A NEW PERSPECTIVE ON THE GOLD STANDARD: INFLATION AS A POPULATION PHENOMENON

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A new perspective on the Gold Standard: 
Inflation as a population phenomenon

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ABSTRACT
The purpose of this paper is to contribute a new model of the Gold Standard, focusing on the interaction between resource scarcity and demographics. In a dynamic micro-founded model we find that: i) prices and equilibrium gold holdings increase with population (a scale effect), but decrease with the population growth rate; ii) that the Gold Standard implies deflation unless extraction resources outstrip population growth; iii) there is no optimal quantity of money. The predictions of the model are examined using a structural VAR. Our results also shed light on debates about the viability of a return to the Gold Standard, and, more generally, on the interaction between policy variables and scarce resources.

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\textit{Keywords:} Gold Standard, Cagan-Keynes, Labor, Extraction, Scarcity, Inflation, Deflation, Population Dynamics.

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1. Introduction

Interest in the Gold Standard (GS) has been a recurrent feature of monetary debates\(^1\), generating a rich and growing literature (for recent contributions see the works of Bordo, Eichengreen, and Dowd).\(^2\) Indeed, one of the consequences of the financial crisis from 2007/08 onwards – and the allied expansion of central banks’ balance sheets – has been the revival of serious interest in commodity-backed standards.

However, one element of the GS – albeit largely neglected in the literature – is that, as a commodity monetary system, it critically depends on the production and extraction of finite gold reserves, and thus on the use of labor. In a simple economy, in which labor is the main (or sole) factor of production, we demonstrate that the study of labor employment (and population dynamics) under such a regime can be highly insightful; it effectively allows one to construct a *demographic* theory of price-level determination.\(^3\)

In a representative agent model of two productive sectors with inelastic labor supply, in which gold satisfies transaction demands, we demonstrate that prices and gold quantities evolve over time as a function of population and labor-force dynamics. This leads to two main conclusions. First, that for finite gold reserves, population changes impart both a scale (i.e., level) and a growth effect on prices which work in opposite ways. Second, by analyzing the dynamic between labor resources and rates of gold extraction, we reveal the conditions under which a GS would ultimately imply deflation. Among other interesting properties of the model, we find that there is no optimum quantity of money, i.e., no Friedman rule.

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\(^1\) Advocates of a return to a commodity standard like the GS are often associated with the “Austrian School” as well as some prominent policy groups, e.g., Cato (2009).


\(^3\) The notion of population dynamics (or biological determinants) of the level of prices and interest, goes back to Samuelson (1958).
As a monetary regime, the GS was thought to provide a robust anchor for long-run price stability; the main reason being that the quantity of money and the price level are determined by competitive market forces, Fisher (1922). Hence, in contrast to fiat currency, there is a separation of price-level determination from government policy, Barro (1979).

A common perception in the literature is that while the GS assures price stability in the long-run, prices could be highly unstable in the short-run. Moreover, economies were vulnerable to real and monetary shocks, and, because governments had little room for discretionary monetary policy, they were less able to stabilize those shocks (e.g., Niehans, 1978). This forced much of the necessary adjustment on to the real side of the economy (e.g., Bordo and Schwartz (1999), Taylor, 1998).

Against this background, our findings, though distinct, echo and synthesize earlier strands of the literature. For instance Keynes (1930 [2009, p. 100]) emphasized that if gold stocks are necessarily scarce given its depletable resource aspect, then there is a risk of deflation. That said, labor not only extracts existing gold deposits, it also prospects for further seams in combination with extraction technologies. In that vein, Chappell and Dowd (1997) emphasized the interaction between the durability and exhaustibility of gold and technical progress and prices. Dowd and Sampson (1993), moreover, examined irreversibility in mining and possible ratchet effects on prices.

Further, the resource costs of a pure Gold Standard have also attracted attention. Friedman (1953) estimated its resource costs to about one half of the annual growth rate of output. Meltzer (1983) revised Friedman’s figure downwards; however the cost of GS still remains high, about 16% of the annual growth rate. All of these elements have a bearing on the functioning of our model.

