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RECONCILING JAIMOVICH-REBELLO PREFERENCES, HABIT IN CONSUMPTION AND LABOR SUPPLY
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Reconciling Jaimovich-Rebello Preferences, Habit in Consumption and Labor Supply

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Abstract

This note studies two forms of a utility function of consumption with habit and leisure that are (a) compatible with long-run balanced growth, (b) hit a steady state observed target for hours worked and (c) are consistent with micro-econometric evidence for the inter-temporal elasticity of substitution and the Frisch elasticity of labor supply. For Jaimovich-Rebello preferences our Theorems 1 and 2 highlight a constraint on the preference parameter needed to target the Frisch elasticity leading to a lower bound for the latter that cannot be reconciled empirically with external habit. Even with internal or no habit, the range of possible values of the Frisch elasticity lie outside empirical results unless we allow for a modest wealth effect. In Theorem 3 we propose a generalized JR utility function that in conjunction with a labor wedge solves the problem.

JEL Classification: E21, E24.

Keywords: Jaimovich-Rebello Preferences, Habit in Consumption, Labor Supply, Frisch Elasticity, Labor Wedge

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1 Introduction

Whether it is in the context of the equity-premium puzzle (see for example Abel (1999)), the savings-growth relation (Carroll and Weil (2000) ) or monetary policy - business cycle analysis (Christiano et al. (2005)), researchers have used the concept of relative preferences to advance their various agendas. In particular, RBC-DSGE models in which a consumer’s utility level not only depends on her consumption level but also how that level compares to a standard set either by her own past consumption levels (internal habit-formation) or the levels of those in her peerage (catching-up with the Joneses’ or external habit) are now ubiquitous in the literature.

At the same time to achieve co-movement of output, hours, consumption and investment modelers turn to preferences proposed by Jaimovich and Rebello (2008) (henceforth JR) that control short-run wealth effects. This note discusses two forms of this utility function, \( U(C, L) \), where \( C \) is consumption modified by habit and \( L = 1 - H \) is leisure, as the proportion of the day, \( H \) being hours. The objective is to choose a form (a) compatible with long-run balanced growth, (b) hits a steady state observed target for \( H \) (c) is consistent with micro-econometric evidence for the inter-temporal elasticity of substitution and the Frisch elasticity of labor supply.

2 The Frisch Elasticity of Labor Supply

We examine a broad class of utility functions that satisfy criterion (a) above proposed by King et al. (1988) – henceforth KPR:

\[
U(C, L) = \left( \frac{1}{1 - \sigma} \right) \exp((\sigma - 1)g(L)); \quad \sigma > 1, \quad g' < 0, \quad g'' > 0
\]  

(1)

Note that (1) is a CRRA function in \( f(L) = -\exp(g(L)) \). Bilbiie (2011) shows that the constant-marginal-utility inverse (Frisch) elasticity of labor supply is given by

\[
\delta = \frac{U_{LC}H}{U_L} \left( \frac{U_{LC}}{U_{CC}} - \frac{U_{LL}}{U_{LC}} \right)
\]

and it is straightforward to show that \( g'' > 0 \) is sufficient for leisure to be a normal good. Differentiating (1) we can substitute for partial derivatives \( U_C, U_{CC}, U_{LC}, U_L \) and \( U_{LL} \) to arrive at the Frisch elasticity for the KPR class of utility functions:

\[
\delta = \frac{H(g''(L) + \frac{\sigma - 1}{\sigma} (g'(L))^2}{g'(L)}
\]  

(2)
Jaimovich and Rebello (2008) propose a utility function of the KPR form in its balanced growth steady state:

\[ U_t = U(C_t, L_t, X_t) = \frac{(C_t - \rho(1 - L_t)^{1+\psi}X_t)^{1-\sigma}}{1-\sigma}; \psi > 0 \]  

(3)

\[ X_t = C_t^{\gamma}X_{t-1}^{1-\gamma}; \quad \gamma \in [0, 1] \]  

(4)

In a zero growth steady state equilibrium with \( \gamma > 0 \) we have \( X_t = C_t \) where for the household decision (as we shall see below) \( C_t \) is a function of \( \gamma \) and

\[ U(C, L) = \frac{C^{1-\sigma}}{1-\sigma}(1 - \rho(1 - L)^{1+\psi})^{1-\sigma} = \frac{C^{1-\sigma}}{1-\sigma} \exp \left( (\sigma - 1) \log(1 - \rho(1 - L)^{1+\psi}) \right) \]

which is of KPR form (1) where \( g(L) = -\log(1 - \rho(1 - L)^{1+\psi}) \). Differentiating \( g(L) \) and using (2), for JR preferences the Frisch elasticity is then given by

\[ \delta_{JR} = \psi + \frac{(2\sigma - 1)(1 + \psi)\rho H^{1+\psi}}{\sigma(1 - \rho H^{1+\psi})} \]  

(5)

Note that in equilibrium \( H = 1 - L \), and therefore \( \delta_{JR} \), are dependent on \( \gamma \) (see Section 3.2).

