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A Fiscal Stimulus with Deep Habits and Optimal Monetary Policy

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Abstract

A New-Keynesian model with deep habits and optimal monetary policy delivers a fiscal multiplier above one and the crowding-in effect on private consumption obtainable in a Real Business Cycle model *à la* Ravn et al. (2006). Optimized Taylor-type or price-level interest rate rules yield results close to optimal policy and dominate a conventional Taylor interest rate rule. Private consumption is crowded out only if the Taylor rule is sub-optimal and then negates the fiscal stimulus by responding strongly to the output gap, or if the ability to commit is absent. At the zero lower bound private consumption is always crowded in across simple rules.

JEL classification: E30, E62.

Keywords: Deep habits, Optimal monetary policy, Price-level rule, Zero lower bound.

1 Introduction

The efficacy of a fiscal stimulus remains a controversial issue in applied macroeconomics. In particular the range of empirical government spending multipliers is wide – Ramey (2011) surveys the literature and argues that this is between 0.8 and 1.5 – and the sign of the effect on private consumption is controversial. In fact, part of the empirical literature finds evidence for a crowding-out of consumption, while many Structural Vector-Autoregressions (SVARs) provide evidence for a crowding-in effect. Canonical Dynamic Stochastic General Equilibrium (DSGE) models typically predict fiscal multipliers well below the empirical range and the crowding-out of private consumption.

A modelling device that has been used to obtain the consumption crowding-in and higher fiscal multipliers in Real Business Cycle (RBC) models is the assumption that external 'deep habits' *à la* Ravn et al. (2006) are formed in private and public consumption, i.e. habits on the average consumption level of each variety of goods. Jacob (2011) shows that in a New-Keynesian (NK) model with deep habits, increasing degrees of price stickiness soften the expansionary effects of a fiscal stimulus and may overturn the results obtainable in a RBC model.

This paper also investigates these issues in a NK model with deep habits but pays particular attention to the subtle interactions between fiscal and monetary policy that determine the outcome of a fiscal stimulus. In particular, we study a boost to government spending alongside a number of possible interest rate policies: first, the welfare-optimal (Ramsey) policy; second, a time-consistent policy; third, a conventional Taylor interest rate rule which prescribes an immediate and strong response to the output gap; fourth, an empirically based rule with a much weaker response to output; and finally an optimized simple Taylor type rule (of which a price-level rule is a special case) that turns out to closely mimic the optimal policy. We also examine the outcome of these simple rules with a zero lower bound constraint for an initial period.

2 Model

The model is a standard NK model with Rotemberg price stickiness and convex investment adjustment costs augmented with deep habit formation.[1]

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[1] To retain a sharp focus on the issue of deep habit we abstract from unemployment. A number of recent papers examine fiscal multipliers having introduced Mortensen-Pissarides search-matching frictions into otherwise standard NK models (but without deep habit) – see Campolmi et al. (2011); Faia et al. (2010); Monacelli et al. (2010).

2.1 Households

A continuum of identical households $j \in [0, 1]$ has preferences over differentiated consumption varieties $i \in [0, 1]$. Following Ravn et al. (2006), households exhibit external deep habit formation in consumption, i.e. on the average consumption level of each variety of good. Their optimisation problem is

$$\max_{\left\{ (X_t^c)^j, K_{t+1}^j, B_{t+1}^j, I_t^j, h_t^j \right\}} E_t \sum_{s=0}^{\infty} \beta^{t+s} U((X_{t+s})^j, 1 - H_{t+s}^j),$$

subject to constraints

$$(X_t^c)^j + \Omega_t + I_t^j + \frac{B_{t+1}^j}{P_t} = \frac{W_t}{P_t} H_t^j + R_t^K K_t^j + \frac{R_{t-1} B_t^j}{P_t} + \int_0^1 J_{it} di - T_t, \tag{1}$$

$$K_{t+1}^j = (1 - \delta) K_t^j + I_t^j \left[1 - S \left(\frac{I_t^j}{I_{t-1}^j} \right) \right], \tag{2}$$

where $\beta \in (0, 1)$ is the discount factor, $(X_t)^j = X\left((X_t^c)^j, X_t^g \right)$ is a composite of habit-adjusted differentiated private and public consumption goods similar to that in Pappa (2009), and H_t^j are hours of work. The private component of $(X_t)^j$ is

$$(X_t^c)^j = \left[\int_0^1 (C_{it}^j - \theta^c S_{it-1}^c)^{1 - \frac{1}{\eta}} di \right]^{\frac{1}{1 - \frac{1}{\eta}}}, \tag{3}$$

where $\theta^c \in (0, 1)$ is the degree of deep habit formation on each variety, and S_{it-1}^c denotes the stock of habit in the consumption of good i, which evolves over time according to

