Common stochastic trends, cycles and sectoral fluctuations: a study of output in the UK*

Anthony Garratt

(Bank of England)

and

Richard G. Pierse

(University of Surrey)

February 1996
(revised May 1996)

Abstract

Two alternative methodologies are compared for identifying common trends and cycles in a set of variables. One, following Harvey, uses an unobserved components structural time series model. The other, following Vahid and Engle, is based on a multivariate Beveridge-Nelson decomposition. Both approaches are applied to a four sector model of output in the UK over the period 1970Q1-1993Q2, producing different trends and cycles. The cycles from the unobserved components model correspond more closely to a reference cycle for aggregate output.

JEL Classification: C32, E32.
Keywords: Cointegration , common trends, common cycles, cofeatures, dissaggregation.

* The authors would like to thank Rob Engle, Clive Granger, Andrew Harvey, João Issler, Siem Koopman, Hashem Pesaran, Neil Shephard, Farshid Vahid, and participants at the Seventh World Congress of the Econometric Society in Tokyo for helpful comments. All errors and omissions are the sole responsibility of the authors.

1 Introduction

The idea of decomposing a variable into trend and cycle components has a long history in economics and the variable that has received the most attention is output. Since the influential work of Nelson and Plosser (1982), it has generally been accepted that output has a unit root so that the trend is stochastic rather than deterministic and the cycle is present in the stationary first difference. Several univariate empirical studies have followed: Campbell and Mankiw (1987) estimated some simple ARIMA processes for US GNP while Harvey (1985), Watson (1986) and Clark (1987) all used Unobserved Components (UC) models to identify trend and cycle.[1] Other studies have adopted a multivariate approach, using additional information from other aggregate variables to help identify the trend and cycle in aggregate output; for example Kydland and Prescott (1988) detrended US GNP and the price level and then showed that there is a negative relationship between the resulting cycles. For other examples see Clark (1989), Evans (1989), Blanchard and Quah (1989), King, Plosser, Stock and Watson (1991) and Evans and Reichlin (1994).

Another source of additional information is that provided by sectoral output data which might for theoretical reasons be expected to move together. Long and Plosser (1983) develop a multisectoral version of a real business cycle model and show that, even if random productivity shocks are independent across sectors, the choices of agents will cause comovement of activity measures for different sectors. Long and Plosser (1987) decompose US output innovations into unobserved common factors or aggregate shocks and a set of independent disturbances unique to each sector. Their results suggest that the explanatory power of a common aggregate disturbance for industrial outputs is significant, but are not very large for most industries. More generally the importance of sectoral information has been shown, for example, by Pesaran, Pierse and Lee (1993) and Lee, Pesaran and Pierse (1992) who estimate multisectoral VAR models of output growth for the US and the UK respectively and investigate the effects of specific identified macroeconomic shocks and unidentified sectoral shocks on output persistence.

The joint analysis of more than one variable allows the possibility that trends and cycles may be common between variables. In fact, as demonstrated by Stock and Watson (1988b), if a set of n variables are cointegrated with r cointegrating vectors, then this implies that there are n-r common trends between them. In a recent paper, Engle and Kozicki (1993) have introduced the general concept of common features, which are defined to be data features that are present in individual series but absent from some linear combination of those series. Cointegration is one example but another common feature of particular interest is that of common serial correlation and this implies common cycles. Engle and Kozicki develop a test for the cofeature rank which is analogous to the Johansen (1988) test for the number of cointegrating vectors. If the cofeature rank is s, then this implies n-s common features.

