$ obtained from the financial institutions. The respective balance sheet is:

$$Q_t k_{i,t+1} = l_{i,t+1} + n_{i,t+1}^E. \quad (5)$$

The entrepreneur transfers the acquired capital to the firm which is owned by her and it yields a marginal product of capital Z_{t+1} . Also, at the end of the period, she sells the undepreciated capital to the capital goods producer at price Q_{t+1} . Therefore, the average return per nominal unit invested in period t is:

$$R_{k,t+1} = k_{t+1}^q \frac{[Z_{t+1} + (1 - \delta)Q_{t+1}]}{Q_t}, \quad (6)$$

In every period t an idiosyncratic shock ψ_i transforms the newly purchased $k_{i,t+1}$ raw units of capital into $\psi_i k_{i,t+1}$ effective units of capital. It is assumed that ψ follows a unit-mean log normal distribution. Following CMR I call the standard deviation of $\log(\psi)$ denoted by σ_t , the *risk shock*. It is the cross sectional dispersion in ψ and it is allowed to vary stochastically over time. This will introduce the uncertainty in model's perturbations. The idiosyncratic shock is drawn from a density $f(\psi_t)$ and the probability of default is then given by:

$$p(\bar{\psi}) = \int_0^{\bar{\psi}} f(\psi) d\psi. \quad (7)$$

A threshold value of ψ_i called $\bar{\psi}_{t+1}$ divides the entrepreneurs that cannot pay back the loan and interest from those who can repay. It is defined by

$$R_{l,t+1} l_{i,t+1} = \bar{\psi}_{t+1} R_{k,t+1} Q_t k_{i,t+1}. \quad (8)$$

$R_{l,t+1}$ is the rate to be decided in the debt contract between the entrepreneur and the banker. When $\psi_i \geq \bar{\psi}_{t+1}$ the entrepreneur repays the bank the amount $R_{l,t+1} l_{i,t+1}$ keeps the profits equal to $\bar{\psi}_{t+1} R_{k,t+1} Q_t k_{i,t+1} - R_{l,t+1} l_{i,t+1}$ and continues the production. If $\psi_i < \bar{\psi}_{t+1}$ the entrepreneur has a negative net worth resulting in bankruptcy and default. An Entrepreneur that defaults, is being monitored by a bank which acquires her assets. The expected net worth of the entrepreneurs is

$$\mathbb{E}_t[(1 - \Gamma_t) \bar{\psi}_{t+1} R_{k,t+1} Q_t k_{i,t+1}], \quad (9)$$

where

$$\Gamma_t(\bar{\psi}_{t+1}) = \int_0^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi + \bar{\psi}_{t+1}(1 - p(\bar{\psi}_{t+1})).$$

and $1 - \Gamma_t(\bar{\psi}_{t+1})$ represents the average weight of the entrepreneurs' gains.

If there was no cost for the banker to observe the idiosyncratic shock $\psi_{i,t}$, then there would have been state-contingent contracts that would perfectly insure the banker. Instead, in order to make default by entrepreneurs costly for the banking sector, ψ_i is costlessly observed by the entrepreneur, but it is not observed by the lender unless he pays a fraction $\mu \in (0, 1]$ of their ex-post revenues. This follows the ‘‘costly state verification’’ illustration proposed by Townsend (1979). In case of the entrepreneur’s default, the financial intermediary must pay a ‘‘monitoring cost’’ to observe the borrower’s realized return on capital. This can be interpreted as legal costs that the banks have to pay in the case of borrowers default. The monitoring costs equals a proportion μ of the gross payoff of the firms capital, i.e. $\mu\psi_{i,t+1}R_{k,t+1}Q_tk_{i,t+1}$.

The optimal contract maximizes the expected profits of the entrepreneur under the condition that the expected return on lending is no less than the opportunity cost of lending. In other words, for the financial intermediary to continue extending credit to entrepreneurs, their expected return from credit must be always greater or equal to the opportunity cost of its funds. The opportunity cost is the riskless rate R_t that banks have to pay the households for their deposits. Hence, the loan contract must satisfy:

$$(1 - \mu)R_{k,t+1}Q_tk_{i,t+1} \int_0^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi + (1 - p(\bar{\psi}_{t+1}))R_{l,t+1}l_{i,t+1} \geq R_t l_{i,t+1}. \quad (10)$$

The left hand side shows the expected gross return that the financial intermediary receives over all realizations of the shock and the right hand side the opportunity cost of lending that the intermediary has.

Using (7) the zero profit condition (10) becomes :

$$R_{k,t+1}Q_tk_{i,t+1}[\Gamma_t(\bar{\psi}_{t+1}) - \mu G_t(\bar{\psi}_{t+1})] \geq R_t(Q_tk_{i,t+1} - n_{i,t+1}^E), \quad (11)$$

where $\mu G_t(\bar{\psi}_{t+1})$ are the monitoring costs:

$$G_t(\bar{\psi}_{t+1}) = \int_0^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi$$

respectively. The optimal contract for the entrepreneur solves the entrepreneur’s expected net worth (9) subject to the zero profit condition (11). The solution is presented in Appendix

B. Combining the first order conditions we arrive at the external finance premium between the interest gain on capital and the riskless rate:

$$\mathbb{E}_t R_{k,t+1} = \mathbb{E}_t \rho(\bar{\psi}_{t+1}) R_{t+1}, \quad (12)$$

$$\rho(\bar{\psi}_{t+1}) = \frac{\Gamma'_t(\bar{\psi}_{t+1})}{[(\Gamma_t(\bar{\psi}_{t+1}) - \mu G_t(\bar{\psi}_{t+1}))\Gamma'_t(\bar{\psi}_{t+1}) + (1 - \Gamma_t(\bar{\psi}_{t+1}))(\Gamma'_t(\bar{\psi}_{t+1}) - \mu G'_t(\bar{\psi}_{t+1}))]}.$$

Aggregation.— At the end of the period a fraction $\sigma_{E,t}$ of entrepreneurs decides to remain and the rest disappear and are replaced by an equal number of workers. This assumption ensures that they will not fund all investments from their own accumulated capital. In contrast with the BGG model, the remain probability of entrepreneurs is not constant and is adjusted taking as given the loan interest rate that they have to pay to the banks. Specifically, it satisfies at every time t the level of leverage that makes the zero profit condition (10) to hold. Exit doesn't necessarily mean default.¹⁰ Thus, $\sigma_{E,t}$ is a time varying probability. The relationship between the probability of default and the remain probability is negative.¹¹

The new entrants receive a start up fund transferred from the old entrepreneurs which is equal to a proportion ξ_E of their wealth. By the law of large numbers the aggregate net worth for every entrepreneurs i at the end of the period t is $(1 - \Gamma_{t-1})\bar{\psi}_t R_{k,t} Q_{t-1} k_{i,t}$. Integrating over all entrepreneurs we get the aggregate net worth at the end of period t . Capital letters denote aggregate variables.

$$N_{t+1}^E = (\sigma_{E,t} + \xi_E)([1 - \Gamma_{t-1}]\bar{\psi}_t R_{k,t} Q_{t-1} K_t).$$

Banks.— Each bank j allocates its funds to credit $l_{j,t+1}$ and reserves $x_{j,t+1}$. It funds its operations by receiving deposit from households $d_{j,t+1}$, emergency funding from the central bank $m_{j,t}$ in turbulent periods and also by raising equity $n_{j,t+1}^B$. It follows that the bank's balance sheet is:

$$l_{j,t+1} + x_{j,t+1} = n_{j,t}^B + d_{j,t+1} + m_{j,t+1}. \quad (13)$$

The bank's net worth evolves as the difference between interest gains on assets and interest payments on liabilities net the cost of holding excess reserves.

$$n_{j,t+1}^B = R_{l,t} l_{j,t} (1 - p(\bar{\psi}_t)) + R_{k,t} k_{j,t} Q_{t-1} (1 - \mu) G_t(\bar{\psi}_t) + R_{x,t} x_{j,t} - R_t d_{j,t} - R_{m,t} m_{j,t} - \Phi(x_t). \quad (14)$$

$R_{x,t}$ is the interest rate of the deposit facility and $R_{m,t}$ the interest rate of the emergency

¹⁰In the BGG model exit happens constantly using the parameter σ_E in order to avoid over-accumulation of the entrepreneurs net worth and self-financing.

¹¹See Appendix C for further details.

funding (LTRO). Banks get repaid the principal plus the interest of the loans from the entrepreneurs with a probability of $(1 - p(\bar{\psi}))$. The first two terms in the right hand side of the equation is the expected return to the bank from the contract averaged over all realizations of the idiosyncratic shock (ψ). Reserve accumulation costs are introduced as in Glocker and Towbin (2012) among others. Banks that hold reserves have to pay an additional fee to the central bank. $\Phi(x_t)$ is the cost of holding reserves and takes the following form:

$$\Phi(x_t) = \left(\frac{\kappa}{2} \Upsilon_t^2 n_t^B + \epsilon \Upsilon_t \right) \zeta_{x,t}.$$

where $\Upsilon_t = x_t/n_t^B$ and $\zeta_{x,t}$ is a transitory reserve penalty shock that follows an autoregressive process of order one. The above formulation implies that as the excess reserves increase, this will increase the penalty that the bank must pay. I allow for the possibility that there could be some efficiency gains in holding excess liquidity (i.e. ϵ can be negative). However, I restrict my attention to calibrations where the banks penalty for reserves increases when reserves increase: at the margin $(\Upsilon_t^2 n_t^B + \epsilon \Upsilon_t)$ is positive. Alike in the case of the entrepreneurs, at the end of the period a fraction σ_B of bankers remains and the rest disappear and are replaced by an equal number of workers. The difference is that now this transition is exogenously determined.