$$S_{it}^c = \varrho^c S_{it-1}^c + (1 - \varrho^c) C_{it}, \tag{4}$$

where $\varrho^c \in (0, 1)$ implies persistence. The optimal level of demand for each variety, C_{it}^j, for a given composite is obtained by minimizing total expenditure $\int_0^1 P_{it} C_{it}^j di$ over C_{it}^j, subject to (3). This leads to

$$C_{it}^j = \left(\frac{P_{it}}{P_t} \right)^{-\eta} (X_t^c)^j + \theta^c S_{it-1}^c, \tag{5}$$

where P_{it} is the price of variety i, $P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}$ is the nominal price index and η is the intratemporal elasticity of substitution. Multiplying (5) by P_{it} and integrating, real consumption expenditure C_t^j can be written as a function of the consumption composite and the stock of habit: $C_t^j = (X_t^c)^j + \Omega_t$, where

$\Omega_t \equiv \theta^c \int_0^1 \frac{P_{it}}{P_t} S_{it-1}^c di$. Households hold K_t^j capital holdings, evolving according to (2) where δ is the capital depreciation rate, I_t^j is investment, and $S(\cdot)$ represents an investment adjustment cost satisfying $S(1) = S'(1) = 0$ and $S''(1) > 0$. Investment is also a composite of goods, i.e. $I_t^j = \left[\int_0^1 \left(I_{it}^j \right)^{1-\frac{1}{\eta}} di \right]^{\frac{1}{1-\frac{1}{\eta}}}$, but does not feature habit formation. Expenditure minimisation leads to the optimal level of demand of private investment goods for each variety i:

$$I_{it}^j = \left(\frac{P_{it}}{P_t} \right)^{-\eta} I_t^j. \tag{6}$$

Households buy consumption goods, C_t^j, invest in investment goods, I_t^j, and nominal bond holdings, B_t^j, receive the hourly wage, W_t, the rental rate of capital, R_t^K, the return on nominal bond holdings, R_t, and firms' profits, $\int_0^1 J_{it} di$; and pay lump-sum taxes T_t.

The first-order condition (FOC) with respect to (w.r.t.) the private consumption composite $(X_t^c)^j$ implies that the Lagrange multiplier on the household's budget constraint (1) is equal to $\Lambda_t^j = U_{X^c,t}^j$, where $U_{X^c,t}^j$ is the marginal utility of the private consumption composite. Let $\Lambda_t^j Q_t^j$ be the multiplier on the capital accumulation equation (2), and Q_t^j represent Tobin's Q. Then, the FOC w.r.t. capital, K_{t+1}^j, implies $Q_t^j = E_t \left\{ D_{t,t+1}^j \left[R_{t+1}^K + (1-\delta) Q_{t+1}^j \right] \right\}$, where $D_{t,t+1}^j \equiv \beta \frac{U_{X^c,t+1}^j}{U_{X^c,t}^j}$ is the stochastic discount factor; the FOC w.r.t. investment I_t^j yields

$$Q_t^j \left(1 - S \left(\frac{I_t^j}{I_{t-1}^j} \right) - S' \left(\frac{I_t^j}{I_{t-1}^j} \right) \frac{I_t^j}{I_{t-1}^j} \right) + E_t \left(D_{t,t+1}^j Q_{t+1}^j S' \left(\frac{I_{t+1}^j}{I_t^j} \right) \left(\frac{I_{t+1}^j}{I_t^j} \right)^2 \right) = 1;$$

the FOC w.r.t. the bond holdings delivers $1 = E_t \left[D_{t,t+1}^j \frac{R_t}{\Pi_{t+1}} \right]$, where $\Pi_t \equiv \frac{P_t}{P_{t+1}}$ is the gross inflation rate. Finally the FOC w.r.t hours implies: $-U_{H,t}^j = U_{X^c,t}^j \frac{W_t}{P_t}$.

2.2 Government

As in Ravn et al. (2006) deep habits are present also in government consumption.[2] In each period t, the government allocates spending $P_t G_t$ over differentiated goods sold by retailers in a monopolistic market to maximize the quantity of a habit-adjusted composite good:

$$X_t^g = \left[\int_0^1 (G_{it} - \theta^g S_{it-1}^g)^{1-\frac{1}{\eta}} di \right]^{\frac{1}{1-\frac{1}{\eta}}},$$

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[2]This can be justified by assuming that households derive habits also on consumption of government provided goods. One can also argue that public goods are local in nature and households care about the provision of individual public goods in their own constituency relative to others; or that procurement relationships are formed between government and firms.