Vahid and Engle (1993) use the framework of the Beveridge-Nelson-Stock-Watson (BNSW) decomposition to identify common trends and cycles. When the number of cointegrating vectors plus the number of common serial correlation features is equal to the number of variables, then this framework allows a very easy recovery of trend and cycle components. Engle and Issler (1995) apply this approach to sectoral output for the US and Calcagnini (1995) to labour productivity for 6 different countries. An alternative approach is provided by the multivariate UC models of Harvey (1989) and Koopman *et al*. (1995) which allow for the imposition of common trends and cycles. One distinction between the two approaches lies in the identifying restrictions used to construct the trend and cycle. The UC approach ensures identification by imposing zero correlation between trend and cycle whereas in Engle and Vahid's approach (EV) this corelation can be non-zero and can actually be estimated. Other significant differences lie in the flexibility each approach allows for the form that the trend and cycle can take. The UC model allows the trend to have a time varying slope, whereas the trend in the EV

[1] A useful survey of this literature is provided by Stock and Watson (1988a).

model is a random walk with drift. However the cycle in the EV model comprises all serial correlation once the trend component has been removed, whereas the UC cycles used by Harvey have an explicit trigonometric representation and are constrained so that the cycle for each variable has the same frequency and damping factor.

In this paper these two approaches are compared and both are applied to a four sector model of quarterly output for the UK. The application provides a comparison with the results of Engle and Issler (1995) who apply the EV model to US sectoral data using 8 sectors and annual data. A plan of the paper is as follows: Section 2 develops both the EV and UC models. Section 3 applies both models to a four sector model of UK output over the period 1970Q1 to 1993Q2. The trends and cycles that arise from the two alternative methodologies are compared. Section 4 presents the conclusions.

2 Methodology

The starting point of this analysis is the trend plus cycle model

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \boldsymbol{\psi}_t \quad , \quad t = 1, \cdots, T \tag{2.1}$$

where the $n \times 1$ vector of variables $\mathbf{y}_t$ is decomposed into two components: a trend, $\boldsymbol{\mu}_t$, and a cycle, $\boldsymbol{\psi}_t$. This representation of a series as the sum of a trend and a cycle has some intuitive appeal and has a long history in statistical modelling. However, there is no unique way to accomplish the decomposition and so several different methods have been suggested, based on different assumptions, and producing models with quite different properties. Two approaches are considered here: one by Engle and Vahid, henceforth denoted as EV, based on the Stock-Watson-Beveridge-Nelson decomposition, the other from a multivariate Unobserved Components model of Harvey (1989) and Koopman *et al.* (1995), henceforth denoted as UC.

2.1 The EV model

Stock and Watson (1988a) provide a multivariate generalisation of the Beveridge and Nelson (1981) decomposition of an ARIMA model into trend and cycle components where the innovations in the two components are perfectly correlated. The starting point is the infinite moving average (Wold) representation of the stationary first difference of $\mathbf{y}_t$:

$$\Delta \mathbf{y}_t = \mathbf{C}(L)\, \mathbf{e}_t = \mathbf{C}(1)\mathbf{e}_t + (1-L)\mathbf{C}^*(L)\, \mathbf{e}_t$$

where, in the second equality, the long run effect of shocks has been separated from the rest. Integrating up this equation defines the Stock-Watson-Beveridge-Nelson (SWBN) decomposition:

$$\mathbf{y}_t = \mathbf{C}(1)\sum_{i=0}^{\infty}\mathbf{e}_{t-i} + \mathbf{C}^*(L)\, \mathbf{e}_t \quad . \tag{2.2}$$

In (2.2) the first term is the trend and the second term, which is stationary, is the cycle. Innovations in the two components are perfectly correlated.

2.2 The Unobserved Components Model

In contrast, in the unobserved components structural models of Harvey (1985, 1989) and Koopman *et al*. (1995) the innovations in trend and cycle are constructed to be independent. The starting point is slightly more general than (2.1) in that it allows an additional component in the equation:

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \boldsymbol{\psi}_t + \boldsymbol{\varepsilon}_t \quad , \quad t = 1, \cdots, T \tag{2.3}$$

where $\boldsymbol{\varepsilon}_t$ is an irregular component with covariance matrix $\boldsymbol{\Sigma}_e$ which is not explained by the model. Separate unobserved components models are built of each of the first two components of (2.3). The trend component is modelled by the local linear trend

$$\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t \tag{2.4}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\xi}_t \tag{2.5}$$

where $\boldsymbol{\mu}_t$ and $\boldsymbol{\beta}_t$ are $n \times 1$ vectors representing the level and slope of the trend respectively, and $\boldsymbol{\eta}_t$ and $\boldsymbol{\xi}_t$ are independent error processes with covariance matrices $\boldsymbol{\Sigma}_\eta$ and $\boldsymbol{\Sigma}_\xi$. In the general case the trend process is second order integrated, $(I(2))$. The slope parameter $\boldsymbol{\beta}_t$ allows this trend to change smoothly but in the special case where $\boldsymbol{\Sigma}_\xi$ is zero, the trend reduces to a random walk with drift term $\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} = \overline{\boldsymbol{\beta}}$. If, in addition, $\boldsymbol{\Sigma}_\eta$ is zero, then the trend becomes deterministic.

The cycle component of the model is trigonometric in form and consists of one or more cycles defined by the pair of equations:

$$\begin{bmatrix} \boldsymbol{\psi}_t \\ \overline{\boldsymbol{\psi}}_t \end{bmatrix} = r \begin{bmatrix} \cos l \, \mathbf{I}_n & \sin l \, \mathbf{I}_n \\ -\sin l \, \mathbf{I}_n & \cos l \, \mathbf{I}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{\psi}_{t-1} \\ \overline{\boldsymbol{\psi}}_{t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}_t \\ \overline{\boldsymbol{\omega}}_t \end{bmatrix} \tag{2.6}$$

where $\boldsymbol{\psi}_t$ and $\overline{\boldsymbol{\psi}}_t$ are $n \times 1$ vectors and $\boldsymbol{\omega}_t$ and $\overline{\boldsymbol{\omega}}_t$ are vector error processes independent of $\boldsymbol{\varepsilon}_t$, $\boldsymbol{\eta}_t$ and $\boldsymbol{\xi}_t$ and with the same covariance matrix $\boldsymbol{\Sigma}_\omega = \boldsymbol{\Sigma}_{\overline{\omega}}$, $\mathbf{I}_n$ is the identity matrix of dimension *n* and the vector process $\overline{\boldsymbol{\psi}}_t$ appears by construction. The scalar parameters r and l (which satisfy the restrictions $0 < r < 1$ and $0 < \lambda < \pi$) represent the cycle damping factor and frequency respectively. In the univariate case ($n = 1$), equation (2.6) corresponds to a restricted *ARMA(2,1)* process where the two autoregressive roots form a complex conjugate pair.[2]

When there is more than one variable, it can be seen from (2.6) that the cycle damping factor and frequency r and l are imposed to be the same for each variable. In the terminology of Koopman *et al.* (1995), this is the assumption of *similar* cycles, and it implies that the cycles for different variables have the same properties - the same autocovariance function and spectrum. This is a very strong assumption and imposes quite a serious restriction on the model, although it does have the advantage of limiting the number of parameters that have to be estimated.

2.3 Cointegration and Cofeatures

Suppose that the $\mathbf{y}_t$ process in (2.1) is cointegrated, having *r* cointegrating vectors given by the $(n \times r)$ matrix $\boldsymbol{a}$. Then, by the definition of cointegration, the vector $\boldsymbol{a}'\mathbf{y}_t$ will be *stationary*. (See Engle and Granger (1987)). Furthermore, Stock and Watson (1988b) showed that this implies that the *n* variables share *n-r* common trends.

[2] It is possible to model more complex dynamics by allowing several cycles of different frequencies, thus allowing a more general specification for the cycle. This is not pursued further here.

Engle and Kozicki (1993) introduced the concept of common features, which are data features that are present in individual series but absent from a linear combination of those series. In particular Vahid and Engle (1993) looked at the feature of common serial correlation among variables which, if it exists, implies common cycles. They used a test developed by Engle and Kozicki to test the cofeature rank. This test is based on canonical correlation analysis along the lines of the Johansen (1988) test for the number of cointegrating vectors. If the serial correlation cofeature rank is *s*, then this implies *n-s* common cycles.

