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Reconciling Jaimovich-Rebello Preferences, Habit in Consumption and Labor Supply

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Abstract

This note studies two forms of a utility function of consumption with habit and leisure that are (a) compatible with long-run balanced growth, (b) hit a steady state observed target for hours worked and (c) are consistent with micro-econometric evidence for the inter-temporal elasticity of substitution and the Frisch elasticity of labor supply. For Jaimovich-Rebello preferences our Theorems 1 and 2 highlight a constraint on the preference parameter needed to target the Frisch elasticity leading to a lower bound for the latter that cannot be reconciled empirically with external habit. Even with internal or no habit, the range of possible values of the Frisch elasticity lie outside empirical results unless we allow for a modest wealth effect. In Theorem 3 we propose a generalized JR utility function that in conjunction with a labor wedge solves the problem.

JEL Classification: E21, E24.

Keywords: Jaimovich-Rebello Preferences, Habit in Consumption, Labor Supply, Frisch Elasticity, Labor Wedge

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Contents

1 Introduction

Whether it is in the context of the equity-premium puzzle (see for example Abel (1999)), the savings-growth relation (Carroll and Weil (2000)) or monetary policy - business cycle analysis (Christiano *et al.* (2005)), researchers have used the concept of relative preferences to advance their various agendas. In particular, RBC-DSGE models in which a consumer's utility level not only depends on her consumption level but also how that level compares to a standard set either by her own past consumption levels (internal habit-formation) or the levels of those in her peerage (catching-up with the Joneses' or external habit) are now ubiquitous in the literature.

At the same time to achieve co-movement of output, hours, consumption and investment modellers turn to preferences proposed by Jaimovich and Rebello (2008) (henceforth JR) that control short-run wealth effects. This note discusses two forms of this utility function, $U(C, L)$, where C is consumption modified by habit and $L = 1 - H$ is leisure, as the proportion of the day, H being hours. The objective is to choose a form (a) compatible with long-run balanced growth, (b) hits a steady state observed target for H (c) is consistent with micro-econometric evidence for the inter-temporal elasticity of substitution and the Frisch elasticity of labor supply.

2 The Frisch Elasticity of Labor Supply

We examine a broad class of utility functions that satisfy criterion (a) above proposed by King *et al.* (1988) – henceforth KPR:

$$U(C, L) = \frac{C^{1-\sigma}}{1-\sigma} \exp((\sigma - 1)g(L)); \ \sigma > 1, \quad g' < 0, \ g'' > 0 \tag{1}$$

Note that (1) is a CRRA function in $f(L) = -\exp(g(L))$. Bilbiie (2011) shows that the constant-marginal-utility inverse (Frisch) elasticity of labor supply is given by

$$\delta = \frac{U_{LC}H}{U_L}\left(\frac{U_{LC}}{U_{CC}} - \frac{U_{LL}}{U_{LC}}\right)$$

and it is straightforward to show that $g'' > 0$ is sufficient for leisure to be a normal good. Differentiating (1) we can substitute for partial derivatives U_C, U_{CC}, U_{LC}, U_L and U_{LL} to arrive at the Frisch elasticity for the KPR class of utility functions:

$$\delta = \frac{H(g''(L) + \frac{\sigma-1}{\sigma}(g'(L))^2}{g'(L)} \tag{2}$$

Jaimovich and Rebello (2008) propose a utility function of the KPR form in its balanced growth steady state:

$$U_t = U(C_t, L_t, X_t) = \frac{(C_t - \varrho(1-L_t)^{1+\psi} X_t)^{1-\sigma}}{1-\sigma}\,;\psi > 0 \tag{3}$$

$$X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}\,;\quad \gamma \in [0,1] \tag{4}$$

In a zero growth steady state equilibrium with $\gamma > 0$ we have $X = C$ where for the household decision (as we shall see below) C is a function of γ and

$$U(C, L) = \frac{C^{1-\sigma}}{1-\sigma}(1 - \varrho(1-L)^{1+\psi})^{1-\sigma} = \frac{C^{1-\sigma}}{1-\sigma}\exp\left(-(\sigma-1)\log(1-\varrho(1-L)^{1+\psi})\right)$$

which is of KPR form (1) where $g(L) = -\log(1 - \varrho(1-L)^{1+\psi})$. Differentiating $g(L)$ and using (2), for JR preferences the Frisch elasticity is then given by

$$\delta_{JR} = \psi + \frac{(2\sigma-1)(1+\psi)\varrho H^{1+\psi}}{\sigma(1-\varrho H^{1+\psi})} \tag{5}$$

Note that in equilibrium $H = 1 - L$, and therefore δ_{JR}, are dependent on γ (see Section 3.2).