subject to the budget constraint $\int_0^1 P_{it}G_{it}di \leq P_tG_t$, where θ^g is the degree of deep habit formation in government spending and S^g_{it-1} denotes the stock of habits for this expenditure, which evolves as:

$$S^g_{it} = \varrho^g S^g_{it-1} + (1 - \varrho^g)G_{it},\tag{7}$$

and exhibits persistence ρ^g. At the optimum

$$G_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} X^g_t + \theta^g S^g_{it-1}.\tag{8}$$

Aggregate real government consumption, G_t, is an autoregressive process

$$\log\left(\frac{G_t}{\bar{G}}\right) = \rho_G \log\left(\frac{G_{t-1}}{\bar{G}}\right) + \epsilon^G_t,\tag{9}$$

where ρ_G is an autoregressive parameter and ϵ^g_t is a mean zero, i.i.d. random shock with standard deviation σ^G. The government budget constraint is simply $G_t = T_t$.

2.3 Firms

A continuum of monopolistically competitive firms indexed by $i \in [0,1]$ rents capital, K_{it}, and hires labour, H_{it} to produce differentiated goods Y_{it} with convex technology $F(H_{it}, K_{it})$, which are sold at price P_{it}. Firms face quadratic price adjustment costs $\frac{\xi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 Y_t$, as in Rotemberg (1982) – where parameter ξ measures the degree of price stickiness – and maximize the following flow of discounted profits:

$$J_{it} = E_t\left\{\sum_{s=0}^{\infty} D_{t,t+s}\left[\begin{array}{c} \frac{P_{it+s}}{P_{t+s}}(C_{it+s} + G_{it+s} + I_{it+s}) \\ -\frac{W_{it+s}}{P_{t+s}}H_{it+s} - R^K_{t+s}K_{it+s} - \frac{\xi}{2}\left(\frac{P_{it+s}}{P_{it+s-1}} - 1\right)^2 Y_t \end{array}\right]\right\},$$

with respect to K^p_{it+s}, H_{it+s}, C_{it+s}, S^c_{it+s}, G_{it+s}, S^g_{it+s} and P_{it+s} subject to (4), (5), (6), (7), (8), and the firm's resource contraint

$$C_{it+s} + G_{it+s} + I_{it+s} = F(H_{it}, K_{it}) - FC = Y_{it},\tag{10}$$

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where FC are fixed production costs, set to ensure that the free entry condition of long-run zero profits is satisfied. The corresponding first-order conditions for this problem are:

$$R_t^K = MC_t F_{K,it},$$

$$\frac{W_t}{P_t} = MC_t F_{H,it},$$

$$\nu_t^c, = \frac{P_{it}}{P_t} - MC_t + (1 - \varrho^c)\lambda_t^c,$$

$$\lambda_t^c = E_t D_{t,t+1}(\theta^c \nu_{t+1}^c + \varrho^c \lambda_{t+1}^c),$$

$$\nu_t^g = \frac{P_{it}}{P_t} - MC_t + (1 - \varrho^g)\lambda_t^g,$$

$$\lambda_t^g = E_t D_{t,t+1}(\theta^g \nu_{t+1}^g + \varrho^g \lambda_{t+1}^g),$$

$$\frac{P_{it}}{P_t}(C_{it} + G_{it}) - \xi\left(\frac{P_{it}}{P_{it-1}} - 1\right)\frac{P_{it}}{P_{it-1}}Y_t + (1 - \eta)\left(\frac{P_{it}}{P_t}\right)^{1-\eta}I_t + \eta MC_t\left(\frac{P_{it}}{P_t}\right)^{-\eta}I_t$$
$$- \eta\nu_t^c\left(\frac{P_{it}}{P_t}\right)^{-\eta}X_t^c - \eta\nu_t^g\left(\frac{P_{it}}{P_t}\right)^{-\eta}X_t^g + \xi E_t D_{t,t+1}\left[\left(\frac{P_{it+1}}{P_{it}} - 1\right)\frac{P_{it+1}}{P_{it}}\right]Y_{t+1} = 0.$$

Variables MC_t, ν_t^c, λ_t^c, ν_t^g, λ_t^g are the Lagrange multipliers associated with constraints (10), (5), (4), (8) and (7) respectively. In particular, MC_t is the shadow value of output and represents the firm's real marginal cost.