Microeconomic and macroeconomic estimates of the Frisch elasticity differ significantly, the former typically ranging from 0 to 0.5 and the latter from 2 to 4 (Peterman, 2016). Estimations of the elasticity of labor supply found using microeconomic data depend on factors such as gender, age, marital status and dependants. Keane (2011) offers a survey of labor supply, restricting the sample to men, finding a range of between 0 to 0.7 with an average of 0.31. Reichling and Whalen (2017) give a thorough review of the estimates found in the literature based on microeconomic data, finding that estimates typically range from 0 to over 1. The higher estimates corresponding to married women with children, whereas the labor supply of men is far lower. Combining the results, Reichling and Whalen (2017) propose a range of between 0.27 and 0.53, with a central point estimate of 0.4. This corresponds to a Frisch coefficient, \( \delta \), between 1.89 and 3.7, with a point estimate of 2.5.

### 3 The Household Problem

Households choose between work and leisure and therefore how much labor they supply. They also own the capital stock which is rented to firms at a rental rate \( r^K_t \) and choose an optimal investment path. The single-period utility is given by JR preferences (3) and (4). In a stochastic environment,
the value function of the representative household at time $t$ is given by

$$V_t = V_t(B_{t-1}, K_{t-1}) = \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s U(C_{t+s}, L_{t+s}, X_{t+s}) \right]$$

For the household’s problem at time $t$ is to choose paths for consumption $\{C_t\}$, labor supply $\{H_t = 1 - L_t\}$, capital stock $\{K_t\}$, investment $\{I_t\}$ and bond holdings to maximize $V_t$ given by (3) given its budget constraint in period $t$

$$B_t = R_{t-1}B_{t-1} + r^K_{t}K_{t-1} + W_tH_t - C_t - I_t - T_t$$

where $B_t$ is the given net stock of financial assets at the end of period $t$, $r^K_t$ is the rental rate, is the wage rate and $R_t$ is the gross interest rate paid on assets held at the beginning of period $t$, $I_t$ is investment and $T_t$ are lump-sum taxes; and given that capital stock accumulates according to

$$K_t = (1 - \delta)K_{t-1} + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t;$$

where $S(\frac{I_t}{I_{t-1}})$ are investment adjustment costs converting $I_t$ units of output converts to $(1 - S(\frac{I_t}{I_{t-1}}))I_t$ of new capital and $S', S'' \geq 0$; $S(1) = S'(1) = 0$. All variables are expressed in real terms relative to the price of output.

### 3.1 Solution of the Household Problem

To solve the household problem we form a Lagrangian

$$\mathcal{L} = \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left( U(C_{t+s}, L_{t+s}, X_{t+s}) + \lambda_{t+s}[R_{t+s-1}B_{t+s-1} + W_{t+s}(1 - L_{t+s}) + r^K_{t+s}K_{t+s-1} - C_{t+s} - I_{t+s} - T_{t+s} - B_{t+s}] + \lambda_{t+s}Q_{t+s}[(1 - \delta)K_{t+s-1} + (1 - S_{t+s}(I_{t+s}/I_{t+s-1}))I_{t+s} - I_{t+s}] + \mu_{t+s}[X_{t+s} - c^\gamma_{t+s}X_{t+s-1}^{1-\gamma}] \right) \right]$$

Defining the stochastic discount factor as $\Lambda_{t,t+1} \equiv \beta \frac{X_{t+1}}{X_t}$, the first order conditions are:

**Euler Consumption** : $1 = R_t \mathbb{E}_t [\Lambda_{t,t+1}]$

where $\lambda_t = U_{C,t} - \gamma \mu_t \frac{X_t}{C_t}$ and $

\mu_t = -U_{X,t} + \beta(1 - \gamma) \mathbb{E}_t \left[ \frac{\mu_{t+1} X_{t+1}}{X_t} \right]$,