Microeconomic and macroeconomic estimates of the Frisch elasticity differ significantly, the former typically ranging from 0 to 0.5 and the latter from 2 to 4 (Peterman, 2016). Estimations of the elasticity of labor supply found using microeconomic data depend on factors such as gender, age, marital status and dependants. Keane (2011) offers a survey of labor supply, restricting the sample to men, finding a range of between 0 to 0.7 with an average of 0.31. Reichling and Whalen (2017) give a thorough review of the estimates found in the literature based on microeconomic data, finding that estimates typically range from 0 to over 1. The higher estimates corresponding to married women with children, whereas the labor supply of men is far lower. Combining the results, Reichling and Whalen (2017) propose a range of between 0.27 and 0.53, with a central point estimate of 0.4. This corresponds to a Frisch coefficient, δ, between 1.89 and 3.7, with a point estimate of 2.5.

3 The Household Problem

Households choose between work and leisure and therefore how much labor they supply. They also own the capital stock which is rented to firms at a rental rate r_t^K and choose an optimal investment path. The single-period utility is given by JR preferences (3) and (4). In a stochastic environment, the value function of the representative household at time t is given by

$$V_t = V_t(B_{t-1}, K_{t-1}) = \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s U(C_{t+s}, L_{t+s}, X_{t+s})\right]$$

For the household's problem at time t is to choose paths for consumption $\{C_t\}$, labor supply $\{H_t = 1 - L_t\}$, capital stock $\{K_t\}$, investment $\{I_t\}$ and bond holdings to maximize V_t given by (3) given its budget constraint in period t

$$B_t = R_{t-1}B_{t-1} + r_t^K K_{t-1} + W_t H_t - C_t - I_t - T_t$$

where B_t is the given net stock of financial assets at the end of period t, r_t^K is the rental rate, is the wage rate and R_t is the gross interest rate paid on assets held at the beginning of period t, I_t is investment and T_t are lump-sum taxes; and given that capital stock accumulates according to

$$K_t \quad = \quad (1-\delta)K_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t\,;$$

where $S(\frac{I_t}{I_{t-1}})$ are investment adjustment costs converting I_t units of output converts to $(1 - S(\frac{I_t}{I_{t-1}}))I_t$ of new capital and S', $S'' \geq 0\,;\ S(1) = S'(1) = 0$. All variables are expressed in real terms relative to the price of output.

3.1 Solution of the Household Problem

To solve the household problem we form a Lagrangian

$$\begin{aligned}
\mathcal{L} \quad = \quad & \mathbb{E}_t\Big[\sum_{s=0}^{\infty}\beta^s\Big(U(C_{t+s}, L_{t+s}, X_{t+s}) \\
+ \quad & \lambda_{t+s}[R_{t+s-1}B_{t+s-1} + W_{t+s}(1 - L_{t+s}) + r_{t+s}^K K_{t+s-1} - C_{t+s} - I_{t+s} - T_{t+s} - B_{t+s}] \\
+ \quad & \lambda_{t+s}Q_{t+s}[(1-\delta)K_{t+s-1} + (1 - S\left(I_{t+s}/I_{t+s-1}\right)) I_{t+s} - I_{t+s}] + \mu_{t+s}[X_{t+s} - C_{t+s}^{\gamma}X_{t+s-1}^{1-\gamma}]\Big)
\end{aligned}$$