2.4 Monetary policy

Monetary policy is set either **(i)** optimally as the solution to a Ramsey problem, in which the monetary authority maximizes households' welfare or **(ii)** to be welfare-optimal subject to a time-consistency constraint or **(iii)** according to a Taylor-type interest-rate rule:

$$\log\left(\frac{R_t}{\bar{R}}\right) = \rho_r \log\left(\frac{R_{t-1}}{\bar{R}}\right) + (1 - \rho_r)\left[\rho_\pi \log\left(\frac{\Pi_t}{\bar{\Pi}}\right) + \rho_y \log\left(\frac{Y_t}{\bar{Y}}\right)\right], \tag{11}$$

where ρ_r is the interest rate smoothing parameter and ρ_π and ρ_y are the monetary responses to inflation and the output gap;[3] or **(iv)** as a price-level rule:

$$\log\left(\frac{R_t}{\bar{R}}\right) = \log\left(\frac{R_{t-1}}{\bar{R}}\right) + \rho_\pi \log\left(\frac{\Pi_t}{\bar{\Pi}}\right) + \rho_y \log\left(\frac{Y_t}{\bar{Y}}\right); \tag{12}$$

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[3]Strictly speaking the output gap is $\frac{Y_t}{Y_t^*}$ where Y_t^* is the flexi-price output, not $\frac{Y_t}{\bar{Y}}$ where $\bar{Y}$ is the deterministic steady state. However none of our results are significantly affected by this feature.

Both sub-optimal and welfare-optimal forms of these simple rules are examined.

2.5 Equilibrium

In equilibrium all markets clear. The resource constraint completes the model:

$$Y_t = C_t + I_t + G_t + \frac{\xi}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 Y_t.$$

3 Functional forms

The utility function specializes as $U(X_t, 1 - H_t) = \frac{\left[X_t^{(1-\varrho)}(1-H_t)^\varrho\right]^{1-\sigma_c}-1}{1-\sigma_c}$, where $\sigma_c > 0$ is the coefficient of relative risk aversion, and ϱ is the elasticity of substitution between leisure and the consumption composite, which in turn is a Cobb-Douglas aggregate of private and public consumption with ν_x representing the share of private consumption in the aggregate. Investment adjustment costs are quadratic: $S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$, $\gamma > 0$, while the production function is Cobb-Douglas: $F(H_t, K_t) = (A_t H_t)^\alpha K_t^{1-\alpha}$, where A_t is a labour-augmenting technology shock and α represents the labour share of income.

4 Parameter choice

Most parameter values are taken directly from the calibration exercise of Ravn et al. (2006): $\beta = 0.9902$, $\alpha = 0.75$, $\eta = 5.3$, $\delta = 0.0253$, $\sigma_c = 2$, $\theta^c = \theta^g = 0.86$, $\rho^c = \rho^g = 0.85$, $\rho_G = 0.9$. Parameters ϱ and ν_x are set to target $h = 0.33$ and $G/Y = 0.20$, respectively, at the steady state and the investment adjustment cost parameter $\gamma = 5$ as estimated by Christiano et al. (2005). The Rotemberg parameter ξ is set equal to 25.304, which corresponds to Calvo contracts of an average duration of 3 quarters. For the conventional Taylor rule (Taylor, 1993) there is no persistence ($\rho_r = 0$) and $\rho_y = 0.5$. Estimated Taylor rules typically reveal considerable persistence and a less aggressive response to output: we choose an empirical rule from Smets and Wouters (2007) (SW) where $\rho_r = 0.81$, $\rho_\pi = 2.04$, $\rho_y = 0.08$. The 'quasi-empirical' rule is a compromise, i.e., the same ρ_r and ρ_π, but $\rho_y = 0.5$ as in the conventional Taylor rule.

5 Results

We report the impulse responses to a government expenditure shock of size 1 percent of steady-state output to be able to read the output response as a fiscal multiplier. First, Table 1 reports the rules set

out in Section 2.4 and welfare outcomes compared with the optimal policy.[4] There are four sources of forward-looking behaviour in our model: the Euler consumption equation, investment, and habit in both consumption and government services. This feature introduces a considerable degree of time inconsistency into the optimal Ramsey policy as can be seen by the substantial welfare loss in percentage terms if the monetary authority cannot commit to some form of interest rate rule. Our optimized simple rules come very close (within 1%) of mimicking the welfare outcome of optimal policy. As discussed before, a conventional Taylor rule involves an instantaneous and over-aggressive response to output compared with optimized rules resulting in a significant welfare loss. The estimated empirical rule by contrast is much closer to being welfare-optimal whilst the quasi-empirical rule is somewhere in between.

Rule	$[\rho_r, \ \rho_\theta, \ \rho_y]$	Welfare Loss (%)
Optimal (Ramsey)	not applicable	0
Time Consistent (TCT)	not applicable	152
Conventional Taylor	[0, 1.5, 0.50]	90.2
Empirical Taylor (SW)	[0.81, 2.04, 0.08]	8.54
Quasi-Empirical Taylor	[0.81, 2.04, 0.50]	24.7
Optimized Simple	[1.00, 0.00587, 0.0137]	0.96
Optimized Price Level	[1.00, 0.00635, 0.00]	0.97