**Labor Supply** : $\frac{U_{H,t}}{\lambda_t} = -\frac{U_{L,t}}{\lambda_t} = -W_t$  \(6\)

**Investment FOC** : $Q_t(1 - S(I_t/I_{t-1}) - \mathbb{E}_t S'(I_t/I_{t-1})) + \mathbb{E}_t \left[ \Lambda_{t,t+1} Q_{t+1} S'(I_{t+1}/I_t)(I_{t+1}/I_t)^2 \right] = 1$
Capital Supply: \( E_t [\Lambda_{t,t+1} R^K_{t+1}] = 1 \)

where \( R^K_t \) is the gross return on capital given by \( R^K_t = \frac{r^K_t + (1 - \delta)Q_t}{Q_{t-1}} \).

The zero-growth steady-state of the above first-order conditions is

\[
\begin{align*}
R & = R^K = \frac{1}{\beta} \quad X = C \quad \Lambda = \beta \\
\lambda & = UC - \gamma \mu \quad \mu = \frac{-UX}{(1 - \beta(1 - \gamma))} \quad Q = 1 \\
W & = -\frac{UH}{\lambda} \quad r^K = \frac{1}{\beta} - 1 + \delta
\end{align*}
\]

### 3.2 JR Preferences

The model up to now is completely general in terms of preferences. It applies to CD and SW preferences by putting \( \mu_t = U_{C,t} = 0 \). With JR preferences given by (3) and (4) we have \( U_{C,t} = (C_t - \varphi H^{1+\psi} X_t)^{-\sigma}, U_{H,t} = -\psi (1 + \psi) H^{1+\psi} X_t U_{C,t} \) and \( U_{X,t} = -\psi H^{1+\psi} U_{C,t} \) if \( \gamma < 1 \) and \( = 0 \) if \( \gamma = 1 \). The steady state then becomes

\[
\begin{align*}
\mu & = \frac{-UX}{(1 - \beta(1 - \gamma))} = \frac{\varphi H^{1+\psi} U_{C}}{(1 - \beta(1 - \gamma))} \\
\lambda & = UC \left( 1 - \frac{\gamma \varphi H^{1+\psi}}{(1 - \beta(1 - \gamma))} \right) \\
W & = -\frac{U_H}{\lambda} = \frac{\varphi H^{1+\psi}}{1 - \frac{\gamma \varphi H^{1+\psi}}{(1 - \beta(1 - \gamma))}} \quad X = C
\end{align*}
\]

With these preferences and the steady-state labor share \( \alpha = \frac{WH}{Y} \) we arrive at

\[
\varphi H^{1+\psi} = \frac{\alpha((1 - \beta(1 - \gamma))}{(1 + \psi) c_y (1 - \beta(1 - \gamma)) + \gamma \alpha) \quad (7)
\]

where \( c_y \equiv \frac{G}{H} \). For a given \( c_y \) and \( H \) (determined in a general equilibrium with a supply side), this pins down \( \varphi \) given the remaining parameters. However a necessary condition for an equilibrium to exist is that \( 0 < \varphi H^{1+\psi} < 1 \). This places the following lower bound on \( \psi \)

\[
\psi > \psi^* = \frac{\alpha(1 - \beta)(1 - \gamma)}{c_y(1 - \beta(1 - \gamma)) + \gamma \alpha} - 1 \quad (8)
\]

For \( \gamma = 0 \) this becomes \( \psi > \frac{\alpha}{c_y} - 1 \) whereas for \( \gamma = 1 \) we have \( \psi > -1 \) and the constraint disappears.

Substituting for \( \varphi H^{1+\psi} \) from (7) into (5) we arrive at the Frisch elasticity as a function of \( \psi \)

\[
\delta_{JR} = \delta_{JR}(\psi) = \psi + \frac{2(2\sigma - 1)(1 + \psi)\alpha(1 - \beta(1 - \gamma))}{\sigma((1 + \psi) c_y - \alpha(1 - \beta(1 - \gamma)) + \gamma \alpha)}
\]
Differentiating $\delta_{JR}(\theta)$ with respect to $\psi$ we find

\[
\frac{d\delta_{JR}}{d\psi} = 1 - \frac{(2\sigma - 1)\alpha^2(1 - \beta(1 - \gamma))(1 - \gamma)(1 - \beta)}{\sigma(c_y(1 - \beta(1 - \gamma))\theta - \alpha(1 - \gamma)(1 - \beta))^2}
\]

\[
\frac{d^2\delta_{JR}}{d\psi^2} = 2\frac{(2\sigma - 1)\alpha^2c_y(1 - \beta(1 - \gamma))^2(1 - \gamma)(1 - \beta)}{\sigma(c_y(1 - \beta(1 - \gamma))\theta - \alpha(1 - \gamma)(1 - \beta))^3} > 0 \text{ if } \sigma > \frac{1}{2}
\]