Defining the stochastic discount factor as $\Lambda_{t,t+1} \equiv \beta\frac{\lambda_{t+1}}{\lambda_t}$ the first order conditions are:

$$\text{Euler Consumption } : 1 = R_t\mathbb{E}_t\left[\Lambda_{t,t+1}\right]$$

where $\lambda_t = U_{C,t} - \gamma\mu_t\frac{X_t}{C_t}$ and $\mu_t = -U_{X,t} + \beta(1-\gamma)\mathbb{E}_t\frac{\mu_{t+1}X_{t+1}}{X_t}$,

$$\text{Labor Supply} \quad : \quad \frac{U_{H,t}}{\lambda_t} = -\frac{U_{L,t}}{\lambda_t} = -W_t \tag{6}$$

$$\text{Investment FOC} \quad : \quad Q_t(1 - S(I_t/I_{t-1}) - \Xi_t S'(I_t/I_{t-1})) + \mathbb{E}_t\left[\Lambda_{t,t+1}\, Q_{t+1}S'(I_{t+1}/I_t)(I_{t+1}/I_t)^2\right] = 1$$

$$\text{Capital Supply} \quad : \quad \mathbb{E}_t\left[\Lambda_{t,t+1}R^K_{t+1}\right] = 1$$

where R^K_t is the gross return on capital given by $R^K_t = \frac{\left[r^K_t + (1-\delta)Q_t\right]}{Q_{t-1}}$.

The zero-growth steady-state of the above first-order conditions is

$$\begin{aligned}
R &= R^K = \frac{1}{\beta}\,; \quad X = C\,; \quad \Lambda = \beta \\
\lambda &= U_C - \gamma\mu\,; \quad \mu = \frac{-U_X}{(1-\beta(1-\gamma))}\,; \quad Q = 1 \\
W &= -\frac{U_H}{\lambda}\,; \quad r^K = \frac{1}{\beta} - 1 + \delta
\end{aligned}$$

3.2 JR Preferences

The model up to now is completely general in terms of preferences. It applies to CD and SW preferences by putting $\mu_t = U_{X,t} = 0$. With JR preferences given by (3) and (4) we have $U_{C,t} = (C_t - \varrho H_t^{1+\psi} X_t)^{-\sigma}$, $U_{H,t} = -\varrho(1+\psi)H_t^{\psi} X_t U_{C,t}$ and $U_{X,t} = -\varrho H_t^{1+\psi} U_{C,t}$ if $\gamma < 1$ and $= 0$ if $\gamma = 1$. The steady state then becomes

$$\begin{aligned}
\mu &= \frac{-U_X}{(1-\beta(1-\gamma))} = \frac{\varrho H^{1+\psi} U_C}{(1-\beta(1-\gamma))}\,; \quad \lambda = U_C\left(1 - \frac{\gamma\varrho H^{1+\psi}}{(1-\beta(1-\gamma))}\right) \\
W &= -\frac{U_H}{\lambda} = \frac{\varrho H^{1+\psi}}{\left(1 - \frac{\gamma\varrho H^{1+\psi}}{(1-\beta(1-\gamma))}\right)}\,; \quad X = C
\end{aligned}$$

With these preferences and the steady-state labor share $\alpha = \frac{WH}{Y}$ we arrive at

$$\varrho H^{1+\psi} = \frac{\alpha((1-\beta(1-\gamma))}{(1+\psi)c_y(1-\beta(1-\gamma)) + \gamma\alpha)} \tag{7}$$

where $c_y \equiv \frac{C}{Y}$. For a given c_y and H (determined in a general equilibrium with a supply side), this pins down ϱ given the remaining parameters. However a necessary condition for an equilibrium to exist is that $0 < \varrho H^{1+\psi} < 1$. This places the following lower bound on ψ

$$\psi > \psi^* = \frac{\alpha(1-\beta)((1-\gamma)}{c_y(1-\beta(1-\gamma))} - 1 \tag{8}$$

For $\gamma = 0$ this becomes $\psi > \frac{\alpha}{c_y} - 1$ whereas for $\gamma = 1$ we have $\psi > -1$ and the constraint disappears. Substituting for $\varrho H^{1+\psi}$ from (7) into (5) we arrive at the Frisch elasticity as a function of ψ

$$\delta_{JR} = \delta_{JR}(\psi) = \psi + \frac{(2\sigma-1)((1+\psi)\alpha(1-\beta(1-\gamma))}{\sigma(((1+\psi)c_y - \alpha(1-\beta(1-\gamma)) + \gamma\alpha)}$$