The banker's objective at the end of period t , is the expected present value of future dividends:

$$V_{j,t} = \mathbb{E}_t \sum_{j=1}^{\infty} (1 - \sigma_B) \sigma_B^{j-1} \Lambda_{t+1} n_{j,t+1}^B. \quad (15)$$

In order to set a limit to the bankers borrowing from either the depositors or the central bank, I introduce an endogenous constraint on the banks ability to borrow in the same fashion as in GK and others. A banker j after collecting deposits from households and liquidity from the central bank may decide to divert a fraction of these funds. This occurs when the bank's value from diverting is higher than its franchise value. It is assumed that the bank can steal a fraction $\theta \in [0, 1]$ of the expected non-defaulting loans net a fraction $\theta\omega \in [0, 1] < \theta$ of the central bank liquidity. The cost of stealing for the banker is that the creditors can force the intermediary into bankruptcy at the beginning of the next period. This sets a limit to the bankers borrowing from either the depositors or the central bank. In order for the banks creditors to continue providing funds to the bank, the following incentive constraint must hold:

$$V_{j,t} \geq \theta[(1 - p(\bar{\psi}))l_{j,t} - \omega m_{j,t}]. \quad (16)$$

That is, bank's value must be greater or at least equal with the value of its divertable assets. When this constraint holds bankers have no incentive to steal from their creditors. In the

case where the constraint binds a spread between the risky and the riskless interest rate emerges. As I will show below this will be the case in times of crisis. A reduction of the banker's net worth will make the constraint to bind and a rise of the spread will occur.

The value of the bank at the end of period $t - 1$ must satisfy the Bellman equation:

$$V_{j,t-1}(l_{j,t-1}, x_{j,t-1}, d_{j,t}, m_{j,t-1}) = E_{t-1} \Lambda_{t-1,t} \sum_{i=1}^{\infty} \{(1 - \sigma_B) n_{j,t}^B + \sigma_B \max_{d_{j,t}} [\max_{l_{j,t}, x_{j,t}, m_{j,t}} V_t(l_{j,t}, x_{j,t}, d_{j,t}, m_{j,t})]\}. \quad (17)$$

Banker's problem is to maximize (15) subject to the balance sheet (13) and liquidity constraint (16).

Proposition 1. A solution to the banker's dynamic program is

$$V_{j,t}(l_{j,t}, x_{j,t}, d_{j,t}, m_{j,t}) = A^B n_{j,t}^B.$$

The marginal value of the banker's net worth A^B is then:

$$A^B = \mu_t \phi_t + \nu_{d,j,t} + \frac{\kappa}{2} \Upsilon_t^2.$$

μ_t is the spread, ϕ_t is the maximum leverage and $\nu_{d,j,t}$ is the marginal loss from deposits.

Proof. See appendix D.

The proposition clarifies the role of the bank's net worth in the model. We can rewrite the incentive constraint using the linearity of the value function as

$$\frac{A^B}{\theta} \geq \frac{[(1 - p(\bar{\psi}_t)) l_{j,t} - \omega m_{j,t}]}{n_{j,t}^B}.$$

The adjusted leverage of a banker cannot be greater than A^B/θ . The right hand side shows that as the net worth of the banker decreases the constraint is more likely to bind. Proposition 1 also implies that even there is heterogeneity in the bankers' holdings and net worth, this does not affect aggregate dynamics. Hence, the transition from the individual to aggregate variables takes place in the same way as in the previous section.

The maximum adjusted leverage ratio of the bank is defined as

$$\phi_{j,t} = \frac{\nu_{d,j,t} + \frac{\kappa}{2} \Upsilon_t^2}{(1 - p(\bar{\psi}_t))\theta - \mu_t}. \quad (18)$$

Maximum adjusted leverage ratio depends positively on the marginal cost of the deposits $\nu_{d,j,t}$

and reserves and on the excess value of bank assets μ_t . As the credit spread increases, banks franchise value V_t increases and the probability of a bank to divert its funds declines. From the other hand as the proportion of assets that a bank can divert, θ increases, the constraint binds more. The detailed model solution shows that when the incentive constraint is not binding, the credit spread between the loan and the deposit rate is zero, while in the case where constraint is binding the spread becomes positive. The latter is the case during a financial crisis.

Aggregation.— Aggregate net worth is the sum of the new bankers' and the existing bankers' equity: $N_{t+1}^B = N_{y,t+1}^B + N_{o,t+1}^B$. Young bankers' net worth is the earnings from loans multiplied by ξ_B which is the fraction of asset gains that being transferred from households to the new bankers

$$N_{y,t+1}^B = \xi_B [R_{l,t} L_t]$$

and the net worth of the old is the probability of survival for an existing banker multiplied by the net earnings from assets and liabilities

$$N_{o,t+1}^B = \sigma_B [R_{l,t} L_t + R_{x,t} X_t - R_m M_t - R_t D_t].$$

2.3. Fiscal, Monetary Policy and Resource Constraint

The government acts as both fiscal and monetary authority. Its fiscal role is limited on collecting lump sum taxes T_t to finance its public expenditures G_t . I assume that the level of the government expenditures is at a fixed level relative to output (γ^G) and subject to a transitory shock g_t that follows an AR(1) process. Hence, $G_t = (\gamma^G Y_t) g_t$. As a monetary authority, it supports the banking liquidity by providing M_t funds at interest rate $R_{m,t}$, it accommodates banks' excess reserves X_t at an interest rate $R_{x,t}$ and issues bonds bought by households $D_{g,t}$ at the interest rate R_t to finance its expenses. The government budget constraint thus is

$$G + M_t - D_{g,t} - X_t = T_t + R_{m,t} M_{t-1} - R_t D_{g,t-1} - R_{x,t} X_{t-1}. \quad (19)$$

The monetary authority's liquidity policy is:

$$\chi_{m,t} = \chi_m + \kappa_m \mathbb{E}_t [(R_{l,t+1} - R_{t+1}) - (R_l^{ss} - R^{ss})], \quad (20)$$

where $\chi_{m,t} = \frac{M_t}{L_t + X_t}$ be the fraction of the total bank assets financed through LTRO and χ_m is its steady state value. $\kappa_m \in [0, 100]$ is the policy coefficient which indicates how strongly the central bank increases the liquidity provision. $(R_{l,t+1} - R_{t+1}) - (R_l^{ss} - R^{ss})$ is the deviation

of the credit spread from its steady-state value.

To complete the model I am introducing the resource constraint

$$Y_t = C_t + C_t^E + I_t[1 + \tilde{f}\left(\frac{I_t}{I_{t-1}}\right)] + G_t + \Phi(x_t) + \mu\psi_t R_{k,t} Q_t K_t.$$

Lastly, in order to enhance intuition on the model mechanism, the flows between agents are summarized in figure 3.

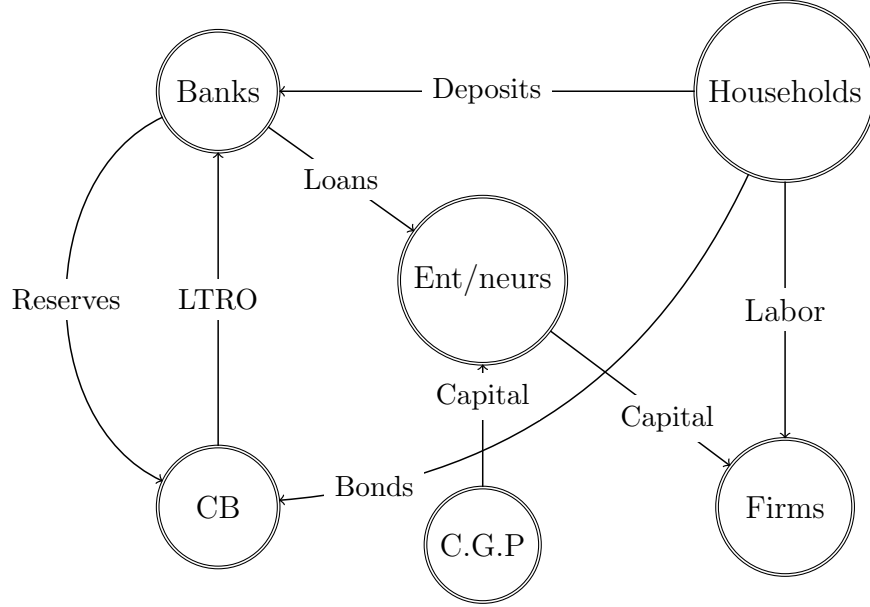


Fig. 3. Model Interactions. CGP: Capital goods producers. CB: Central bank

3. Banker's Optimal Asset Allocation

Due to the linearity of the value function the optimization problem can be rewritten as

$$V_{j,t}(s_{j,t}, x_{j,t}, d_{j,t}, m_{j,t}) = E_t \Lambda_{t,t+1} \Omega_{t+1} n_{t+1}^B \quad (21)$$

The banker's problem is to maximize its discounted net worth subject to the incentive compatibility constraint and the balance sheet constraint.

Ω_{t+1} is the marginal value of the banker's net worth:

$$\Omega_{t+1} = (1 - \sigma_B) + \sigma_B (\mu_{t+1} \phi_{t+1} + \nu_{d,t+1} + \frac{\kappa}{2} \Upsilon_t^2). \quad (22)$$

Taking the first order conditions yields the optimal allocation for excess reserves and

loans. At optimum, the demand for excess reserves for the bank is such that the marginal benefit for investing in one unit of reserves, $\nu_{x,j,t}$, equals the marginal cost from using one unit of short term debt $\nu_{d,j,t}$ and the marginal cost of raising one unit of reserves.

$$\nu_{x,j,t} = \nu_{d,j,t} + \Phi'(x_{j,t}).$$

The bank's credit supply to non-financial firms is derived by solution of the maximization problem and is:

$$l_{j,t} = \phi_{j,t} n_{j,t}^B + \underbrace{\frac{1}{1 - p(\bar{\psi}_t)}}_{\text{risk-shifting}} (\omega m_{j,t}). \quad (23)$$

Available credit depends on two components: the bank's own funds and the liquidity received by the central bank. When the liquidity policy is absent ($m_{j,t} = 0$), then the bank adjust its loan supply according to the product of its leverage $\phi_{j,t}$ and its net worth. At turbulent times, when the central bank injects liquidity into the system ($m_{j,t} > 0$) banks that receive LTRO funds will increase their lending compared to the no liquidity case but they engage in risky lending. Banks search for yield and increase the lending to the non-financial firms which during crises have a higher likelihood of default. I denote this as the *risk-shifting* component. Search for yield occurs using the central bank funds and this captures the risk-shifting channel of monetary policy.