Hence $\delta_{JR}(\psi)$ is convex and putting $\frac{d\delta_{JR}}{d\psi} = 0$ we arrive at:

**Theorem 1**

Restricting ourselves to the case $\sigma > \frac{1}{2}$, $\delta_{JR}(\psi)$ is bounded below at a value $\delta_{JR}(\psi)$ where

\[
\psi = \alpha \left[ (1 - \gamma)(1 - \beta) + \sqrt{\frac{(2\sigma - 1)}{\sigma}(1 - \beta(1 - \gamma))(1 - \gamma)(1 - \beta)} \right] / c_y(1 - \beta(1 - \gamma)) - 1 > \psi^\star
\]

A sting in the tail arises if we introduce *external habit* with $C_t$ in the utility function replaced by $C_t - \chi C_{t-1}$. Then $c_y$ is replaced with $c_y(1 - \chi)$ pushing the constraint on $\psi$ into an implausible range. This we now show can be mitigated by making habit *internal rather than external*.

### 3.3 External versus Internal Habit

With external habit in consumption, household $j$ has a single-period utility

\[
U^j_t = \left( C^j_t - \chi C_{t-1} - \phi(H^j_t)^{1+\psi} X^j_t \right)^{1-\sigma}; \quad \chi \in [0, 1)
\]

\[
X^j_t = \left( C^j_t - \chi C_{t-1} \right)^{\gamma} (X^j_{t-1})^{1-\gamma}; \quad \gamma \in [0, 1]
\]

where $C_{t-1}$ is aggregate per capita consumption whereas with internal habit we have

\[
U^j_t = \left( C^j_t - \chi C_{t-1} - \phi(H^j_t)^{1+\psi} X^j_t \right)^{1-\sigma}; \quad \chi \in [0, 1)
\]

\[
X^j_t = \left( C^j_t - C^j_{t-1} \right)^{\gamma} (X^j_{t-1})^{1-\gamma}; \quad \gamma \in [0, 1]
\]

In a symmetric equilibrium, the household first-order conditions are as before with marginal utility

\[
U_{C,t} = (C_t - \chi C_{t-1} - \phi H_t^{1+\psi} X_t)^{-\sigma}
\]

and for external habit and internal habit respectively we have

\[
\lambda_t = U_{C,t} - \frac{\gamma \mu_t X_t}{(C_t - \chi C_{t-1})}
\]

\[
\lambda_t = U_{C,t} - \beta \chi E_t[U_{C,t+1}] - \gamma \left( \frac{\mu_t X_t}{(C_t - \chi C_{t-1})} - \beta \chi E_t \left[ \frac{\mu_{t+1} X_{t+1}}{(C_{t+1} - \chi C_t)} \right] \right)
\]
The zero-growth steady state then becomes:

\[ U_C = (C(1 - \chi) - \rho H^{1+\psi}X)^{-\sigma}, \quad \lambda = U_C - \frac{\gamma \mu X}{(C(1 - \chi))} \]

for external habit and

\[ \lambda = U_C(1 - \beta \chi) - \frac{\gamma(1 - \delta \chi) \mu X}{(C(1 - \chi))} \]

for internal habit. These results lead to:

**Theorem 2**

The results of Theorem 1 apply to habit in consumption with \(cy\) replaced with \(cy(1 - \chi)\) for external habit and \(cy \frac{1 - \chi}{1 - \beta \chi}\) for internal habit.

### 3.4 Numerical Illustration

Table 1 illustrates the analysis. Parameter values are \(\alpha = 0.7, \ c_y = 0.6, \ \beta = .99, \ \sigma = 2.0\) and stated values for \(\gamma, \ \chi.\)

We can now assess the empirical plausibility of JR preferences with habit in consumption. From our discussion in Section 2 we wish to calibrate \(\psi\) to hit an inverse elasticity \(\delta_{JR} \in [1.89, 3.70]\) with a central value 2.50. From our numerical results for the lower bound \(\delta_{JR}(\psi)\), this rules out external habit if we are to choose JR preferences that allow for only weak wealth effects \((\gamma\) very small). But even without habit, or with internal habit, it is difficult to reconcile the extreme choice of \(\gamma\) almost zero with a Frisch elasticity within this empirical range.