Differentiating $\delta_{JR}(\theta)$ with respect to ψ we find

$$\begin{aligned}
\frac{d\delta_{JR}}{d\psi} &= 1-\frac{(2\sigma-1)\alpha^2(1-\beta(1-\gamma))(1-\gamma)(1-\beta)}{\sigma(c_y(1-\beta(1-\gamma)\theta-\alpha(1-\gamma)(1-\beta))^2} \\
\frac{d^2\delta_{JR}}{d\theta^2} &= 2\frac{(2\sigma-1)\alpha^2c_y(1-\beta(1-\gamma))^2(1-\gamma)(1-\beta)}{\sigma(c_y(1-\beta(1-\gamma)\theta-\alpha(1-\gamma)(1-\beta))^3} > 0 \text{ if } \sigma > \frac{1}{2}
\end{aligned}$$

Hence $\delta_{JR}(\psi)$ is convex and putting $\frac{d\delta_{JR}}{d\psi}=0$ we arrive at:

Theorem 1

Restricting ourselves to the case $\sigma > \frac{1}{2}$, $\delta_{JR}(\psi)$ is bounded below at a value $\delta_{JR}(\underline{\psi})$ where

$$\underline{\psi} = \frac{\alpha\left[(1-\gamma)(1-\beta)+\sqrt{\frac{(2\sigma-1)}{\sigma}(1-\beta(1-\gamma))(1-\gamma)(1-\beta)}\right]}{c_y(1-\beta(1-\gamma))} - 1 > \psi^*$$

A sting in the tail arises if we introduce *external habit* with C_t in the utility function replaced by $C_t-\chi C_{t-1}$. Then c_y is replaced with $c_y(1-\chi)$ pushing the constraint on ψ into an implausible range. This we now show can be mitigated by making habit *internal rather than external.*

3.3 External versus Internal Habit

With external habit in consumption, household j has a single-period utility

$$\begin{aligned}
U_t^j &= \frac{(C_t^j-\chi C_{t-1}-\varrho(H_t^j)^{1+\psi}X_t^j)^{1-\sigma}}{1-\sigma}; \quad \chi\in[0,1) \\
X_t^j &= (C_t^j-\chi C_{t-1})^{\gamma}(X_{t-1}^j)^{1-\gamma}; \quad \gamma\in[0,1]
\end{aligned}$$

where C_{t-1} is aggregate per capita consumption whereas with internal habit we have

$$\begin{aligned}
U_t^j &= \frac{(C_t^j-\chi C_{t-1}^j-\varrho(H_t^j)^{1+\psi}X_t^j)^{1-\sigma}}{1-\sigma}; \quad \chi\in[0,1) \\
X_t^j &= (C_t^j-\chi C_{t-1}^j)^{\gamma}(X_{t-1}^j)^{1-\gamma}; \quad \gamma\in[0,1]
\end{aligned}$$

In a symmetric equilibrium, the household first-order conditions are as before with marginal utility

$$U_{C,t} = (C_t-\chi C_{t-1}-\varrho H_t^{1+\psi}X_t)^{-\sigma}$$

and for external habit and internal habit respectively we have

$$\begin{aligned}
\lambda_t &= U_{C,t}-\frac{\gamma\mu_tX_t}{(C_t-\chi C_{t-1})} \\
\lambda_t &= U_{C,t}-\beta\chi\mathbb{E}_t[U_{C,t+1}]-\gamma\left(\frac{\mu_tX_t}{(C_t-\chi C_{t-1})}-\beta\chi\mathbb{E}_t\left[\frac{\mu_{t+1}X_{t+1}}{(C_{t+1}-\chi C_t)}\right]\right)
\end{aligned}$$

The zero-growth steady state then becomes $U_C = (C(1-\chi) - \varrho H^{1+\psi} X)^{-\sigma}$, $\lambda = U_C - \frac{\gamma\mu X}{(C(1-\chi))}$ for external habit and $\lambda = U_C(1-\beta\chi) - \frac{\gamma(1-\beta\chi)\mu X}{(C(1-\chi))}$ for internal habit. These results lead to:

Theorem 2

The results of Theorem 1 apply to habit in consumption with c_y replaced with $c_y(1-\chi)$ for external habit and $c_y\frac{1-\chi}{1-\beta\chi}$ for internal habit