The bank's demand for loans is determined from the expected loan rate that charges to the firms.

$$\mathbb{E}_t R_{l,t+1} = \underbrace{\frac{\lambda_t}{(1 + \lambda_t)} \frac{\theta}{\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1}}}_{\text{liquidity component}} + \underbrace{\mathbb{E}_t R_{t+1} \frac{1}{1 - p(\psi_t)}}_{\text{risk component}}$$

Two components determine the expected loan rate. The first, is due to the binding funding constraints for the bankers. When the constraint binds, bankers cannot get new funding to explore new profitable activities. Hence they adjust the loan rate. This will be referred as the *liquidity component*. The second one reflects the compensation that bankers demand when the firms' probability of default increases. This is the *risk component*. This result is in line with Bocola (2016). Using another source of uncertainty (an increase of future sovereign default) instead of the firms' default, shows the existence of the same two sources of frictions between the loan and the risk free rates.

The interest rate of LTRO funding is endogenously calculated as follows:

$$R_{m,t} = \omega R_{l,t} + \left(1 - \omega \frac{1}{1 - p(\bar{\psi}_t)}\right) R_t.$$

The liquidity funding interest rate is a weighted average of the loan rate and the deposit rate. I calibrate the parameter values in order to have a liquidity funding interest rate below the loan rate but slightly above the riskless rate. Lastly the interest rate on reserves is defined as a function of the riskless rate $R_{x,t} = \tau R_t$.

4. Calibration & Data

In this section I present the model calibration and evaluation. I proceed with the calibration section, providing the parameter values. Next, I compare the model's statistics with the Euro Area data and I present two exercises that help measuring the model's empirical performance.

4.1. Calibration

The model's calibration has been designed in order to match the moments of the Eurozone data. To parametrize the model I use values used in the literature associated with the Euro area data, where many of them have been estimated using Bayesian techniques. For the parameters that are not defined in the literature, I choose the ones appropriate to match the model with the data.

One period in the model is one quarter. All the calibrated values are presented in Table 1. The values for the share of capital α and the depreciation rate δ are chosen to 0.3 and 0.025 respectively as in Gerali, Neri, Sessa, and Signoretti (2010), Gelain (2010). They are included also in the span of values typically used in the literature. I choose the value of β to be 0.99, in order to yield a quarterly discount rate $R = 1.01$ which is equivalent to a 4% annual interest rate, a value close to the historical time series of the interest rate and also in line with several contributions for the Euro Area. The steady state fraction of government expenditures to output is set to 0.2, a value close to the data and to many papers that estimate this parameter for the Euro Area (see Christoffel and Schabert (2015) for example).

Regarding the consumers, the relative utility weight of labor χ is chosen to ensure a level of labor close to 1/3 in steady state, a fairly common benchmark in the literature (see Corsetti,

Parameters	Value	Definition
Households		
β	0.99	Discount rate
γ	0.70	Habit parameter for consumption
χ	5.584	Relative utility weight of labor
ϵ^B	0.333	Inverse Frisch elasticity of labor supply
Banks		
θ	0.23	Divertable fraction of loans
ω	0.3	Divertable fraction of LTRO
ξ_B	0.009	Entering bankers initial capital
σ_B	0.955	Bankers' survival rate
κ	13.41	Reserves cost function parameter
ϵ	-0.2	Gains from reserves
δ	0.025	Depreciation of capital
α	0.33	Capital share
τ	1	Interest on reserves relative to the riskless rate
Entrepreneurs		
μ	0.21	Monitoring costs
σ_{ss}	0.26	Steady state of idiosyncratic variance
ξ_E	0.005	Entering entrepreneurs initial capital
Resource constraint and government policy		
δ	0.025	Depreciation of capital
α	0.33	Capital share
η	10.09	Inverse elasticity of net investment to the price of capital
γ^G	0.2	Steady state fraction of government expenditures to output
κ_m	20	Central bank's response to the spread deviation
χ_m	0.001	Steady state value of the LTRO

Table 1: Parameter Values

Kuester, Meier, and Müller (2014), and Poutineau and Vermandel (2015) for instance). The habit parameter for consumption γ is chosen to be 0.7, a value close the estimated value in Gerali et al. (2010), Christoffel and Schabert (2015) and Christiano, Motto, and Rostagno (2010). The inverse Frisch elasticity of labor supply ϵ^B is 0.33 similar to Christoffel and Schabert (2015). The inverse elasticity of net investment to the price of capital η equal to 10.09 as the estimated value from Gerali et al. (2010) for the Eurozone.

For the banking sector, I set the values for parameters $\theta, \omega, \xi, \sigma_B$ such that the model yields a steady state leverage (ϕ) equal to 4 for the banks and a bank capital to lending ratio of 0.25 close to the value suggested by Christoffel and Schabert (2015). Also with these parameter values, the model yields a steady state excess return on private securities of 100 basis points which is also close to the aforementioned reference, a steady state penalty rate of 50 basis points and a steady state excess return on the deposit facility less that the penalty

rate. I define τ equal to one as I set the rate on reserves equal to the rate of the riskless asset which is the case according to the pre-2009 Euro data. I choose the parameters of the reserve penalty function as in Gertler et al. (2012). Firstly I target a marginally positive level of excess reserves in steady state and secondly by assigning a negative value to ϵ , I allow for some liquidity management gains from holding reserves.

The choice of idiosyncratic risk I choose a value of monitoring costs (μ) equal to 0.21. It has been estimated by Queijo von Heideken (2009) that in the Euro area the monitoring costs are about 27%. The parameter for transfers to the new entrepreneurs (ξ^E) is set to 0.005. Under this value the model gives an equity to debt ratio of 1.31. This is included in the interval of 1.08 – 2.19 that Christiano et al. (2010) observe for the Euro data. Lastly, the value for the steady state entrepreneur’s idiosyncratic variance is 0.26 close to the findings of Queijo von Heideken (2009) for the Euro Area.

The value chosen for χ_m replicates the real value of the LTRO over the other bank’s assets before the crisis. I set κ_m equal to 20. Under this calibration, I can simulate the increase of the total banks’ assets financed by the LTRO in the Eurozone which was roughly 20%. Finally, I choose the values of the autoregressive parameters and the transitory shocks in order to match the second moments of the Euro Area data as shown below.

4.2. Model Evaluation

In the following section I try to shed light on the performance of the model relative to the Euro Area data. I present two tables reporting steady state statistics and business cycle moments. Prior to the analysis I transform the data as follows. In all the variables apart from the credit spread and interest rate I firstly transform them to real variables by dividing with the GDP deflator. Then they are expressed as per capita terms by dividing them with the active labor force. Data used is in quarterly frequency. Table 2 reports the non-stochastic steady state properties of the model when the parameters are set to their calibrated values and also the corresponding values in the data. The data values are calculated as the average of each variable relative to the average level of output. The model manages to deliver well the ratios of different variables. Consumption, investment, government spending and reserves follow closely the data values. Credit to output is capturing the fact that is far above all the other statistics but the model overestimates its value. Lastly, the non-risk bearing interest rate is overestimated by 1/3 of the actual value in the data.

Table 3 presents the second moments of selected variables and their correlation with output. Also reports their corresponding counterparts in the data. To perform this exercise I take the logarithmic first difference and subtract the mean of the transformed per capita,

Variable	Model	Data
C/Y	0.550	0.561
I/Y	0.223	0.216
L/Y	4.13	1.68
G/Y	0.200	0.182
X/Y	0.041	0.011
R	4.040	2.759

Table 2: Steady State Properties vs. Euro Data. Data sources: ECB Warehouse, St.Louis FED - FRED and EUROSTAT

real variables. Overall, the values seem to follow closely the data. The standard deviation of output in the model is 0.47 and it's true value in the data 0.7. Also, the model captures the fact that consumption volatility is closely following output volatility. Investment is much more volatile than output and this is clearly the case with a value of 1.77 in the model and 1.7 in the data. Credit standard deviation is almost a half of the value seen in the data. The model seems to underestimate the credit spread volatility which is 2/3 below its empirical counterpart.

In terms of business cycles cyclicalilty, the model again performs relatively well. It manages to deliver the correct cyclicalilty of all variables. Investment and credit are characterized by strong procyclicalilty with correlation with output of 0.84 and 0.68 respectively. The credit spread is following the counter-cyclicalilty suggested by the data with a value close to the one we observe in the data. Finally, consumption correlation with output is about a 1/3 lower than the level of the data shows.

5. Quantitatively Analysis

This section illustrates the policy recommendations that the model can provide by performing two different sets of experiments. In what follows I present the impulse response functions to a number of model's structural shocks and then I estimate the welfare gains (or costs) from a number of different policy actions. To solve the model I apply an approximation to the policy functions. The numerical strategy is based on perturbation methods as in Schmitt-Grohé and Uribe (2004) and is well-suited for the specific modeling framework, given the large number of state variables.