### 4 A Resolution: Generalized JR Utility and Labor Wedge

To get round the empirical problem posed, we propose a generalization of JR preferences to replace (3) with a generalized form:

\[ U_t(C_t, C_{t-1}, H_t, Z_t) = \frac{(C_t - \chi C_{t-1})(1 - \phi H_t^{1+\psi} Z_t)^{\theta}}{1 - \sigma}; \quad \psi > 0, \ \theta \in (0, 1] \]

\[ Z_t \equiv \frac{X_t}{C_t - \chi C_{t-1}} = Z_t^{1-\gamma} \left( \frac{C_{t-1} - \chi C_{t-2}}{C_t - \chi C_{t-1}} \right)^{1-\gamma}; \quad \gamma \in [0, 1] \]

which reduces to JR preferences with \(\theta = 1\) and Cobb-Douglas preferences with \(\phi = \gamma = 1\) and \(\psi = 0\). Proceeding as before the steady-state equilibrium (7) becomes

\[ \phi H^{1+\psi} = \frac{\alpha((1 - \beta(1 - \gamma))}{(\theta(1 + \psi)c_y + (1 - \theta)\alpha)(1 - \beta(1 - \gamma)) + \gamma \alpha \theta)} \]

\(^1\)In fact \(\gamma > 0\) is required for balanced growth, but \(\gamma\) can be very small.
where \( c_y \equiv \frac{\gamma}{\lambda} \). Again for a given \( H \) this pins down \( \varrho \) given the remaining parameters. As before a necessary condition for an equilibrium to exist is that \( 0 < \varrho H^{1+\psi} < 1 \). Following some algebra this places the same bound on \( \psi \) given by (8), independent of \( \theta \), but the Frisch elasticity becomes

\[
\delta_{JR} = \psi + \frac{(\sigma + \theta(\sigma - 1))(1 + \psi)\varrho H^{1+\psi}}{\sigma(1 - \varrho H^{1+\psi})}
\]

Then for \( \theta \in (0,1] \) Theorem 1 generalizes to:

**Theorem 3**

Restricting ourselves to the case \( \delta > \frac{1}{2} \), \( \delta_{JR}(\psi) \) is bounded below at a value \( \delta_{JR}(\psi(\theta)) \) where

\[
\psi(\theta) = \frac{\alpha}{c_y} \left[ (1 - \gamma)(1 - \beta) + \sqrt{\frac{\sigma + \theta(\sigma - 1)}{\sigma}(1 - \beta)(1 - \gamma))} \right] - 1 > \psi^* \quad \theta \in (0,1] \tag{9}
\]

Thus the lower bound on both \( \psi \) and \( \delta_{F} \) falls as \( \theta \) drops below its value \( \theta = 1 \) for JR preferences.

Our second modification is to introduce a labor wedge into the household problem (as in Shimer (2009)). Then (6) becomes \( \frac{U_{H,t}}{X_t} = -\frac{U_{L,t}}{X_t} = -W_t(1 - \tau) \) where \( \tau \in [0.27,0.37] \) is the wedge and \( \alpha \) in (9) is replaced with \( \alpha(1 - \tau) \). Figure 1 shows that a combination of our generalized JR utility and an empirically supported wedge, we can calibrate parameters to achieve the desired empirical value for \( \delta_{JR} \).

![Figure 1](image.png)

(a) \( \gamma = 0, \chi = 0.5, \tau = 0 \)  
(b) \( \gamma = 0, \chi = 0.5, \tau = 0.3 \)

**Figure 1**: Frisch Elasticity: Generalized JR Utility, Internal Habit and a Labor Wedge.

5 Conclusions

This note has reviewed a utility function commonly used in RBC-DSGE models that is non-separable in habit-adjusted consumption and leisure, compatible with balanced growth and elim-
inates counterfactual wealth effects highlighted by Jaimovich and Rebello (2008). Our main contributions are first, Theorems 1 and 2 that highlight a constraint on the preference parameter $\psi$ needed to target the Frisch elasticity. This leads to a lower bound for the latter that cannot be reconciled empirically with external habit. Even with internal or no habit, the range of possible values of the Frisch elasticity lie outside empirical results unless we allow for a modest wealth effect. Second, in Theorem 3 we propose a generalized JR utility function that together with the introduction of a labor wedge, as proposed by Shimer (2009), solves the problem.

References