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4 Fagan et al. (2012) compare the classical GS period with the US “Great Moderation” (1984-2007) and find that the standard deviation of output, inflation and nominal money growth under the former to be respectively around four, ten and three times as volatile.

5 Bordo et al. (2009) examine the deflationary regimes over the GS period: “...The period 1880-1914 was characterized by two decades of secular deflation followed by two decades of secular inflation. (p1) … the deflation in the late nineteenth century gold standard era in three key countries [US, UK, and Germany] reflected both positive aggregate supply and negative money supply shocks. Yet the negative money shock had only a minor effect on output. [...] this suggests that deflation in the late nineteenth century was primarily good” (p15).
The paper is organized as follows. The next section presents some empirical background relevant to our study. Thereafter, we present the theoretical model and in Section 4 perform some comparative statics. Section 5 provides some associated Structural Vector Auto Regression (SVAR) evidence. Section 6 concludes.

2. The Historical Gold Standard: Some Background

In his seminal study Cagan (1965) showed that movements in the stock of base money during the GS were largely due to a few key elements: changes in the gold stock (reflecting new discoveries and improved extraction techniques) and capital flows. Changes in broader measures of money (e.g. M2) reflected, in addition to change in base money, movements in the currency and reserve ratios.

These aspects are reflected in Fig. 1 using US data over the “classical” Gold Standard, 1880-1914. First, we observe the strong co-movement between growth in gold supply and the monetary base, Panel A. In turn, Panel B shows the responsiveness of the price level to gold discoveries around the world. Prior to 1850 gold was a relatively precious metal (with Russia the largest producer) but from 1851-1900, propelled by various discoveries around the world, there was a near ten-fold increase. Following these discoveries, the level-shift effect on the price level (albeit with a lag) was dramatic. However, interestingly we also see, Panel C, that the role of base money (i.e., gold) as a means of exchange was continuously declining over the period.

– Insert Fig. 1 Here –

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6 Our data are annual and span 1870 to 1914 and are for the most part taken or derived from Balke and Gordon (1989). Population data are from the US Department of Commerce (Historical Statistics of the United States) and Monetary Gold Stock from the NBER Macro History Database (series 14076).

7 Namely, the California Gold Rush (US, 1848-52), the Victorian Gold Rush (Australia, 1851) and the Klondike Gold Rush (Canada, 1897-99). More specifically, world gold production from 1800-1850 totaled around 1,200 metric tonnes but from 1851-1990 it increased to 10,400 metric tons, Green (1999).
Moreover, Fig. 2 (Panel A) shows that whilst the inflation rate was around zero on average, sustained spells of inflation and deflation were common. This led to some very high and volatile real interest rates, Panel B. Finally, Panel C shows that population growth was also volatile and with an apparent downward trend. The importance of these dynamic features for our model will become clear below.

– Insert Fig. 2 Here –

3. The Model

We now derive a representative consumer problem along the lines of Sidrauski (1967a) that captures some of these important features of the GS regime. The steady-state solution of the problem allows us to unveil a relationship between the growth rate of prices, total population size and population growth, which is our main result.

Following Barro (1979) we assume a closed economy that can represent either a single country or the world economy under fixed exchange rates. Under a 100% reserve Gold Standard, the quantity of money is the stock of gold that has already been mined, extracted and minted. We further assume that there is a fixed known upper limit for extractable gold reserves and, for simplicity, that there is no non-monetary (consumption) demand for gold, and that gold holdings have negligible depreciation rates.

Many GS models rely on switches between the monetary and non-monetary demands for gold to allow some form of monetary authority intervention, Barro (1979), Barsky and Summers (1988). In the present set up, there need be no explicit monetary authority. In addition, we assume that labor is the sole factor of production but that there are two production sectors: besides production of gold, there is production of a consumption good, $C$. Money, or rather gold, enters in the economy to
satisfy transaction motives. On the production side, un-mined gold reserves combine with labor resources in the production function of the gold sector.