Variable		SD (%)	Corr. with output
Output	Model	0.466	1.000
	Data	0.704	1.000
Investment	Model	1.765	0.841
	Data	1.705	0.953
Consumption	Model	0.450	0.594
	Data	0.737	0.997
Credit	Model	0.745	0.676
	Data	1.437	0.922
Credit Spread	Model	0.341	-0.158
	Data	1.039	-0.229
Government Spending	Model	0.230	0.171
	Data	0.696	0.953

Table 3: Business Cycle Statistics, Model Versus Data. Data sources: ECB Warehouse, St.Louis FED - FRED and EUROSTAT

5.1. Impulse Response Functions

The first objective is to simulate the big downturn of the Euro economy as occurred in the end of 2007 and to see how the model economy responds without an intervention from the central bank. Then, I show what happens when the central bank takes action by supplying liquidity. To implement this, I follow Christiano et al. (2014) definition of the risk shock. The risk shock is defined as an increase in the volatility of the entrepreneurs distribution of good and bad signals. Specifically, there is an increase in the standard deviation σ_ψ of the idiosyncratic shock ψ that the entrepreneurs receive. I provide the impulse response functions to a 1% standard deviation increase of the risk shock. I apply the same exercise for two different cases. In the first one there is no policy response by the central bank, whilst in the second case the central bank follows the feedback rule described in the model section and supplies liquidity to the banking system:

Results are reported in Figure 4. The blue line shows the responses to an 1% standard deviation increase of the risk shock when the central bank does not provide liquidity to stimulate the banking system. The responses show that as the riskiness of the entrepreneurial project increases banks charge higher interest rates to cover the costs, thus the spread increases. As expected the default probability of entrepreneurs becomes higher as they cannot repay back the loans. Entrepreneurs borrow less and credit drops. With fewer financial re-

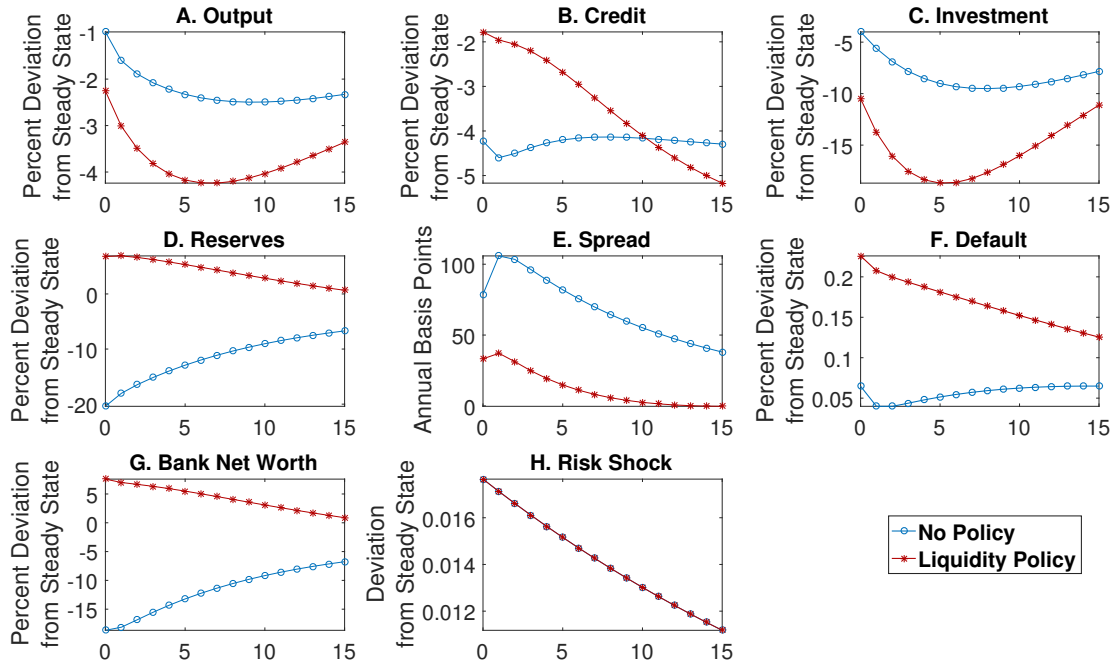


Fig. 4. Impulse responses to an increase of the risk shock

sources, entrepreneurs purchase less capital, which leads to an investment fall. The drop in investment then leads to a fall in output and consumption. The fall in investment produces a fall in the price of capital, which reduces the net worth of entrepreneurs, and this magnifies the impact of the jump in risk through accelerator effects.

The red line displays the responses when the central bank follows the feedback rule and intervenes with liquidity in order to reduce the spread. In this case the interest rate spread falls from above than 100 basis points to less than 50 basis points when the risk shock hits. Extra liquidity provides more funds for the banks and thus they reduce the interest rate for the non-financial firms. Banks now reduce their aversion to supply new credit but they increase their reserve holdings as they use a portion of the fresh liquidity to invest in the safe asset. The central bank policy improves the health of the financial institutions and that can be seen by the increase in their net worth. A first takeaway from this exercise is that an increase of the firms' project volatility combined with new liquidity ejected from the central bank could be the answer to the recent unprecedented reserve accumulation.

Staying on the liquidity regime graph, real variables such as investment, output and capital are worse off compared to the no policy case. This is due to the risk-taking channel of monetary policy. The liquidity provided by the central bank is driving excessive risk-

taking from the banks as the riskiness of the firms has increased and banks face moral hazard problems. The likelihood of default by firms now increases as they leverage more due to the lower cost of credit and banks expand credit to more insolvent firms. The net worth of the entrepreneurs is affected which leads to less capital purchase and a higher drop in investment than in the no policy regime. This describes a potential problem of the open market operations mechanism in turbulent times. Even if banks spend the liquidity injected to new credit, this credit ends up to insolvent non-financial corporations.

Having described the risk shifting effect, I proceed with an exercise trying to capture the effect of the negative rates on reserves. Here this is implemented by an increase of the penalty rate for holding reserves. At the same time the central bank supplies liquidity to the banking system. In other words, now banks have to pay more in order to accumulate excess reserves. This tries to replicate the recent European Central Bank policy of charging fees to reserves. Here, I depart from an increase in the riskiness of the entrepreneurial projects. Therefore, this exercise describes the banking behavior when they don't have an incentive to cut off lending and is used as a plain experiment of assessing the disincentives from holding reserves. The following figure shows the response of variables of importance to an 1% standard deviation increase in the reserves' penalty level.

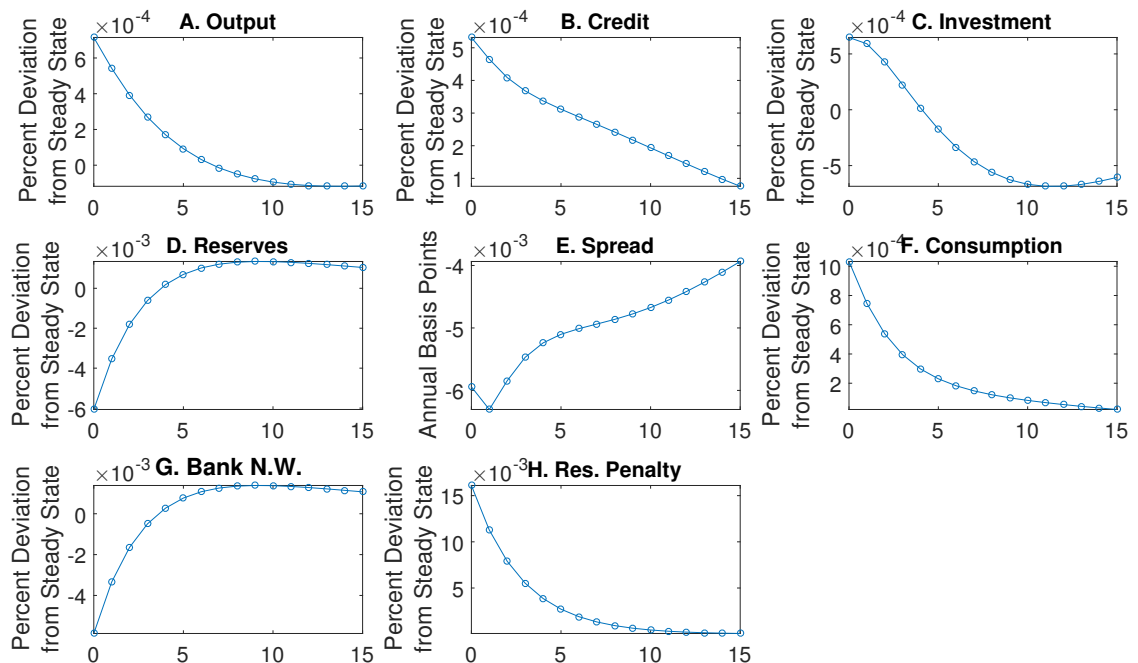


Fig. 5. Impulse responses to an increase of the reserve penalty

It is clear from the graph that as the penalty for reserves increases, banks want at least in the short run want to hold less reserves. At the same time a smaller increase in credit is taking place. That gives a push to the economy. Entrepreneurs now borrow more and hence they invest more in capital. This has an immediate consequence on output and consumption which both increase. Hence, this presents some evidence that the recently announced policy of the European Central Bank to charge fees on reserves is on the right direction. But even if on the one hand, qualitatively the policy produces the desired outcome, on the other hand although banks are forced to increase credit there could be a risk shifting channel problem as I described in the previous exercise.

5.2. *Measuring Welfare Costs*

In order to conduct policy analysis, I will now present the welfare costs (or gains) in terms of consumption units between i) the adoption of aggressive liquidity supply scheme by the central bank and ii) the no policy rule.

Since the non-stochastic steady state for the two different regimes is different, using the unconditional expectation of welfare will leave out the dynamics associated with the stochastic steady state. Therefore, as in Schmitt-Grohé and Uribe (2007) I proceed with the welfare conditional on the initial state being the non-stochastic steady state. At time zero, the state vector is the same for both policies, in other words all state variables equal their steady states. This ensures that in both regimes we start from the same initial values. Given that in a first order approximation the welfare \mathcal{W}_t equals to its non-stochastic steady state I will proceed with a second order approximation to determine the effects of different regimes on lifetime utility.