2.4 Common Trends and Cycles in the EV Model

In equation (2.2) cointegration implies that $\boldsymbol{a}\,'\mathbf{C}(1)=\mathbf{0}$ so that $\mathbf{C}(1)$ can be written as the product $\mathbf{C}(1)=\boldsymbol{d}\,\boldsymbol{g}\,'$ where $\boldsymbol{d}$ and $\boldsymbol{g}$ are both $(n\times n-r)$ matrices and $\boldsymbol{a}\,'\boldsymbol{d}=\mathbf{0}$. Then the first term in (2.2) becomes

$$\mathbf{C}(1)\sum_{i=0}^{\infty}\mathbf{e}_{t-i}=\boldsymbol{d}\;\boldsymbol{g}\,'\sum_{i=0}^{\infty}\mathbf{e}_{t-i}=\boldsymbol{d}\;\boldsymbol{t}_{t} \qquad (2.7)$$

where $\boldsymbol{t}_{t}$ is a vector of common trends that follows the random walk process

$$\boldsymbol{t}_{t}=\boldsymbol{t}_{t-1}+\boldsymbol{g}\,'\mathbf{e}_{t}\,.$$

Similarly, if the sectors exhibit common serial correlation features then, for some $(n\times s)$ matrix of common features, $\tilde{\boldsymbol{a}}$, it follows that $\tilde{\boldsymbol{a}}\,'\mathbf{C}^{*}(L)=\mathbf{0}$
and

$$\mathbf{C}^{*}(L)=\mathbf{\Phi}\boldsymbol{l}\;(L)$$

where $\mathbf{\Phi}$ is an $(n\times n-s)$ matrix of coefficients with $\tilde{\boldsymbol{a}}\,'\mathbf{\Phi}=\mathbf{0}$ and $\boldsymbol{l}\;(L)$ is an $(n-s\times n)$ matrix in the lag operator. Then the second term in (2.2) becomes

$$\mathbf{C}^{*}(L)\mathbf{e}_{t}=\mathbf{\Phi}\boldsymbol{l}\;(L)\,\mathbf{e}_{t}=\mathbf{\Phi}\boldsymbol{c}_{t} \qquad (2.8)$$

where $\boldsymbol{c}_{t}$ is a vector of common cycles.

The set of cofeature vectors $\tilde{\boldsymbol{a}}$ must be linearly independent of the cointegration vectors $\boldsymbol{a}$ and it has to be the case that $\mathrm{r+s\le n}$. If $\mathrm{r+s<n}$ then the trend and cycle decomposition in the Vahid and Engle framework is not unique. However, in the special case that $r+s=n$, the matrix

$$A=\begin{bmatrix}\tilde{\boldsymbol{a}}\,'\\ \boldsymbol{a}\,'\end{bmatrix}$$

is square and of full rank, with a conformably partitioned inverse defined by

$$A^{-1}=\begin{bmatrix}\tilde{\boldsymbol{a}}^{-} & \boldsymbol{a}^{-}\end{bmatrix},$$

and it follows that

$$y_{t}=A^{-1}Ay_{t}=\tilde{\boldsymbol{a}}^{-}\tilde{\boldsymbol{a}}\,'y_{t}+\boldsymbol{a}^{-}\boldsymbol{a}\,'y_{t}=\mathbf{P}\boldsymbol{d}\;\boldsymbol{t}_{t}+(\mathbf{I}-\mathbf{P})\,\mathbf{\Phi}\boldsymbol{c}_{t} \qquad (2.9)$$

where $\mathbf{P}\equiv\tilde{\boldsymbol{a}}^{-}\tilde{\boldsymbol{a}}\,'$ is an idempotent projection matrix and $\boldsymbol{a}^{-}\boldsymbol{a}\,'\equiv\mathbf{I}-\mathbf{P}$. Thus the condition $r+s=n$ allows a simple and unique decomposition that is independent of the normalisation of the cointegration and cofeature vectors.