3.1 Gold Production and Discovery

Assume that total known gold reserves underground at time $t=0$ is $R_0$, that $R_t$ is the amount of gold reserves at time $t$, and that $\dot{M}$ is the extraction of gold, where $\dot{M} = \frac{dM}{dt}$. All gold that is extracted subsequently enters in the economy as money. Naturally, the extraction of gold reduces the amount of known gold reserves. Thus, we have the following law of motion for gold reserves,

$$R_{t+1} - R_t = -\dot{M}_t.$$  

Note we can re-express equation (1) as, $R_{t+1} = R_t - \dot{M}_T = R_0 - \sum_{i=0}^{T} \dot{M}_i$. Assume that it is optimal to extract all known gold reserves at time $t=T+1$, so that $R_{T+1} = 0$. It then follows that

$$R_T = R_{T+1} - \dot{M}_T = R_0 - \sum_{i=0}^{T} \dot{M}_i = 0 \Rightarrow R_0 = \sum_{i=0}^{T} \dot{M}_i.$$  

Notice that if all known gold reserves are exhausted, the amount of gold circulating in the economy as money is at its maximum, $\bar{M}$, i.e.,

$$R_0 = \sum_{i=0}^{T} \dot{M}_i = \bar{M}.$$  

In order to extract gold at time $t$, $\dot{M}_t$, it is necessary to apply and combine labor resources with known gold reserves:

$$\dot{M}_t = f(N_{g}, R_t) = z_{g}N_{g}^{\alpha}R_t = z_{g}N_{g}^{\alpha}\left(R_0 - \sum_{i=0}^{t-1} \dot{M}_i \right) = z_{g}N_{g}^{\alpha}(R_0 - M_t)$$  

(1a)
Where $N_g$ is the working population employed in mining gold, $0 < \alpha \leq 1$ is a production elasticity, and $z_g$ is exogenous technical progress in the gold-mining sector.\(^8\) Thus, the “production function” for gold naturally combines two inputs: labor resources and un-mined gold reserves (along with exogenous technical improvements in resource extraction) to extract and mint gold. The last equality in (1a) comes from the fact that the stock of money at $t$ in the economy, $M_t$, is the sum of all gold that was extracted before.

### 3.2 The Maximization Problem

We now analyze a decentralized economy. There is only one factor market since we assume that labor is the only factor of production employed to produce both the consumption good and gold; given labor mobility, a common salary is paid in both sectors. Total working population, $N$, is therefore employed either in the gold-mining sector or in production of the consumption good: $N \equiv N_g + N_c$.

At any time disposable income (labor earnings plus money holdings) equals consumption plus saving (held in the form of gold). The nominal budget constraint is given by:

\[
pC + p_g M = pwN + p_g z_g N_g \alpha (R_0 - M)
\]

Where $N, C, M$ are (household) population size, consumption and holdings of gold, respectively; $w$ is the real wage, $p$ is the price of the consumption good and $p_g$ is the price of gold. There are many identical families and to find the individual budget constraint, we divide both sides by $pN$, and

\(^8\) Since labor is the only production factor, technical progress is Harrod neutral and compatible with balanced growth (see the discussions in León-Ledesma et al. (2010)). Sustained technical improvements in the mining sectors have historically been very important, e.g., Hustrulid and Bullock (2001).
denoting per-capita variables by lowercase letters, using \( \frac{M}{pN} = m + m(\pi + n) \), where \( n = \frac{\dot{N}}{N} \) is the population growth rate, \( \frac{\dot{p}}{p} = \pi \) is the growth rate of the consumption price deflator, and \( m = \frac{M}{pN} \) denotes real per-capita money balances, we derive the individual budget constraint:

\[
\dot{m} = \frac{w-c}{p_g} + \frac{z_g N_g}{pN} - R_0 - m(z_g N_g + \pi + n)
\]

Equation (3) describes the rate of change of total per-capita wealth as the gap between income and total consumption, where consumption is the sum of two terms, \( c \) and \( m(\pi + n + z_g N_g) \).