I define the welfare associated with the no policy scheme conditional on a particular state of the economy in period 0 as:

$$\mathcal{W}_0^n = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^n, N_t^n)$$

where the C_t^n, N_t^n denote the consumption units and labor hours spend under the no policy scheme. In a similar way I define the conditional welfare associated with the liquidity supply scheme as:

$$\mathcal{W}_0^l = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^l, N_t^l)$$

where again, where the C_t^l, N_t^l denote the consumption units and labor hours spend under the liquidity supply scheme.

Let λ^c be the conditional welfare cost (or gain) for the consumer of adopting a liquidity policy rather than a no action policy by the central bank. In other words λ^c is the fraction of consumption that the household would need each period in the liquidity supply regime to yield the same welfare as would be achieved in the no policy regime. Formally λ^c is chosen to solve

$$\mathcal{W}_0^l = E_0 \sum_{t=0}^{\infty} \beta^t U((1 + \lambda^c)C_t^n, N_t^n).$$

A positive value for λ^c means that the household prefers the liquidity policy regime - i.e. it would need extra consumption when the liquidity regime is on to be indifferent between the two regimes. In contrast, a negative value of λ^c means that the household prefers the no policy regime.

Substituting the utility function given in equation (1) we can rewrite the above expression as:

$$\begin{aligned} \mathcal{W}_0^l &= E_t \sum_{i=0}^{\infty} \beta^i [\ln((C_{t+i} - \gamma C_{t+i-1})(1 + \lambda^c)) - \frac{\chi}{1 + \epsilon} N_{t+i}^{1+\epsilon}] \\ &= \frac{\ln(1 - \lambda^c)}{1 - \beta} + \mathcal{W}_0^n. \end{aligned}$$

Solving for λ^c we have

$$\lambda^c = \exp\{(\mathcal{W}_0^l - \mathcal{W}_0^n)(1 - \beta)\} - 1. \quad (24)$$

Table 4 shows the welfare analysis results. It includes the total value of conditional welfare in the liquidity policy and in the no policy rule and also the consumption equivalent metric that yields from the transition between the two policies. The consumption equivalence is measured in percentage terms. This metric is an indication of how much consumption units in percent are lost or gained from the transition to the new policy. The conditional welfare as is reported in Table 4 increases as we move from the no policy regime to the liquidity policy regime. The gain is about 4.9 % of consumption units. Hence, the liquidity policy is considered to be welfare improving.

Additional to the conditional welfare comparisons I present the second moments of selected variables for the two different policy regimes. As expected, consumption volatility reduces to almost a half after the liquidity policy, from 0.83 to 0.45. Output and credit volatility behave in a similar manner and also the discount rate and the credit spread as the liquidity policy stabilizes and reduces the spread. The standard deviation of investment on

the other hand more than doubles after the change in the policy regime.

	No Policy	Liquidity Policy
Welfare		
Conditional Welfare	-304.61	-299.85
Conditional Welfare Cost (Gain)	0	4.8752
Standard Deviation		
Output	0.5064	0.4664
Consumption	0.8307	0.4502
Investment	0.7792	1.7649
Credit	0.6795	0.3452
Spread	0.8393	0.3405
Discount Rate	0.5742	0.2256

Table 4: Welfare Costs and Second Moments

Having observed the welfare benefits of the new liquidity accommodating policy I proceed to two exercises associated with the recent negative interest rates policy that has been established in the ECB and in a number of other central banks. As it was illustrated in the model section, there is a penalty for reserves that the banks have to pay if they accumulate reserves. The central bank in the model can adjust the cost of excess reserves. In the first experiment I study the the welfare effects of different penalties associated with the reserve holdings. I estimate the welfare value function for different levels of the reserve penalty under the liquidity policy regime and present below the conditional cost (or gain). Results are presented in Figure 6. It's noticeable from the graph that welfare is an increasing function to the level of reserves' penalty. As the fee that banks have to pay increases, banks marginal benefit from holding excess reserves decreases and banks find it more profitable to extend their credit instead.

From a consumer's perspective the higher the penalty rates for holding reserves, the higher the welfare gains she enjoys. For the same values of the penalty parameter at Figure 7 below are the stochastic steady state paths of reserves and credit. As the penalty rates increase, banks decide to hold less reserves and expand their credit to non-financial corporations thus increasing the welfare gains. This comes in line with the unprecedented policy of the ECB to charge the banks of the Euro Area for holding reserves. As the costs of reserves increase banks will extend credit supply and this has a positive impact on welfare.

The last exercise focuses on the stochastic steady state levels of credit and reserves. Specifically, I find how the paths of stochastic steady states for credit and excess reserves evolve in response to an increase to the penalty rate. Figure 7 displays the results.

The figure shows that as the cost of reserves increases, banks as expected will reduce their reserves and increase their credit. At the same time, in order to achieve the reserves reduction

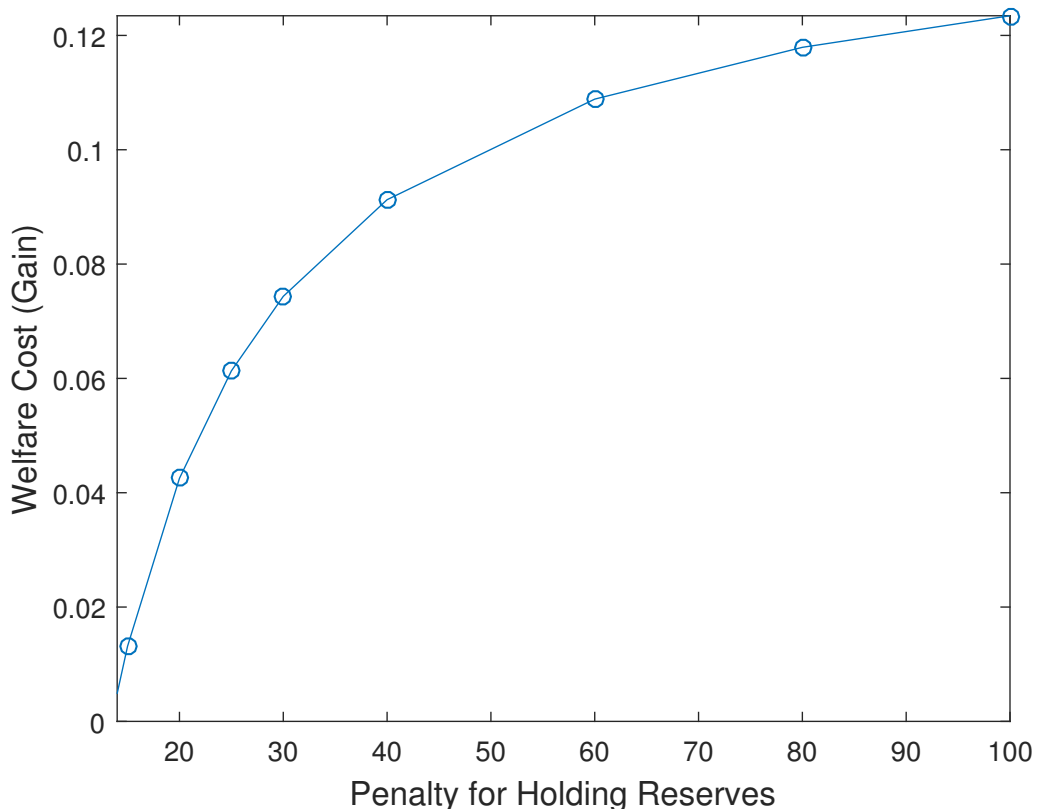


Fig. 6. Conditional and Unconditional Welfare Relative to Reserve Penalty

to a substantial level, the penalty must increase to almost ten times the initial steady state value. Trying to bring the above results to the recent central bank unconventional measures, the general intake is that, implementing negative interest rates policy will make the banks adverse to increase their credit but only when the rates charged will be negative enough. In other words, only when the fees that the banks will have to pay is high enough.

6. Conclusion

The impact of recent policy actions by the European Central Bank is on debate for several years since the start of the financial crisis. Although the ECB has significantly increased its balance sheet in order to provide liquidity to financial institutions, the macroeconomic environment has still not yet revived from the crisis. Banks have increased significantly their reserves holdings while credit growth is still in very low levels. To examine these developments, I build a DSGE model with financial frictions on the demand and the supply side of credit and I calibrate it to the Euro Area data. I conduct a set of different exercises