Table 1. Optimal and ad hoc Monetary Rules Compared

Notes: The welfare loss is reported as a % increase of that under optimal policy. For integral simple rules with $\rho_r = 1$, the rule is expressed as $\log\left(\frac{R_t}{\bar{R}}\right) = \rho_r \log\left(\frac{R_{t-1}}{\bar{R}}\right) + \rho_\pi \log\left(\frac{\Pi_t}{\bar{\Pi}}\right) + \rho_y \log\left(\frac{Y_t}{\bar{Y}}\right).$

Figure 1 shows the impulse response functions (equivalent to fiscal multipliers) to a fiscal shock when monetary policy is either ex ante optimal, time-consistent or conducted using either the optimized or the conventional Taylor simple commitment rule reported in Table 1. We see that the model delivers a fiscal multiplier above one for a prolonged period and the crowding-in effect on private consumption *if the monetary authority can commit to some ex ante optimal rule*. If it cannot commit, then the model provides some support for fiscal stimulus pessimism with a crowding-out effect on private consumption. The same applies to a fiscal stimulus alongside the conventional Taylor rule.

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[4]See Levine et al. (2008) for details of these three monetary policy regimes. Note that these optimized simple rules are shock-dependent and here only apply to a fiscal shock with the assumed persistence. In a stochastic environment facing many shocks they need to be redesigned and will be dependent on the relative persistence and variances of all shocks. It then becomes important to estimate the model, including the properties of the shocks, before proceeding to the design of such rules.

Figure 2 compares the optimized simple Taylor-type rule (which gives outcomes almost identical to both the optimized price-level rule and the Ramsey policy) with the two ad hoc rules with and without the imposition of a Zero Lower Bound (ZLB) constraint. The ZLB is imposed for an arbitrary number quarters (four) following the approach of Cogan et al. (2010). In line with the results of Christiano et al. (2009) and Woodford (2011) for a standard NK model, also in a NK model where habits are 'deep' a fiscal stimulus is more expansionary at the ZLB. The highest output multiplication effect obtained under an optimized simple rule is substantial.

6   Conclusion

This paper shows that (i) for an empirically relevant degree of price stickiness, when a RBC *à la* Ravn et al. (2006) is turned into a NK model and monetary policy is set optimally, the model delivers a fiscal multiplier above one and the crowding-in effect on private consumption obtainable in a RBC; (ii) an optimized simple Taylor-type interest-rate rule yield results close to optimal policy and dominates a conventional Taylor rule; (iii) private consumption is crowded out and the fiscal multiplier experiences a sizeable contraction if the Taylor rule negates the fiscal stimulus with an immediate and high response to the output gap that, we show, is implausible from both a normative and positive perspective, or if the government cannot commit; (iv) at the zero lower bound private consumption is *always* crowded in across all our simple rules.