The representative household’s problem is:

\[
\text{Max} \int_0^T U(c,m)e^{-\theta t} dt
\]

subject to her budget constraint, equation (3). Where \( \theta \) is the rate of time preference, and \( U(\cdot) \) is the instantaneous utility function with the following properties,

\[
U_c, U_m > 0, U_{cc}, U_{mm} < 0, U_{cm} \geq 0
\]

The above representative agent problem has a clear Sidrauskian flavor (Sidrauski (1967a)) because of the money-in-the-utility-function (MIUF) approach used. Feenstra (1986) shows the equivalence between MIUF and money as a medium of exchange that minimizes transactions costs. Since we argued that gold is used as money to satisfy transaction needs, MIUF is fully consistent with our formulation. Note, though, that in contrast to Sidrauski’s model where money is a control variable, here money is a \textit{state} variable.

The Hamiltonian associated with this problem is:

\[\text{9} \] Samuelson (1947) was a pioneer of this approach – see also Samuelson (1968).
\[ U(c, m) + \lambda \left[ \frac{w - c}{p_g} + \frac{z_g N_g^a}{pN} R_0 - m \left( z_g N_g^a + \pi + n \right) \right] \]  

(6)

where \( \lambda \) is the shadow price of per-capita gold holdings. The first-order conditions are,

\[ \dot{\lambda} = p_g U_c(c, m) \]  

(7)

\[ \dot{\lambda} - \theta \dot{\lambda} = -U_m(c, m) + \lambda \left[ z_g N_g^a + \pi + n \right] \]  

(8)

\[ \lim_{t \to \infty} m \dot{\lambda} e^{-\alpha t} = 0 \]  

(9)

Differentiating (7) with respect to time, and assuming linear separability between money and consumption in the utility function, \( U_{cm} = 0 \), yields:

\[ \dot{\lambda} = p_g U_c(c, m) + p_g U_{cc}(c, m) \cdot c \]  

(10)

Combining (8) and (10) yields a differential equation for per-capita consumption:10

\[ \ddot{c} = \frac{U_c(c, m)}{U_{cc}(c, m)} \left[ \theta + \pi - \frac{p_g}{p_g} + n + z_g N_g^a \right] - \frac{1}{p_g} \frac{U_m(c, m)}{U_c(c, m)} \]  

(11)

3.2.1 Equilibrium

We now determine the steady-state equilibrium values of the endogenous variables. There are three endogenous variables in this model: the control variable \( c \), the state variable \( m \) and the co-state variable (i.e., the shadow price of per-capita gold holdings) \( \lambda \). These variables are determined by equations (3) and (11) with \( m = c = 0 \), and equation (7).

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10 This equation is thus the Keynes-Ramsey rule in the context of our GS model.
Considering a steady state in which all prices grow at a common rate, \( \pi = \pi' = \pi / \pi_g \), we have from equation (3) with \( m = 0 \), two conditions to fulfill:

\[
c = w \quad \text{(12)}
\]

and

\[
\frac{z_g N_g^\alpha}{pN} R_0 = m(z_g N_g^\alpha + \pi + n) \quad \text{(13)}
\]

To determine the steady-state consumption, \( c^* \), assume that production of the consumption good sector is given by the following constant-returns production function, \( C = f(z_c N_c) = z_c N_c \), where \( z_c \) is technical progress in the consumption-good sector.\(^\text{11}\) Profit maximization in this sector implies \( w = z_c \). Given labor mobility, the same salary, \( w \), is paid in both sectors in equilibrium and therefore the marginal productivity of labor is common across sectors:\(^\text{12}\)

\[
z_c = w = \alpha z_g N_g^{\alpha-1} (R_0 - M) = [z_g (R_0 - M) \text{ for } \alpha = 1] \quad \text{(14)}
\]

Steady-state equilibrium consumption is found through equation (12):

\[
c^* = w \Rightarrow c^* = z_c \quad \text{(15)}
\]

Equation (15) uses equations (14) and (2).

\(^\text{11}\) In the gold sector, if the wage equals the marginal productivity of labor we have,

\[
w = \frac{\partial M}{\partial N_g} = \alpha z_g N_g^{\alpha-1} (R_0 - M).
\]

For \( wN_g \) to equal \( \dot{M} \) requires \( \alpha = 1 \). This is the condition for the Euler equation to hold in the gold sector, i.e.,

\[
wN_g = \dot{M}
\]

and so all the production is exhausted paying wages and there is no profit.