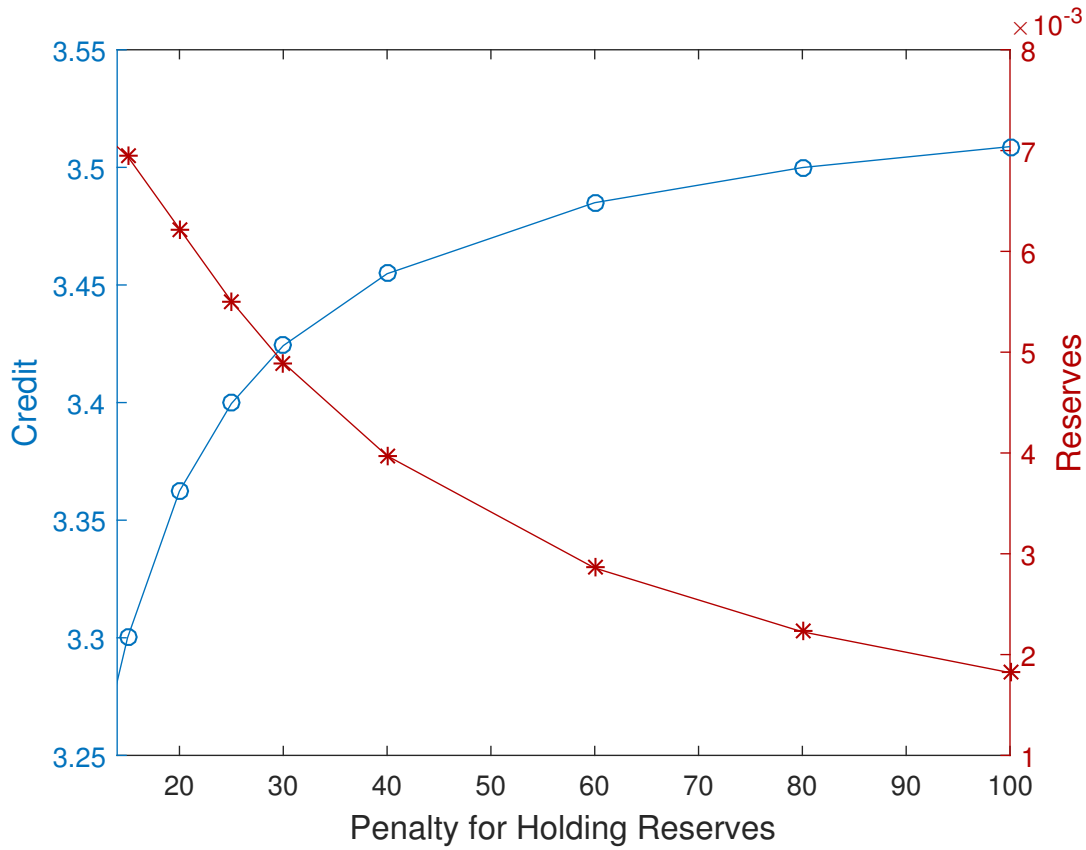


Fig. 7. Stochastic Steady State Path of Credit and Reserves

to explore questions rising from the recent policies adapted in the Eurozone. Why banks decide to hold reserves and reduce credit? Has the LTRO policy improved economy and the banking sector health? Were these measures welfare improving? What can the ECB do in order to make banks reduce their reserve holdings and expand credit?

The main finding is that the LTRO liquidity policy followed by the ECB improved the banks' health but at the same time the macroeconomy would have been better off should the liquidity policy haven't taken place. This result is due to the risk-shifting channel of unconventional monetary policy. It is also in line with recent empirical findings on the monetary policy transmission channels. The paper also shows that an increase in the riskiness of the non-financial corporations is making banks to reduce new credit and instead accumulate more reserves when the central bank provides liquidity. Hence, the recent increase can be addressed to a credit demand shock. To test the welfare implications of the recent liquidity policies, I employ the consumption equivalence metric that measures the conditional welfare change between the counter-factual no policy regime and the liquidity policy. I find that the policy adapted by the ECB improves welfare. Lastly, in an attempt to study the recent

negative interest rates policy of the ECB, I find that as the interest rates become negative enough then banks will start reducing their accumulated reserves.

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Appendix A Household's Problem

Let $u_{c,t}$ denote the marginal utility of consumption and $\Lambda_{t,t+1}$ denote the household's stochastic discount factor (the intertemporal marginal rate of substitution):

$$\Lambda_{t,t+1} \equiv \beta \frac{u_{c,t+1}}{u_{c,t}}, \quad (\text{A.1})$$

$$u_{c,t} = (C_t - \gamma C_{t-1})^{-1} - \beta \mathbb{E}_t \gamma (C_{t+1} - \gamma C_t)^{-1}.$$

Let λ be the Lagrange multiplier associated with the household problem, the Lagrangian is

$$\mathcal{L} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\epsilon} N_{t+i}^{1+\epsilon} + \lambda_t [W_t N_t + \Pi_t + R_t D_{h,t} - (C_t + T_t + D_{h,t+1})] \right\}.$$

The first order conditions yield:

$$\frac{\theta \mathcal{L}}{\theta C_t} : u_{c,t} - \lambda_t = 0 \quad (\text{A.2})$$

$$\frac{\theta \mathcal{L}}{\theta D_{h,t+1}} : -\lambda_t + \beta \lambda_{t+1} (R_{t+1}) = 0 \quad (\text{A.3})$$

$$\frac{\theta \mathcal{L}}{\theta N_t} : -\chi N_t^\epsilon + \lambda_t W_t = 0 \quad (\text{A.4})$$

Combining (A.2) and (A.3) we get the Euler equation

$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1} = 1$$

and by combining (A.2) and (A.4) we get the optimality condition for labor supply

$$u_{c,t} W_t = \chi N_t^\epsilon$$

Appendix B Entrepreneur's Problem

Let \mathcal{L} be the Lagrangian of the maximization problem and λ_t^ϵ the Lagrange multiplier associated with the zero profit condition.

$$\mathcal{L} = [1 - \Gamma(\overline{\psi}_{t+1}) R_{k,t+1} Q_t K_{t+1}] + \lambda_t^\epsilon [R_{k,t} Q_t K_t [\Gamma(\overline{\psi}_{t+1}) - \mu G(\overline{\psi}_{t+1})] - R_{t+1} (Q_t K_t - N_t^\epsilon)].$$

The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta \mathcal{L}}{\theta K_t} : 1 - \Gamma(\bar{\psi}_{t+1})R_{k,t+1} + \lambda_t^e[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})R_{k,t+1} - R_{t+1}] = 0 \quad (\text{B.1})$$

$$\frac{\theta \mathcal{L}}{\theta \bar{\psi}_{t+1}} : -\Gamma'(\bar{\psi}_{t+1}) + \lambda_t^e[\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1})] = 0 \quad (\text{B.2})$$

From equation B.2 we get

$$\lambda_t = \frac{\Gamma'(\bar{\psi}_{t+1})}{\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1})}. \quad (\text{B.3})$$

Inserting B.3 to B.1 we get:

$$R_{k,t} = \frac{\Gamma'(\bar{\psi}_{t+1})}{(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1 - \Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))} R_{t+1},$$

which gives the external finance premium as shown in the BGG:

$$\mathbb{E}_t R_{k,t+1} = \mathbb{E}_t \rho(\bar{\psi}_{t+1}) R_{t+1}$$

where $\rho(\bar{\psi}_{t+1})$ is given by

$$\rho(\bar{\psi}_{t+1}) = \frac{\Gamma'(\bar{\psi}_{t+1})}{[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))\Gamma'(\bar{\psi}_{t+1}) + (1 - \Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))]}.$$

Appendix C Entrepreneur's choice of remain

Proof. The zero profit condition is

$$R_{k,t} Q_t K_t [\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})] \geq R_{t+1} (Q_t K_t - N_t^e)$$

and divided by N_t^e becomes

$$R_{k,t} \frac{Q_t K_t}{N_t^e} [\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})] \geq R_{t+1} \left(\frac{Q_t K_t}{N_t^e} - 1 \right).$$

Substituting the definition of N_t^e

$$R_{k,t} \frac{Q_t K_t}{(\sigma_E + \xi)(1 - \Gamma(\bar{\psi}_t)) R_{k,t} Q_{t-1} K_{t-1}} [\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})] \geq R_{t+1} \left(\frac{Q_t K_t}{(\sigma_E + \xi)(1 - \Gamma(\bar{\psi}_t)) R_{k,t} Q_{t-1} K_{t-1}} - 1 \right)$$

we have

$$\frac{[\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})]}{(\sigma_E + \xi)(1 - \Gamma(\bar{\psi}_t))} \geq R_{t+1} \left(\frac{1}{(\sigma_E + \xi)(1 - \Gamma(\bar{\psi}_t))R_{k,t}} - 1 \right)$$

and we get the equation for σ_t^e

$$\sigma_t^e = \frac{1}{R_k(1 - \Gamma(\bar{\psi}_t))} - \frac{\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})}{R_t(1 - \Gamma(\bar{\psi}_t))} - \xi$$

and the derivative with respect to $\bar{\psi}$

$$\frac{\partial \sigma_t^e}{\partial \bar{\psi}} = \frac{\Gamma'(\bar{\psi}_t)R_k}{[R_k(1 - \Gamma'(\bar{\psi}_t))]^2} - \frac{\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1})}{R_t(1 - \Gamma'(\bar{\psi}_t))} - \frac{\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})R_t\Gamma'(\bar{\psi}_{t+1})}{[R_t((1 - \Gamma'(\bar{\psi}_t)))]^2}.$$

The $\sigma_{E,t}$ the values of $[0, 1]$ (so it is actually a probability measure), when $\bar{\psi} \in [0.49, 0.65]$, everything else remain constant. In the calibration there should be a restriction in the values of $\bar{\psi}$. That is in the variance of $\bar{\psi}$, σ_ψ .

For those values of $\bar{\psi}$ as ψ increases, $\sigma_{E,t}$ decreases. Hence the derivative is negative for those values. The path of $\sigma_{E,t}$ for the values of $\bar{\psi}$ is shown in Figure 8.