2.5 Common Trends and Cycles in the UC Model

Common trends in the general UC model of (2.4)-(2.6) can arise either through common levels, or common slopes or both. For comparison with the EV model, here only the special case is considered where the slope parameter is fixed so that $\boldsymbol{\Sigma}_x = \mathbf{0}$ and $\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} = \bar{\boldsymbol{\beta}}$. In this case, the trend and hence $\mathbf{y}_t$ will be integrated of order one and common trends implies that

$$\boldsymbol{\mu}_t = \Theta_m \tilde{\boldsymbol{\mu}}_t + \boldsymbol{\mu}_0 \qquad (2.10)$$

where $\tilde{\boldsymbol{\mu}}_t$ is the $(n-r\times 1)$ vector of *common trends*, $\boldsymbol{\mu}_0$ is a vector of fixed values and Θ_m is an $(n\times n-r)$ factor loading matrix satisfying the restriction $\boldsymbol{a}'\Theta_m = \mathbf{0}$. The variance covariance matrix $\boldsymbol{\Sigma}_\eta$ will also be singular.
Similarly, the existence of common cycles in (2.6) implies that

$$\boldsymbol{\psi}_t = \Theta_y \tilde{\boldsymbol{\psi}}_t \qquad (2.11)$$

where $\tilde{\boldsymbol{\psi}}_t$ is the $(n-s\times 1)$ vector of *common cycles* and the $(n\times n-s)$ factor loading matrix Θ_y satisfies the restriction $\tilde{\boldsymbol{a}}'\Theta_y = \mathbf{0}$. Note that no vector of fixed values is needed here because the cycle has zero mean.
The system (2.3)-(2.6) can then be rewritten in terms of the common trends and cycles as

$$\mathbf{y}_t = \Theta_m \tilde{\boldsymbol{\mu}}_t + \boldsymbol{\mu}_0 + \Theta_y \tilde{\boldsymbol{\psi}}_t + \boldsymbol{\varepsilon}_t \quad , \quad t = 1,\cdots,T$$

$$\tilde{\boldsymbol{\mu}}_t = \tilde{\boldsymbol{\mu}}_{t-1} + \tilde{\boldsymbol{\beta}} + \tilde{\boldsymbol{\eta}}_t \qquad (2.12)$$

$$\begin{bmatrix} \tilde{\boldsymbol{\psi}}_t \\ \tilde{\tilde{\boldsymbol{\psi}}}_t \end{bmatrix} = r \begin{bmatrix} \cos l\, \mathbf{I}_n & \sin l\, \mathbf{I}_n \\ -\sin l\, \mathbf{I}_n & \cos l\, \mathbf{I}_n \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{\psi}}_{t-1} \\ \tilde{\tilde{\boldsymbol{\psi}}}_{t-1} \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{\omega}}_t \\ \tilde{\tilde{\boldsymbol{\omega}}}_t \end{bmatrix}$$

where the error processes $\tilde{\boldsymbol{\eta}}_t$, $\tilde{\boldsymbol{\omega}}_t$ and $\tilde{\tilde{\boldsymbol{\omega}}}_t$ are of dimensions $n-r\times 1$, $n-s\times 1$, and $n-s\times 1$ respectively with nonsingular covariance matrices $\boldsymbol{\Sigma}_{\tilde{h}}$ and $\boldsymbol{\Sigma}_{\tilde{w}} = \boldsymbol{\Sigma}_{\tilde{\tilde{w}}}$. $\tilde{\boldsymbol{\beta}}$ is an $n-r\times 1$ vector of drift terms in the common trends. The system (2.12) is the version of the UC model with common trends and cycles that is used in the estimation in Section 3.

The factor loading matrices Θ_m and Θ_y in (2.12) are not uniquely defined unless some conditions are imposed to ensure identification. The standard identification restrictions impose lower triangularity so that $\Theta_{ij} = 0$, for $\forall \mathrm{j} > \mathrm{i}$ and $\Theta_{ii} = 1$, for $\forall i$, and the associated covariance matrices $\boldsymbol{\Sigma}_{\tilde{h}}$ and $\boldsymbol{\Sigma}_{\tilde{e}}$ are diagonal. The last condition ensures that the common trends (cycles) are all uncorrelated with each other and the first two conditions imply that only the first common trend (cycle) affects the first sector, the first two common trends (cycles) the second sector and so on for the first $n-r$ $(n-s)$ of the n sectors. These restrictions merely ensure identification; once the model parameters have been estimated, the common trends and cycles can then be transformed by premultiplication by any orthogonal matrix. This is called *factor rotation* and may allow the transformed common factors to be given a more useful interpretation.