\(^\text{12}\) Notice there need be no presumption that technical progress is common across sectors.
Note that equation (3) with \( m = 0 \) generates two equations, namely equations (12) and (13), rather than just one. Therefore we have an additional equation, i.e., (13), which allows us to relate the growth rate of prices (i.e., inflation) to population growth rate, \( n \), and total population size, \( N \).

Before doing this we use equation (11) to derive the optimal gold holdings in the steady state, \( m^* \), provided that optimal consumption is determined by equation (15), as:

\[
\frac{\dot{c}}{c} = 0 \Rightarrow \frac{U_m(c^*, m^*)}{U_c(c^*, m^*)} = p_g [\theta + n + z_g N^u_g]
\]  

(11a)

The marginal rate of substitution between consumption and gold holdings is thus proportional to time preference, population growth plus the effective labor used to produce gold.

To derive a closed form expression for optimal gold holdings requires an explicit utility function. To illustrate, assume a simple linear utility function: \( U(c, m) = c^a + m^b \), \( 0 < a, b < 1 \). Accordingly, solving from (11a), yields:

\[
m^* = \left( \frac{b c^{a-1}}{a p_g (\theta + n + z_g N^u_g)} \right)^{1/(1-b)}
\]

(16)

Given the equilibrium values of \( c \) and \( m \), from (7) the equilibrium shadow price of per-capita gold holdings is,

\[
\lambda^* = p_g U_c(c^*, m^*)
\]

(17)

The relevant dynamic information of our model is given by the system represented by equations (3) and (11). Linearizing this system around its steady state, and evaluating all derivatives at the steady state yields,

\[
\begin{bmatrix}
\dot{m} \\
\dot{c}
\end{bmatrix} = \begin{bmatrix}
-z_g N^u_g - n - 1/p_g \\
-U_{mn}/p_g U_{cc} & U_m/p_g U_c
\end{bmatrix} \begin{bmatrix}
m - m^* \\
c - c^*
\end{bmatrix}
\]

(18)
Since the Jacobian determinant is negative, i.e., \(- (z_g N_g^\alpha + \pi + n)(U_m/p_g U_e) - U_{mn}/p_g U_{ee} < 0\), the system is saddle-path stable.

3.3 Inflation and Population: Scale and Growth Effects

Note that in equation (13) we can exploit the fact that \( R_0 = \sum_{i=0}^{T} \tilde{M}_i = \tilde{M} \), so we have

\[
\frac{R_0}{pN} = \frac{\bar{M}}{pN} = \bar{m},
\]

where \( \bar{m} \) is thus the maximum per-capita gold-holdings. Substituting \( \bar{m} \) and equation (16) into (13) and solving for the inflation rate yields:

\[
\pi = z_g N_g^\alpha \left( \frac{\bar{m} - m^*}{m^*} \right) - n = z_g N_g^\alpha \left\{ \frac{\bar{m}}{b(c^*)^{-\alpha}} \left( \frac{1}{\alpha p_g (\theta + n + z_g N_g^\alpha)} \right) \right\} - n
\]

Equation (13a) gives a curious *biological* interpretation to the inflation process over both the short and long run. A short-run increase in population growth (i.e., a one-off change in the working population) imparts a positive effect on inflation:

\[
\frac{d\pi}{dN} = \alpha z_g N_g^\alpha \left( \frac{\bar{m} - m^*}{m^*} \right) \frac{dN_g}{dN} + z_g N_g^\alpha \left( - \frac{\bar{m}^*}{(m^*)^\alpha} \frac{dm^*}{dN} \right) > 0
\]

since \( \bar{m}^* > m^* \) and \( \frac{dm^*}{dN} < 0 \).