3.4 Numeral Illustration

Table 1 illustates the analysis. Parameter values are $\alpha = 0.7$, $c_y = 0.6$, $\beta = .99$, $\sigma = 2.0$ and stated values for γ, χ.[1] We can now assess the empirical plausibility of JR preferences with habit

γ	No Habit ($\chi = 0.0$)	External Habit ($\chi = 0.5$)	Internal Habit ($\chi = 0.5$)	External Habit ($\chi = 0.8$)	Internal Habit ($\chi = 0.8$)
0.5	1.046	3.094	1.066	9.230	1.128
0.1	1.668	4.335	1.694	12.340	1.774
0	4.774	10.549	4.832	27.870	5.005

Table 1: Lower Bound $\delta_{JR}(\underline{\psi})$ with Habit and Constraints on JR Preferences

in consumption. From our discussion in Section 2 we wish to calibrate ψ to hit an inverse elasticity $\delta_{JR} \in [1.89, 3.70]$ with a central value 2.50. From our numerical results for the lower bound $\delta_{JR}(\underline{\psi})$, this rules out external habit if we are to choose JR preferences that allow for only weak wealth effects (γ very small). But even without habit, or with internal habit, it is difficult to reconcile the extreme choice of γ almost zero with a Frisch elasticity within this empirical range.

4 A Resolution: Generalized JR Utility and Labor Wedge

To get round the empirical problem posed, we propose a generalization of JR preferences to replace (3) with a generalized form:

$$\begin{aligned} U_t(C_t, C_{t-1}, H_t, Z_t) &= \frac{\left((C_t - \chi C_{t-1})(1 - \varrho H_t^{1+\psi}\, Z_t)^{\theta}\right)^{1-\sigma}}{1-\sigma}; \quad \psi > 0, \theta \in (0,1] \\ Z_t \equiv \frac{X_t}{C_t - \chi C_{t-1}} &= Z_{t-1}^{1-\gamma}\left(\frac{C_{t-1} - \chi C_{t-2}}{C_t - \chi C_{t-1}}\right)^{1-\gamma}; \quad \gamma \in [0,1] \end{aligned}$$

which reduces to JR preferences with $\theta = 1$ and Cobb-Douglas preferences with $\varrho = \gamma = 1$ and $\psi = 0$. Proceeding as before the steady-state equilibrium (7) becomes

$$\varrho H^{1+\psi} = \frac{\alpha((1-\beta(1-\gamma))}{(\theta(1+\psi)c_y + (1-\theta)\alpha)(1-\beta(1-\gamma)) + \gamma\alpha\theta)}$$

[1]In fact $\gamma > 0$ is required for balanced growth, but γ can be very small.

where $c_y \equiv \frac{C}{Y}$. Again for a given H this pins down ϱ given the remaining parameters. As before a necessary condition for an equilibrium to exist is that $0 < \varrho H^{1+\psi} < 1$. Following some algebra this places the *same* bound on ψ given by (8), independent of θ, but the Frisch elasticity becomes

$$\delta_{JR} \quad = \quad \psi + \frac{(\sigma + \theta(\sigma-1))(1+\psi)\varrho H^{1+\psi}}{\sigma(1-\varrho H^{1+\psi})}$$

Then for $\theta \in (0, 1]$ Theorem 1 generalizes to:

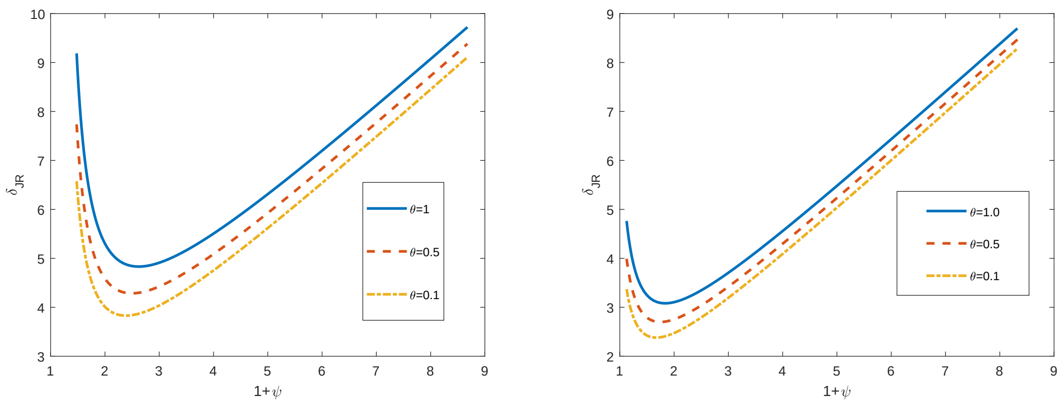
Theorem 3

Restricting ourselves to the case $\delta > \frac{1}{2}$, $\delta_{JR}(\psi)$ is bounded below at a value $\delta_{JR}(\underline{\psi}(\theta))$ where