3. The Results

The methodologies outlined in the previous section are applied to model Gross Domestic Product (GDP) for the United Kingdom disaggregated into four sectors (listed in order of size with their 1994 weight in parentheses): Services (629), Production (280), Construction (72) and Agriculture (19). These sectors represent the highest level breakdown of the UK Standard Industrial Classification. The data come from Table 2.5 of the Central Statistical Office (CSO) Blue Book; the observations are quarterly seasonally adjusted indices at constant factor cost (1990=100) for the period 1970Q1 to 1993Q2 (94 observations).

3.1 Time series properties of the data

The data are plotted in natural logarithms as the unbroken lines in Figures 1 and 2, and all analysis uses the logarithmic transformation. The largest sector, Services, is the most smoothly trended series and the one that most closely resembles aggregate GDP (of which it forms 63%). Production is noticeably more volatile than Services and shows a flatter trajectory for the 1970s, with clear falls in output in the recession of the early 1980s.[3] Construction is even more volatile, and despite having the general appearance of an upward trend, shows periods of sharp decline in the late 1970s and early 1980s. The smallest sector, Agriculture, is clearly trended but displays some volatility.

Table 1: Orders of Integration: Augmented Dickey-Fuller Tests

Sector	*Levels*		*First differences*	
Services	-0.99	-2.53	-2.87	-2.91
Production	-1.36	-1.87	-4.73	- 4.72
Construction	-1.60	-2.18	-3.02	-2.99
Agriculture	-1.09	-1.83	-4.50	-4.48

Notes: The first column is the 4th order Dickey-Fuller statistic, ADF(4), without time trend, the second with time trend. The 95% critical value without time trend is -2.87, and with time trend is -3.436. The sample period is 1970Q1-1993Q2.

The first step in a more formal analysis is to test the order of integration of the series through augmented Dickey-Fuller tests. A likelihood ratio test on an unrestricted VAR suggested an optimal lag length of 4 and all remaining tests are contructed using this assumption. Table 1 reports the results of ADF(4) tests for each sector both in levels and first differences, and computed both with and without an included time trend. In all cases the sectors fail to reject a unit root in levels. In first differences, the unit root null hypothesis is clearly rejected in all sectors except Services, which appears to be borderline I(2). However, ADF statistics for lag lengths one through three for this sector clearly reject a unit root in first differences as do tests computed using longer sample periods. All four sectors are thus taken to be I(1).

Turning to examine the cointegrating properties of the data using the Johansen methodology, the first step is to identify the appropriate model. A LR test of whether to include a constant in the VAR (equivalently a time trend in the levels formulation) gave a value of 27.8 (4 degrees of freedom) which rejects the null of no constant. A further test of the restriction that the constant is present only inside the error correction term gave a LR statistic of 21.2 which rejects the restriction. The model chosen thus included a unrestricted constant in the VAR.

[3] The two sharp falls in the 1970s correspond to a mining strike in 1972Q1 and the three day week of 1974Q1. To take account of these two events, two dummy variables were created and used in all subsequent analysis.

Table 2a. Cointegration Analysis: Johansen Tests

Null	*Alternative*	*Test statistic*	*95% Critical Value*	*90% Critical Value*
r = 0	r = 1	32.27	27.07	24.78
r <= 1	r = 2	11.23	20.97	18.90
r <= 2	r = 3	6.21	14.07	12.91
r <= 3	r = 4	2.83	3.76	6.50

Notes: 90 observations from 1971Q1to 1993Q2. Maximum lag in the VAR=4. Unrestricted intercept model. Cointegration LR test based on maximal eigenvalues of the stochastic matrix.