By contrast, the equilibrium growth rate of prices decrease one-to-one with a sustained (i.e., long-run) increase in population growth,

\[
\frac{d\pi}{dn} = -1
\]
Thus, we can define a positive (short run) scale effect and a negative (long run) growth effect of population on inflation under a GS. The intuition is that a one-off increase in the working population provides a direct impact on gold extraction through the production function. In the long-run, however, transactions are proportional to the population growth, and inflation falls. Viewed in this light, we see that under the Gold Standard inflation is a population phenomenon. This is the first of our key results.

Moreover, (13a) states that technical progress in the gold production sector increases inflation:

\[ \frac{d\pi}{dz_g} = N_g^\alpha \left( \frac{m - m^*}{m^*} \right) > 0 \]

Our prior might normally be that technological improvements reduce prices, but a technology shock in the mining sector expands the monetary base under a 100% reserve standard and full employment, and this naturally increases prices.

We have already noted that historically the GS regime was characterized by periods of deflation. Accordingly, note that derivative of (13a) with respect to time yields:

\[ \pi^* = \left[ z_g N_g^\alpha + \alpha z_g N_g^\alpha \cdot \frac{\bar{m} - m^*}{m^*} \right] + z_g N_g^\alpha \left( - \frac{\bar{m}}{m} \right) - n \]

Thus, the GS regime is deflationary if,

\[ \dot{\pi} < 0 \iff \left[ z_g N_g^\alpha + \alpha z_g N_g^\alpha \cdot \frac{\bar{m} - m^*}{m^*} \right] < z_g N_g^\alpha \left( - \frac{\bar{m}}{m} \right) + n \]

The right hand side of the inequality depends on the growth rate of per-capita gold holdings \( \frac{m^*}{m} \), and acceleration of population growth, \( n \), the left hand side depends on the time variation of the number
of workers in the gold sector, \( \dot{N}_g \), and time variation of technology, \( \dot{z}_g \). As long as the growth rate of per-capita gold holdings \( \frac{\dot{m}^*}{m^*} \), and acceleration of population growth, \( \dot{n} \), are higher than the time variation of the number of workers in the gold sector, \( \dot{N}_g \), and time variation of technology, \( \dot{z}_g \), deflation characterizes the GS.

4. **Comparative Statics**

The comparative statics analysis of \( c^* \) and \( m^* \), given by equations (14) and (16), provide several interesting results, besides the population properties of inflation. As noted before, according to equation (15), the equilibrium value of per-capita consumption, \( c^* \), is independent of the rate of growth of money [gold], i.e., \( dc^*/d \left( \frac{\dot{M}}{M} \right) = 0 \). Therefore, money [gold] is super-neutral in this model.

As a consequence of this super-neutrality, the “Tobin effect”\(^{13} \) (Tobin (1965)) is ruled out in equilibrium in this model, and the result is close to the typical super-neutrality result obtained by Sidrauski (1967a). Notice that the Tobin effect holds in an extended version of the Sidrauski (1967a) model with elastic labor supply, and Pareto substitutability between money and consumption \( U_{cm}(c,m,l) < 0 \), where \( l \) is leisure, and Pareto complementarity between leisure and money \( U_{lm}(c,m,l) > 0 \) (Brock (1974), Wang and Yip (1992)). In our formulation labor is supplied

\(^{13} \) The Tobin effect is characterized by the positive impact of the rate of money growth on the real economy (see also Johnson (1966) and Sidrauski (1967b)).
inelastically\textsuperscript{14}, and there is a possible complementarity between money and consumption in utility, recall (5). A possible extension of our model therefore includes an examination of an elastic labor supply and substitutability between money and consumption.

As the system of equations (15)-(17) is block recursive, and equilibrium consumption is determined prior to equilibrium money holdings, the comparative statics analysis shows that equilibrium per-capita quantity of gold held as money depends positively on the quantity of the consumption good, from (11a) it follows that,

\[
\frac{dm^*}{dc^*} = p_s \left[ \theta + n + z \sigma g \right] \frac{U^m}{U^*_m} > 0
\]

The steady state quantity of gold held as money decreases with the time preference,

\[
\frac{dm^*}{d\theta} = p_s \frac{U^m}{U^*_m} < 0
\]

This is entirely intuitive: the more impatient the individual, less real money balances she is willing to hold. This is also consistent with money as a store of value, as reflected in the speculative motive of money demand. It is important to stress that the only endogenous variable in this model affected by the rate of time preference in the steady state is gold holdings.