$$\underline{\psi}(\theta) = \frac{\alpha\left[(1-\gamma)(1-\beta) + \sqrt{\frac{(\sigma+\theta(\sigma-1)}{\sigma}(1-\beta(1-\gamma))(1-\gamma)(1-\beta)}\right]}{c_y(1-\beta(1-\gamma))} - 1 > \psi^* \quad \theta \in (0, 1] \qquad (9)$$

Thus the lower bound on both ψ and δ_F *falls* as θ drops below its value $\theta = 1$ for JR preferences.

Our second modification is to introduce a labor wedge into the household problem (as in Shimer (2009). Then (6) becomes $\frac{U_{H,t}}{\lambda_t} = -\frac{U_{L,t}}{\lambda_t} = -W_t(1-\tau)$ where $\tau \in [0.27, 0.37]$ is the wedge and α in (9) is replaced with $\alpha(1-\tau)$. Figure 1 shows that a combination of our generalized JR utility and an empirically supported wedge, we can calibrate parameters to achieve the desired empirical value for δ_{JR}.



(a) $\gamma = 0$, $\chi = 0.5$, $\tau = 0$ (b) $\gamma = 0$, $\chi = 0.5$, $\tau = 0.3$

Figure 1: **Frisch Elasticity: Generalized JR Utility, Internal Habit and a Labor Wedge.**

5 Conclusions

This note has reviewed a utility function commonly used in RBC-DSGE models that is non-separable in habit-adjusted consumption and leisure, compatible with balanced growth and eliminates counterfactual wealth effects highlighted by Jaimovich and Rebello (2008). Our main contributions are first, Theorems 1 and 2 that highlight a constraint on the preference parameter ψ needed to target the Frisch elasticity. This leads to a lower bound for the latter that cannot be reconciled empirically with external habit. Even with internal or no habit, the range of possible values of the Frisch elasticity lie outside empirical results unless we allow for a modest wealth effect. Second, in Theorem 3 we propose a generalized JR utility function that together with the introduction of a labor wedge, as proposed by Shimer (2009), solves the problem.

References

Abel, A. B. (1999). Risk premia and term premia in general equilibrium. *Journal of Monetary Economics*, **43**, 3–33.

Bilbiie, F. (2011). Nonseparable preferences, Frisch labor supply and the consumption multiplier of government spending: one solution to the fiscal policy puzzle. *Journal of Money, Credit and Banking*, **43**(1), 221–251.

Carroll, Christopher D., J. O. and Weil, D. N. (2000). Saving and growth with habit formation. *American Economic Review*, **90**(3), 341–355.

Christiano, L., Eichenbaum, M., and Evans, C. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, **118**(1), 1–45.

Jaimovich, N. and Rebello, S. (2008). News and business cycles in open economies. *Journal of Money, Credit and Banking*, **40**(8), 1699–1710.

Keane, M. P. (2011). Labor supply and taxes: a survey. *Journal of Economic Literature*, **494**, 961–1075.

King, R., Plosser, C., and Rebelo, S. (1988). Production, growth and business cycles I: The basic neoclassical model. *Journal of Monetary Economics*, **21**, 195–231.

Peterman, W. B. (2016). Reconciling micro and macro estimates of the Frisch labor supply elasticity. *Economic Inquiry*, **54**(1), 100–120.

Reichling, F. and Whalen, C. (2017). Review of stimates of the Frisch elasticity of labor supply. *Eastern Economic Journal*, **43**(1), 37–42.

Shimer, R. (2009). Convergence in macroeconomics: The labor wedge. *American Economic Journal: Macroeconomics*, **1**(1), 280–297.