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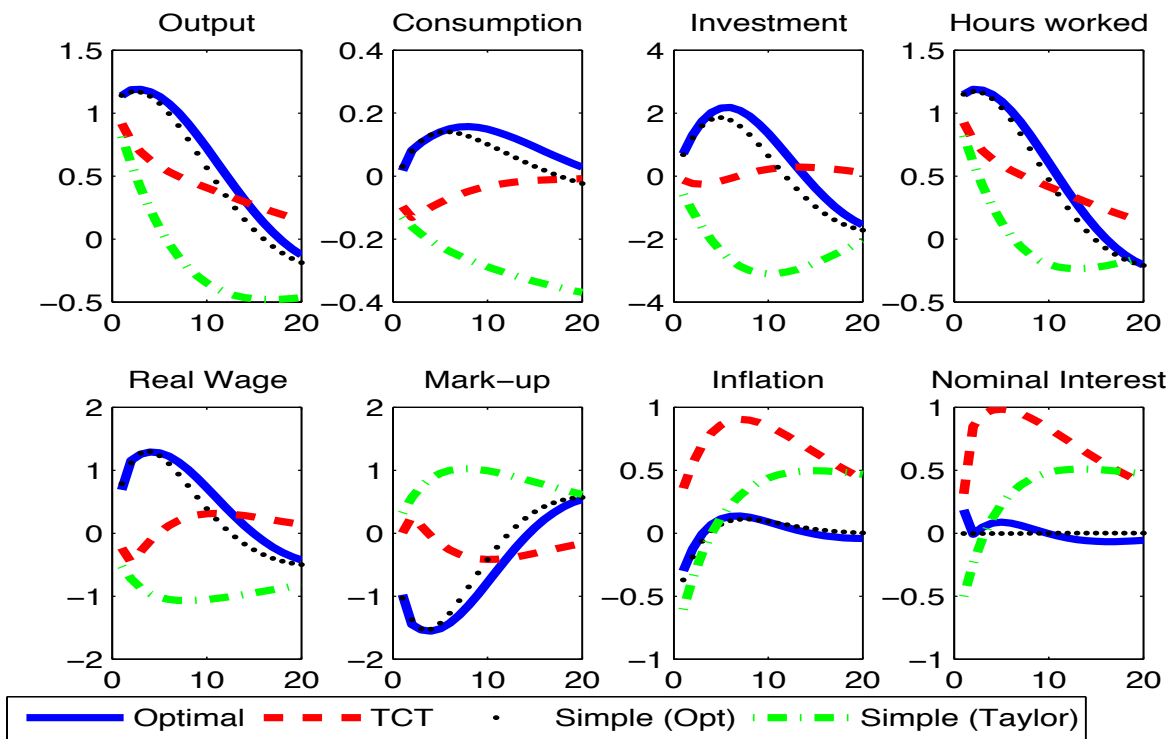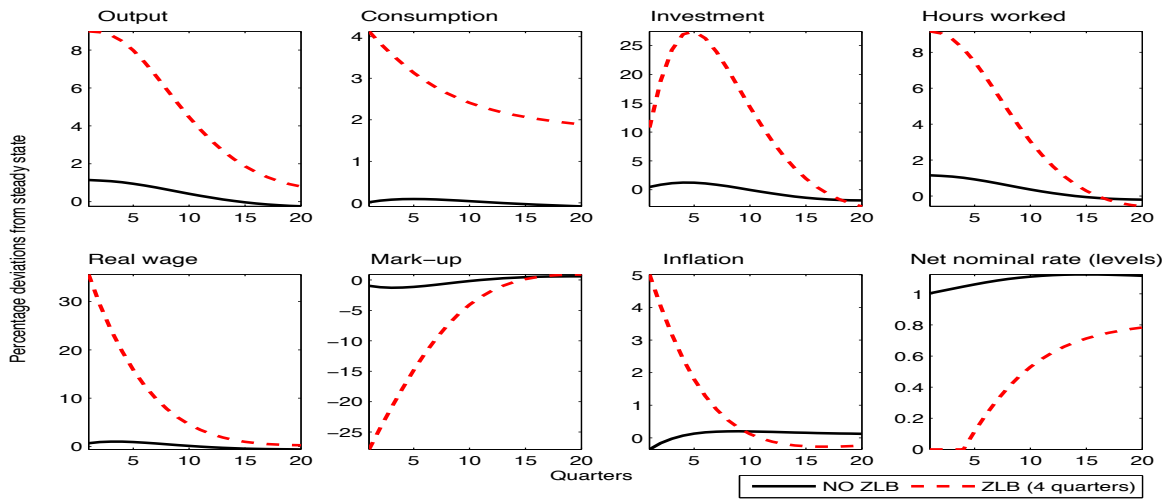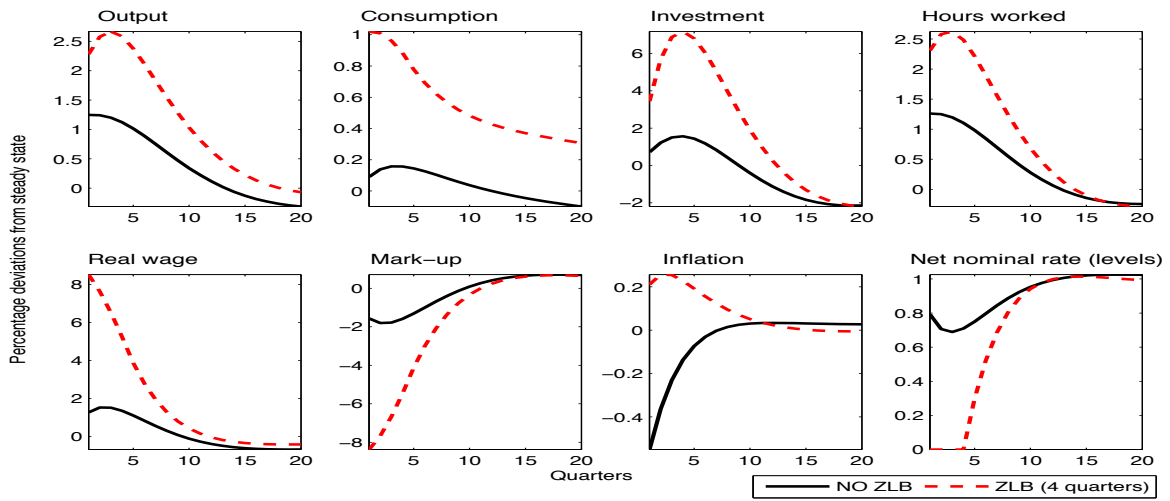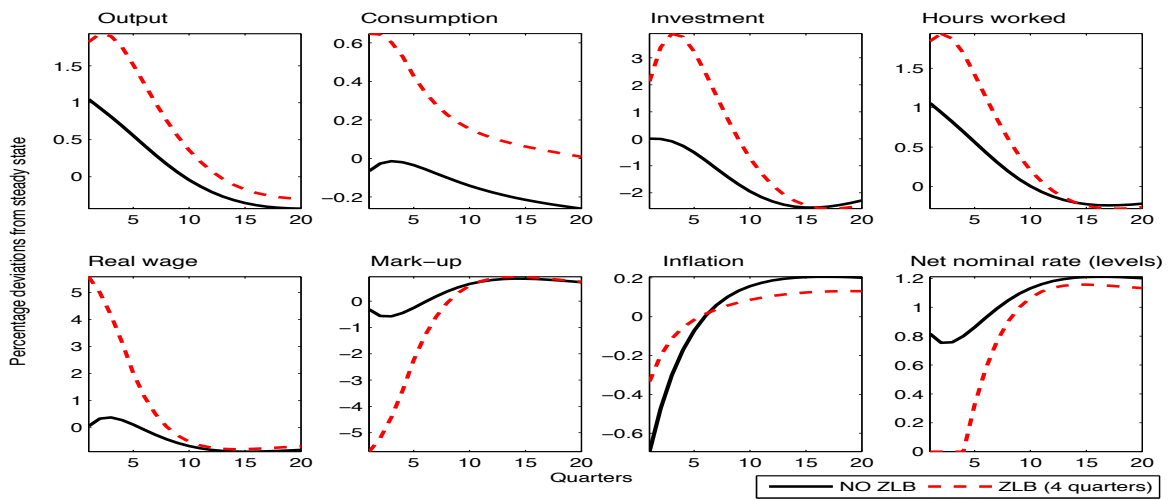
Figure 1: A government spending expansion under alternative monetary regimes