As $\bar{\psi}$ increases the probability of default increase too. It is much more likely for $\psi \leq \bar{\psi}$. Therefore, as the probability of default increases, the remain probability decrease up to the point it becomes zero. \square

Appendix D Bank's Problem

This appendix describes the method used for solving the banker's problem. I solve this, with the method of undetermined coefficient in the same fashion as in Gertler and Kiyotaki (2010). I conjecture that a value function has the following linear form:

$$V_t(l_{j,t}, d_{j,t}, x_{j,t}, m_{j,t}) = \nu_{l,j,t}l_{j,t}(1 - p) + \nu_{x,j,t}x_{j,t} - \nu_{d,j,t}d_{j,t} - \nu_{m,j,t}m_{j,t} - \Phi(x_t), \quad (\text{D.1})$$

where $\nu_{s,j,t}$ is the marginal value from credit for bank j , $\nu_{d,t}$ the marginal cost of deposits, $\nu_{x,j,t}$ the marginal value from the deposit facility and $\nu_{m,j,t}$ the marginal cost of the emergency funding. The banker's decision problem is to choose $s_{j,t}, x_{j,t}, d_{j,t}, m_{j,t}$ to maximize $V_{j,t}$ subject to the incentive constraint (16) and the balance sheet constraint (13). Using (13) we can eliminate $d_{j,t}$ from the value function. This yields:

$$V_{j,t} = l_{j,t}(\nu_{l,t}(1 - p) - \nu_{d,t}) + x_{j,t}(\nu_{x,j,t} - \nu_{d,j,t}) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{k,j,t}Q_t k_t + \nu_{d,t}n_{j,t}^B - \Phi(x_t).$$

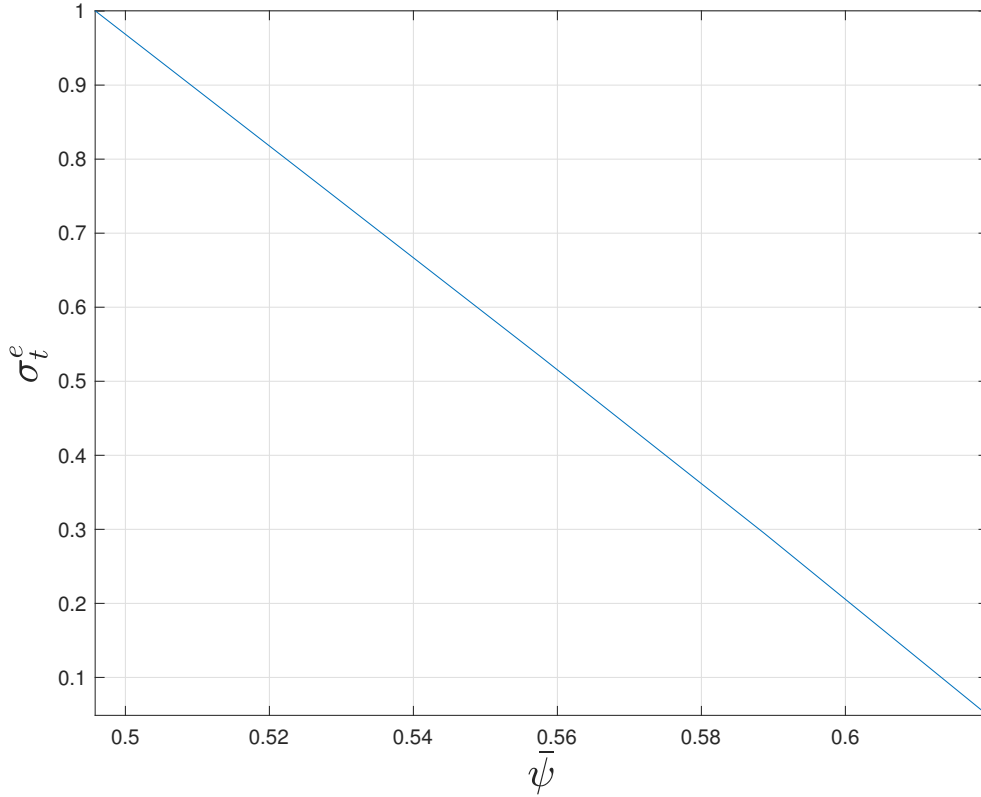


Fig. 8. Path of $\sigma_{E,t}$ for the values of $\bar{\psi}$

I define the ratio of excess liquidity to the net worth as

$$\Upsilon_t = \frac{x_t}{n_t^B}$$

and assume that the reserves penalty function has the following form:

$$\Phi(x_t) = \left(\frac{\kappa}{2} \Upsilon_t^2 n_t^B + \epsilon \Upsilon_t \right) \zeta_t.$$

Let \mathcal{L} be the Lagrangian of the maximization problem and λ_t the Lagrange multiplier.

$$\mathcal{L} = V_t + \lambda_t [V_t - \theta((1-p)l_t - \omega m_t)] = (1 + \lambda_t)V_t - \lambda_t \theta((1-p)l_t - \omega m_t).$$

The first order and Kuhn-Tucker conditions for the maximization problem are:

$$\frac{\theta \mathcal{L}}{\theta l_{j,t}} : (1 + \lambda_t)(\nu_{l,j,t}(1 - p) - \nu_{d,t}) = \lambda_t(1 - p)\theta \quad (\text{D.2})$$

$$\frac{\theta \mathcal{L}}{\theta \chi_{j,t}} : (1 + \lambda_t)((\nu_{x,j,t} - \nu_{d,t})n_t - \kappa \Upsilon_t n_t) = 0 \quad (\text{D.3})$$

$$\frac{\theta \mathcal{L}}{\theta m_{j,t}} : (1 + \lambda_t)(\nu_{m,t} - \nu_{d,j,t}) = \omega \lambda_t \theta \quad (\text{D.4})$$

$$\frac{\theta \mathcal{L}}{\theta k_{j,t}} : (1 + \lambda_t)\nu_{k,j,t}Q_t = 0 \quad (\text{D.5})$$

Equation (D.3) shows the optimal rule for the reserves' supply of the bank:

$$\nu_{x,j,t} - \nu_{d,j,t} = \kappa \Upsilon_t - \epsilon.$$

The Kuhn-Tucker condition yields:

$$\begin{aligned} KT : \lambda_t [l_{j,t}(\nu_{l,j,t}(1 - p) - \nu_{d,t}) + x_{j,t}(\nu_{x,j,t} - \nu_{d,j,t}) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) \\ + \nu_{d,j,t}n_{j,t}^B - \Phi_t - \theta((1 - p)l_{j,t} - \omega m_{j,t})] = 0. \end{aligned} \quad (\text{D.6})$$

I define the excess value of bank's financial claim holdings as

$$\mu_t = \nu_{l,j,t}(1 - p) - \nu_{d,j,t}. \quad (\text{D.7})$$

The excess cost to a bank of LTRO credit relative to deposits

$$\mu_t^m = \nu_{m,j,t} - \nu_{d,j,t}.$$

Then from the first order conditions we have:

$$\mu_t^m = \omega \mu_t \frac{1}{1 - p}. \quad (\text{D.8})$$

From (D.6) and (D.8) when the constraint is binding ($\lambda_t > 0$) we get:

$$\begin{aligned}
l_{j,t}(\nu_{l,t}(1-p) - \nu_{d,t}) + x_{j,t}(\nu_{x,j,t} - \nu_{d,j,t}) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} - \Phi_t &= \theta((1-p)l_t - \omega m_t) \\
l_{j,t}(\nu_{l,t}(1-p) - \nu_{d,t}) + \Upsilon_t n_t (\kappa \Upsilon_t) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} - \frac{\kappa}{2}\Upsilon_t^2 n_t &= \theta((1-p)l_t - \omega m_t) \\
l_{j,t}(\nu_{l,t}(1-p) - \nu_{d,t}) - m_{j,t}(\nu_{m,j,t} - \nu_{d,j,t}) + \nu_{d,t}n_{j,t} + \frac{\kappa}{2}\Upsilon_t^2 n_t &= \theta((1-p)l_t - \omega m_t) \\
l_{j,t}(\theta(1-p) - \mu_t) - m_{j,t}(\omega\theta - \mu_t^m) &= \nu_{d,t}n_{j,t} + \frac{\kappa}{2}\Upsilon_t^2 n_t \\
l_{j,t}(\theta(1-p) - \mu_t) - m_{j,t}(\omega\theta - \omega\mu_t \frac{1}{1-p}) &= \nu_{d,t}n_{j,t} + \frac{\kappa}{2}\Upsilon_t^2 n_t \\
l_{j,t}(\theta(1-p) - \mu_t) - \frac{1}{1-p}\omega m_{j,t}(\theta(1-p) - \mu_t) &= \nu_{d,t}n_{j,t} + \frac{\kappa}{2}\Upsilon_t^2 n_t
\end{aligned}$$

and by rearranging terms, we get equation (23) on the main text :

$$l_{j,t} - \frac{1}{1-p}(\omega m_{j,t}) = \frac{(\nu_{d,j,t} + \frac{\kappa}{2}\Upsilon_t^2)n_t}{\theta(1-p) - \mu_t},$$

which gives the bank asset funding. It is given by the constraint at equality, where ϕ_t is the maximum leverage allowed for the bank. The constraint limits the portfolio size to the point where the bank's incentive to cheat is exactly balanced by the cost of losing the franchise value. Hence, in times of crisis, where a deterioration of banks' net worth takes place, supply for assets will decline.

Now, in order to find the unknown coefficients I return to the guessed value function

$$V_{j,t} = l_{j,t}(\mu_t) + x_{j,t}(\nu_{x,j,t} - \nu_{d,j,t}) - m_{j,t}(\mu_t^m) + \nu_{d,t}n_{j,t}^B - \Phi_t. \quad (\text{D.9})$$

Substituting (23) into the guessed value function yields:

$$\begin{aligned}
V_t &= (n_{j,t}\phi_t + \frac{1}{1-p}(\omega m_{j,t}))\mu_t + x_{j,t}\kappa\Upsilon_t - m_{j,t}\mu_t^m + \nu_{d,j,t}n_{j,t} - \Phi_t \Leftrightarrow \quad (\text{D.10}) \\
V_t &= (n_{j,t}\phi_t + \frac{1}{1-p}(\omega m_{j,t}))\mu_t + \kappa\Upsilon_t^2 n_t - m_{j,t}\mu_t^m + \nu_{d,j,t}n_{j,t} - \frac{\kappa}{2}\Upsilon_t^2 n_t \Leftrightarrow \\
\Leftrightarrow V_t &= n_{j,t}(\phi_t\mu_t + \nu_{d,j,t} + \frac{\kappa}{2}\Upsilon_t^2) - m_{j,t}(\mu_t^m - \omega\mu_t \frac{1}{1-p})
\end{aligned}$$

and by (D.8) the guessed value function (D.10) becomes:

$$V_t = n_{j,t}^B(\phi_t\mu_t + \nu_{d,j,t} + \frac{\kappa}{2}\Upsilon_t^2).$$

Given the linearity of the value function we get that

$$A^B = \phi_t \mu_t + \nu_{d,j,t} + \frac{\kappa}{2} \Upsilon_t^2. \quad (\text{D.11})$$