4.1 The “Friedman Rule” and the Shadow Price of Gold Holdings

The Friedman (1969) rule (FR) applies Pareto efficiency criteria to the provision of money: namely, that the opportunity cost of holding money faced by agents should equal the social cost of creating additional money. Under a fiat regime, the latter cost is essentially zero. Under a GS – as noted earlier – the extraction costs are non trivial.

\textsuperscript{14}Chappell and Dowd (1997) examine a model of the GS in which the representative agent has to decide how to allocate his time between producing a consumption good and gold.
The answer as to whether it is optimal to satiate individuals with money in this model is thus fundamentally no\textsuperscript{15}. It is easy to see this from Equation (11a): $U_m(c^*, m^*) > 0$ since $p_g[\theta + n + z_g N_g^\alpha U_c(c^*, m^*)] > 0$.

Concerning the equilibrium shadow price of per-capita gold holdings it decreases with consumption: $\frac{d \lambda^*}{dc^*} = p_g U_{cc}(c^*, m^*) < 0$. It is interesting to notice that $\lambda^*$ is independent of the equilibrium quantity of money if utility is separable in money and consumption.

5. **An SVAR Analysis**

Our model posits a relationship under a GS between inflation, population growth and the extraction of finite gold deposits. Although stylized, the insights from the model can be brought to bear on data from the era. Accordingly, we examine impulse responses from a SVAR (structural VAR) (e.g., Amisano (2012)) informed by the predictions of the model. The data – as in Section 2 – are annual, taken mostly from Balke and Gordon (1989) and span 1870 to 1914 and comprise real GDP ($GDP$), the GDP deflator ($P$), base money ($M$), population ($POP$) and labor productivity ($PROD=GDP/POP$).

To proceed, we estimate a structural VAR in $m$, $p$, $pop$ and $prod$ (lower case denotes logs).\textsuperscript{16} We tested and failed to reject unit roots for all series, failed to accept co-integration and thus enter these series in first differences. Consider the following $j$-lag VAR:

$$\Phi(L)Y_t = \varepsilon_t$$  \hspace{1cm} (19)

\textsuperscript{15} This question raises two related issues: i) whether there is an optimum quantity of money (OQM); ii) whether there is a need of a monetary authority to control money supply in a GS regime. The OQM arises in Friedman model because paper money is costless to produce, so clearly we have no reason to expect it to hold under a GS. Moreover, a centralized monetary authority controlling money supply is not necessary. As stressed by Garrison (1985) the GS in its purest form neither requires nor permits the State to exercise control over the money supply. In fact for most “Austrian” economists the absence of centralized, discretionary monetary control constitutes its primary benefit.

\textsuperscript{16} All series are I (1).
where \( Y_i = \begin{bmatrix} \Delta m_i \\ \Delta p_i \\ \Delta \text{pop}_i \\ \Delta \text{prod}_i \end{bmatrix} \), \( \varepsilon_i = \begin{bmatrix} \varepsilon_i^m \\ \varepsilon_i^{\text{pop}} \\ \varepsilon_i^{\text{prod}} \\ \varepsilon_i^t \end{bmatrix} \) and \( \Phi(L) \) is a \( j \)th order matrix polynomial in the lag operator. The VAR can be re-written in its moving average representation:

\[
Y_i = C(L)\varepsilon_i
\]  

(20)

where \( C(L) = \Phi(L)^{-1} \) is an infinite polynomial matrix in the lag operator. In terms of the structural interpretation of the innovations in (19), these can be considered a technology shock (i.e., a productivity innovation), a shock to population (labor supply), a cost push shock and a shock to money supply (e.g., a gold discovery or an exogenous improvement in extraction technologies).\(^{17}\)

The zero restrictions which we place on the long-run response matrix \( C \) (see Table 1) follow the logic of the model:

**Table 1**

SVAR Restrictions

<table>
<thead>
<tr>
<th>Variables</th>
<th>Innovations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>( p )</td>
</tr>
<tr>
<td>( M )</td>
<td>( - )</td>
</tr>
<tr>
<td>( p )</td>
<td>( - )</td>
</tr>
<tr>
<td>( \text{pop} )</td>
<td>0</td>
</tr>
<tr>
<td>( \text{prod} )</td>
<td>0</td>
</tr>
</tbody>
</table>

**Note:** “0” denotes a zero restriction and “.” an unrestricted case.

These zero restrictions can be read in the following manner. We assume productivity and population are independent of nominal innovations (hence the lower diagonal zero block), and that base money predetermines prices. The monetary shock (representing, to repeat, either gold discoveries or improved gold extraction technologies) affects the monetary base and prices. An

\(^{17}\) Thus, through the lens of the model, \( \varepsilon_i^{\text{prod}} \) and \( \varepsilon_i^m \), can be rationalized as mapping to \( z_c \) and \( z_g \), respectively.
economy-wide technology shock also affects long-run prices through an expansion of aggregate supply. Note, that with these restrictions the VAR is exactly identified (information criteria suggested a VAR lag-length of 1).

Fig. 3 shows the accumulated impulse responses. Highlights include the fact that an innovation to population initially increases money and prices but, if sustained, eventually implies long-run deflation under a GS, which – recalling equation (13a) – is consistent with our model. Figures 2 A and C provide a startling confirmation of this: the strong population growth in the first half of the sample coincided with a period of sustained deflation; when population growth stabilized thereafter, a period of positive inflation followed.

An innovation in the money supply (e.g., a gold discovery) feeds through immediately to money supply and has a permanent level effect on prices and inflation but, by construction, no effect on the real economy. Again this is consistent with the predictions of our model. An economy-wide positive technology shock expands the monetary base but – consistent with what we know about technology shocks generally – has an initially negative impact on prices.

6. Conclusion

This paper offered a new perspective on the Gold Standard. Under a Gold Standard, base money comprises existing gold stocks and evolves according to new but finite deposits extracted by labor and improvements in extraction technologies. Our interest has been to enhance the contribution of labor to the functioning and stability of the system allied to the inherent scarcity of gold reserves. We developed a micro-founded model with two productive sectors, in which gold satisfies transaction demands, and showed that prices and gold volumes evolve over time as a function of
population dynamics. We unveiled a relationship between inflation, population and population growth, in which inflation grows with population (a scale effect) and decreases with population growth (a growth effect). In short, under a Gold Standard, inflation is a population phenomenon.

Our model also suggests that such a monetary regime would ultimately imply deflation if population growth rates dominates rates of gold extraction. In effect, gold extraction rates could only keep pace if an increasing fraction of the labor force were transferred to the gold-mining sector or if technical improvements in the economy were increasingly biased towards extraction techniques. The SVAR analysis conducted bolsters our analysis in respect of both of these respects.

To avoid systematic deflation, another possibility (implicit from Figure 1C) is that the system would have to systematically depart from a 100% reserve standard. However, since the quantity of gold currently falls significantly short of global activity this would imply an ever widening money-to-gold ratio and raise of specter of convertibility crises.\textsuperscript{18}

Given renewed interest in the Gold Standard, our analysis also contributes to debates about the viability of a return to the Gold Standard. Moreover, our framework is tractable yet rich enough to be extended to investigate a number of related issues pertaining to the Gold Standard such as the effects of an elastic labor supply, imperfect labor mobility, nominal rigidities, the inclusion of capital in productive sectors, the consideration of paper money and a central authority responsible for its issue.

\textsuperscript{18} The last of these, for the US, being the “Nixon Shock”.
References


Fig. 1. Gold, Prices and the Monetary Multiplier

Note: All data refer to the US except global gold production in panel B. This is data interpolated from Greene (1999) alongside a 5-year moving average. Gold data are in metric tons. US CPI (1982-84 = 100).
Fig. 2. Selected Gold Standard era variables
Fig. 3. Gold Standard Impulse Responses (Accumulated Responses)