(a) Optimized Simple Monetary Rule

(b) Empirical Taylor (SW) Rule

(c) Conventional Taylor Rule

Figure 2: A government spending expansion at the zero lower bound.

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Appendix (Not for Journal Publication)

Symmetric equilibrium

$$U_t = \frac{\left[X_t^{(1-\varrho)}(1-H_t)^\varrho\right]^{1-\sigma_c} - 1}{1 - \sigma_c}$$

$$X_t = \left\{ \nu_x^{\frac{1}{\sigma_x}} \left[X_t^c\right]^{\frac{\sigma_x-1}{\sigma_x}} + (1-\nu_x)^{\frac{1}{\sigma_x}} \left[X_t^g\right]^{\frac{\sigma_x-1}{\sigma_x}} \right\}^{\frac{\sigma_x}{\sigma_x-1}}$$

$$U_{X^c,t} = \nu_x^{\frac{1}{\sigma_x}} (1-\varrho) X_t^{(1-\varrho)(1-\sigma_c)-1} (1-H_t)^{\varrho(1-\sigma_c)} \left(\frac{X_t}{X_t^c}\right)^{\frac{1}{\sigma_x}}$$

$$U_{H,t} = -\varrho X_t^{(1-\varrho)(1-\sigma_c)} (1-H_t)^{\varrho(1-\sigma_c)-1}$$

$$1 = E_t \left[D_{t,t+1} \frac{R_t}{\Pi_{t+1}} \right]$$

$$D_{t,t+1} = \beta \frac{U_{X^c,t+1}}{U_{X^c,t}}$$

$$-U_{H,t} = U_{X^c,t} \frac{W_t}{P_t}$$

$$K_{t+1} = (1-\delta)K_t + I_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right]$$

$$Q_t = E_t \left\{ D_{t,t+1} \left[R_{t+1}^K + (1-\delta)Q_{t+1} \right] \right\}$$

$$Q_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} \right) + E_t \left(D_{t,t+1}Q_{t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \right) = 1$$

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

$$S'\left(\frac{I_t}{I_{t-1}}\right) = \gamma\left(\frac{I_t}{I_{t-1}} - 1\right)$$

$$C_t = X_t^c + \theta^c S_{t-1}^c$$

$$S_t^c = \varrho^c S_{t-1}^c + (1-\varrho^c)C_t$$

$$G_t = X_t^g + \theta^g S_{t-1}^g$$

$$S_t^g = \varrho^g S_{t-1}^g + (1-\varrho^g)G_t$$

$$F(H_t, K_t) = (A_t H_t)^\alpha K_t^{1-\alpha}$$

$$Y_t = F(H_t, K_t) - FC$$

$$F_{H,t} = \alpha \frac{F(H_t, K_t)}{H_t}$$

$$F_{K,t} = (1 - \alpha) \frac{F(H_t, K_t)}{K_t}$$

$$R_t^K = MC_t F_{K,t}$$

$$\frac{W_t}{P_t} = MC_t F_{H,t}$$

$$\nu_t^c = 1 - MC_t + (1 - \varrho^c)\lambda_t^c$$

$$\lambda_t^c = E_t D_{t,t+1}(\theta^c \nu_{t+1}^c + \varrho^c \lambda_{t+1}^c)$$