The Bellman equation (17) now is:

$$\begin{aligned} V_{j,t-1}(s_{j,t-1}, x_{j,t-1}, d_{j,t}, m_{j,t-1}) &= \mathbb{E}_{t-1} \Lambda_{t-1,t} \sum_{i=1}^{\infty} \{(1 - \sigma_B) n_{j,t}^B \\ &+ \sigma_B (\phi_t \mu_t + \nu_{d,j,t} + \frac{\kappa}{2} \Upsilon_t^2) n_{j,t}^B\}. \end{aligned} \quad (\text{D.12})$$

By collecting terms with $n_{j,t}$ the common factor and defining the variable Ω_t as the marginal value of net worth:

$$\Omega_{t+1} = (1 - \sigma_B) + \sigma_B (\mu_{t+1} \phi_{t+1} + \nu_{d,t+1} + \frac{\kappa}{2} \Upsilon_t^2). \quad (\text{D.13})$$

The Bellman equation becomes:

$$\begin{aligned} V_{j,t}(s_{j,t}, x_{j,t}, d_{j,t}, m_{j,t}) &= E_t \Lambda_{t,t+1} \Omega_{t+1} n_{t+1}^B = \\ &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{k,t} l_{j,t-1} (1 - p) + R_{x,t} x_{j,t} - R_t d_{j,t} - R_{m,t} m_{j,t} - \Phi_t]. \end{aligned} \quad (\text{D.14})$$

The marginal value of net worth implies the following: Bankers who exit with probability $(1 - \sigma_B)$ have a marginal net worth value of 1. Bankers who survive and continue with probability σ_B , by gaining one more unit of net worth, they can increase their assets by ϕ_t and have a net profit of μ_t per assets. By this action they acquire also the marginal cost of deposits $\nu_{d,t}$ which is saved by the extra amount of net worth instead of an additional unit of deposits and also the additional cost of reserves $\frac{\kappa}{2} \Upsilon_t^2$. Using the method of undetermined coefficients and comparing (D.1) with (D.14) we have the final solutions for the coefficients:

$$\begin{aligned} \nu_{l,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{l,t+1} \\ \nu_{x,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{x,t+1} \\ \nu_{m,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{m,t+1} \\ \nu_{d,j,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \\ \mu_t &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{l,t+1} (1 - p) - R_{t+1}] \\ \mu_t^x &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{x,t+1} - R_{t+1}] \end{aligned} \quad (\text{D.15})$$

$$\mu_t^m = E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{m,t+1} - R_{t+1}] \quad (\text{D.16})$$

The first order condition (D.2) implies that when the incentive constraint is not binding ($\lambda_t = 0$), $\mu_t = 0$ the spread is zero, but in the case where constraint is binding ($\lambda_t > 0$) excess value of assets is positive $\mu_t > 0$. The same follows for μ_t^x and μ_t^m by equations (D.3) and (D.4) respectively. An important feature is that two effects take place to form the marginal value of the loans for the bank. The one is the case of the binding constraint and the other is the case of increased default probability. Taking equations (D.7) and the FOC (D.2) we have that

$$\nu_{l,j,t} = \frac{\lambda_t}{(1 + \lambda_t)}\theta + \nu_{d,j,t}\frac{1}{1 - p}.$$

The marginal value from extending a unit of loan is equal to the marginal cost from getting deposits which is increasing in default (as the banks' net worth is decreasing), plus the cost from the binding constraint.

From (D.9) we can get the following relationship between the expected loan rate, the riskless rate and the default probability.

$$E_t\Lambda_{t,t+1}\Omega_{t+1}R_{l,t+1} = \frac{\lambda_t}{(1 + \lambda_t)}\theta + E_t\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}\frac{1}{1 - p(\psi_t)} \quad (\text{D.17})$$

This shows the two effects on the expected loan rate. The first, is due to the binding funding constraints for the bankers. This can be referred as the liquidity component. The second one reflects the compensation that bankers demand when the firms' probability of default increases. This can be called as risk component.

Appendix E Model Equations

$$\begin{aligned}
K_{t+1} &= k_{t+1}^q [I_t + (1 - \delta)K_t] \\
Y_t &= A_t K_t^\alpha L_t^{1-\alpha} \\
u_{c,t} &= (C_t - \gamma C_{t-1})^{-1} - \beta \mathbb{E}_t \gamma (C_{t+1} - \gamma C_t)^{-1} \\
\Lambda_{t,t+1} &\equiv \beta \frac{u_{c,t+1}}{u_{c,t}} \\
\mathbb{E}_t \Lambda_{t,t+1} R_{t+1} &= 1 \\
u_{c,t} W_t &= \chi N_t^\epsilon \\
W_t &= (1 - \alpha) \left(\frac{K_t}{L_t} \right)^\alpha \\
Z_t &= \alpha \left(\frac{L_t}{K_t} \right)^{1-\alpha} \\
\Omega_{t+1} &= (1 - \sigma_B) + \sigma_B (\mu_{t+1} \phi_{t+1} + \nu_{d,t+1} + \frac{\kappa}{2} \Upsilon_t^2) \\
L_t + X_t &= N_t + D_t + M_t \\
L_t &= \phi N_t^B + \frac{1}{1-p} (\omega M_t) \\
\phi_t &= \frac{\nu_{d,j,t} + \frac{\kappa}{2} \Upsilon_t^2}{(1-p)\theta - \mu_t} \\
N_{y,t} &= \xi [R_{l,t} L_{t-1}] \\
N_{o,t} &= \sigma_B [R_{l,t} L_{t-1} + R_{x,t} X_{t-1} - R_m M_{t-1} R_t D_{t-1}] \\
N_t^E &= N_{y,t} + N_{o,t} \\
\mu_t &= E_t \Lambda_{t,t+1} \Omega_{t+1} [R_{l,t+1} (1-p) - R_{t+1}] \\
\nu_{d,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \\
\nu_{l,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{l,t+1} \\
\chi_{m,t} &= \chi_m + \kappa_m E_t ((R_{l,t+1}) - R_{t+1} - (R_l - R)) \\
\chi_{m,t} &= \frac{M_t}{L_t + \Delta X_t} \\
R_{x,t} &= \tau R_t \\
R_{m,t} &= \omega R_{l,t} + (1 - \omega \frac{1}{1-p}) R_t \\
\nu_{x,t} &= E_t \Lambda_{t,t+1} \Omega_{t+1} R_{x,t+1}
\end{aligned}$$

$$\begin{aligned}
X_t &= \Upsilon_t N_t \\
\nu_{x,j,t} - \nu_{d,j,t} &= \kappa \Upsilon_t - \epsilon \Phi(x_t) = \left(\frac{\kappa}{2} \Upsilon_t^2 n_t + \epsilon \Upsilon_t \right) \zeta_t \\
Q_t K_{i,t} &= L_{i,t} + N_{i,t}^E \\
N_{i,t}^E &= R_{k,t} Q_t K_{i,t} - R_{l,t} L_{i,t} \\
R_{k,t+1} &= k_{t+1}^q \frac{[Z_{t+1} + (1 - \delta) Q_{t+1}]}{Q_t} \\
R_{l,t} l_{i,t} &= \bar{\psi}_t R_{k,t} Q_t k_{i,t} \\
N_t^E &= (\sigma_{E,t} + \xi^e) (1 - \Gamma(\bar{\psi}_{t+1})) R_{k,t+1} Q_t K_{t+1} \\
N_t^E \phi_t^E &= Q_t K_t \\
R_{k,t} Q_t K_{i,t} [\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})] &\geq R_{t+1} (Q_t K_{i,t} - N_{i,t}^E) \\
\mathbb{E}_t R_{k,t+1} &= \mathbb{E}_t \rho(\bar{\psi}_{t+1}) R_{t+1} \\
Q_t &= 1 + \left(\chi \frac{I_\tau}{I_{\tau-1}} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right) + \frac{\chi}{2} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right)^2 - \chi \Lambda_{t,\tau} \frac{I_{\tau+1}^2}{I_\tau^2} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right) \right) \\
G + M_t - D_{g,t} - X_t &= T_t + R_{m,t} M_{t-1} - R_t D_{g,t-1} - R_{x,t} X_{t-1} \\
Y_t &= C_t + C_t^E + I_t \left[1 + f \left(\frac{I_t}{I_{t-1}} \right) \right] + G + \Phi_t + \mu \psi_t R_{k,t} Q_t K_t \\
G_t &= \gamma^G Y_t g_t
\end{aligned}$$

$$\rho(\bar{\psi}_{t+1}) = \frac{\Gamma'(\bar{\psi}_{t+1})}{[(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1})) \Gamma'(\bar{\psi}_{t+1}) + (1 - \Gamma(\bar{\psi}_{t+1})) (\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1}))]}$$

where

$$p(\bar{\psi}_t) = \int_0^{\bar{\psi}_t} f(\psi, -0.5(\sigma_\psi)^2, \sigma_\psi^2) d\psi$$

$$\Gamma(\bar{\psi}_t) = G(\bar{\psi}_t) + \bar{\psi}_t (1 - p)$$

$$G(\bar{\psi}_t) = \int_0^{\bar{\psi}_t} \psi f(\psi, -0.5(\sigma_\psi)^2, \sigma_\psi^2) d\psi$$

$$\Gamma'(\bar{\psi}_t) = (1 - p \bar{\psi}_t)$$

$$G'(\bar{\psi}_t) = \frac{1}{\sigma_\psi \sqrt{\pi}} \exp \left[- \frac{(\log(\bar{\psi}) + 0.5 \sigma_\psi^2)^2}{2 \sigma_\psi^2} \right]$$