$$\nu_t^g = 1 - MC_t + (1 - \varrho^g)\lambda_t^g$$

$$\lambda_t^g = E_t D_{t,t+1}(\theta^g \nu_{t+1}^g + \varrho^g \lambda_{t+1}^g)$$

$$C_t + G_t + (1 - \eta)I_t + \eta MC_t I_t - \eta \nu_t^c X_t^c - \eta \nu_t^g X_t^g + \xi E_t D_{t,t+1}\left[(\Pi_{t+1} - 1)\Pi_{t+1}\right]Y_{t+1} - \xi(\Pi_t - 1)\Pi_t Y_t = 0$$

$$Y_t = C_t + I_t + G_t + \frac{\xi}{2}(\Pi_t - 1)^2 Y_t$$

$$\log\left(\frac{G_t}{\bar{G}}\right) = \rho_G \log\left(\frac{G_{t-1}}{\bar{G}}\right) + \epsilon_t^G$$

$$\log\left(\frac{A_t}{\bar{A}}\right) = \rho_A \log\left(\frac{A_{t-1}}{\bar{A}}\right) + \epsilon_t^A$$

$$\log\left(\frac{R_t}{\bar{R}}\right) = \rho_R \log\left(\frac{R_t}{\bar{R}}\right) + (1 - \rho_R)\left[\rho_\Pi \log\left(\frac{\Pi_t}{\bar{\Pi}}\right) + \rho_Y \log\left(\frac{Y_t}{Y_t^f}\right)\right]$$

Steady state

H, MC and ν_x solve simultaneously:

$$Y = C + I + G + \frac{\xi}{2}(\Pi - 1)^2 Y$$

$$C + G + (1 - \eta)I + \eta MC\, I - \eta \nu^c X^c - \eta \nu^g X^g + \xi D(\Pi - 1)\Pi Y - \xi(\Pi - 1)\Pi Y = 0$$

$$\nu_x = \frac{X^c}{X^c + X^g}$$

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while all other variables solve the following system of recurvive equations:

$$A = \bar{A}$$

$$\Pi = \bar{\Pi}$$

$$Q = 1$$

$$S = 0$$

$$S' = 0$$

$$D = \beta$$

$$R = \frac{\Pi}{\beta}$$

$$R^K = \frac{1}{D} - (1 - \delta)$$

$$F_K = \frac{R^K}{MC}$$

$$\frac{K}{F(H,K)} = \frac{1 - \alpha}{F_K}$$

$$F(H,K) = A \left(\frac{1}{MC} \right)^{\frac{1}{\alpha}} \left(\frac{K}{Y} \right)^{\frac{1-\alpha}{\alpha}} H$$

$$K = \frac{K}{F(H,K)} F(H,K)$$

$$I = \delta K$$

$$FC = (1 - MC)F(H,K)$$

$$Y = F(H,K) - FC$$

$$G = \frac{G}{Y} Y$$

$$S^g = G$$

$$X^g = (1 - \theta^g) G$$

$$F_H = \alpha \frac{F(H,K)}{H}$$

$$\frac{W}{P} = MC\, F_H$$

$$X = \frac{1 - \varrho}{\varrho}\frac{W}{P}\left(1 - H\right)$$

$$X^c = \left(\frac{X^{\frac{\sigma_x - 1}{\sigma_x}} - (1 - \nu_x)^{\frac{1}{\sigma_x}}\left[X^g\right]^{\frac{\sigma_x - 1}{\sigma_x}}}{\nu_x^{\frac{1}{\sigma_x}}}\right)^{\frac{\sigma_x - 1}{\sigma_x}}$$

$$C = \frac{X^c}{1 - \theta^c}$$

$$S^c = C$$

$$\lambda^c = \frac{D\theta^c(1 - MC)}{1 - D\theta^c\left(1 - \rho^c\right) - D\rho^c}$$

$$\nu^c = 1 - MC + (1 - \varrho^c)\lambda^c$$

$$\lambda^g = \frac{D\theta^g(1 - MC)}{1 - D\theta^g(1 - \rho^g) - D\rho^g}$$

$$\nu^g = 1 - MC + (1 - \varrho^g)\lambda^g$$

$$U_{X^c} = (1 - \varrho)\, X^{(1 - \varrho)(1 - \sigma_c) - 1}\left(1 - H\right)^{\varrho(1 - \sigma_c)}$$

$$U_H = -\varrho X^{(1 - \varrho)(1 - \sigma_c)}\left(1 - H\right)^{\varrho(1 - \sigma_c) - 1}$$