The results of the Johansen analysis are reported in Table 2a. They indicate one cointegrating vector and hence three common stochastic trends.

The possibility of common (synchronous) cycles was then investigated using the cofeature test described in Engle and Kozicki (1993). This is based on the canonical correlations of the first differences of the data with their lags (four) and the error correction term lagged once. The squared canonical correlations and the value of the test statistic for the number of cofeature vector are reported in Table 2b.

Table 2b. Cofeature Analysis: Vahid and Engle tests

Null	*Squared correlation*	*Test statistic*	*Degrees of freedom*	*p-value*
s > 0	0.119	10.88	10	0.3672
s > 1	0.231	33.48	22	0.0554
s > 2	0.383	75.03	36	0.0001
s > 3	0.411	120.53	52	0.0000

Notes: 90 observations from 1971Q1 to 1993Q2. Maximum lag in the VAR=4.

At the 5 per cent level of significance, the test suggests two cofeature vectors implying two common cycles. The conclusion of the data analysis is therefore that the four sectors exhibit three common trends and two common cycles. This is a problem since it does not satisfy the restriction ($r + s = n$) that allows the simple Vahid and Engle decomposition and in order to make a comparison between the two approaches we are therefore obliged to violate one of the results of the data analysis. We chose to accept the results of the cointegration analysis but to impose a single common cycle. This is consistent with Engle and Issler (1995) who find many sector specific trends but relatively few cycles for the US. All subsquent analysis is therefore based on the assumption that the four sectors exhibit three common trends and one common cycle.[4]

3.2 Constructing Sectoral Trends and Cycles

This section compares the sets of sectoral trends and cycles which result from the two models outlined in Section 2, constructed imposing the restriction of three common trends and one common cycle. In both models, the trend is a random walk with drift, enabling a direct comparison of the two approaches.

[4] We also estimated the UC model with the data consistent common factor restrictions of three common trends and two common cycles. The results were very similar to those reported in the text.

The main differences between the two approaches are the identifying assumptions and the definition and specification of the cycle.[5]

3.2.1 EV Model Results

Figure 1 plots the EV trends along with the actual series. The trends in all sectors fit reasonably closely with the possible exception of Services where the series itself is relatively smooth but the estimated trend is much more variable. More generally, as was found by Engle and Issler (1995) for US sectoral output, the estimated trends are more volatile than the actual series, which is arguably an undesireable feature.[6]

Figure 3 graphs the estimated EV cycles. As there is only one common cycle, the cyclical pattern is the same in all sectors although the amplitudes are different and in Construction and Agriculture the pattern is inverted. Construction has the highest variance (0.0020), compared to 0.0011 and 0.0016 for Production and Services respectively. This ranking is consistent with the conventional view of Construction as the most volatile sector. Agriculture has a very small variance (0.00022) which is a little surprising.

An interesting question is how far do the sectoral cycles match cyclical movements in aggregate economic activity. For the UK, there is no established set of dated cycles or turning points in aggregate output (unlike the NBER reference cycle for the US) so that a number of reference points have to be used. Quah (1994) identified three peaks at 1974Q3, 1979Q4, and 1990Q1. Artis, Bladen-Hovell and Zhang (1994) identified peaks (P) and troughs (T) in months 1972M2(T), 1973M6(P), 1975M8(T), 1979M6(P), 1981M5(T), 1984M1(P), 1984M8(T), 1989M4(P) and 1992M5(T). The synchronisation of these two sets of peaks is not exact but does give a reasonable indication of turning points, which ought to some degree be reflected in the disaggregate sector cycles.[7]

Generally the timing of the peaks and troughs of the EV cycles does not correspond particularly well with the reference points. For example although Services and Production do show a marked peak in 1974, this corresponds to a trough for Construction and Agriculture. The later reference peaks of Quah (1994) are not apparent in any of the sectors. Focusing on the two largest sectors, Services and Production, the EV cycles suggest big troughs in 1976 and 1983/4 with a peak in 1985. None of these turning points matches the reference turning points.

In summary, the EV trends perform reasonably well but the cycles do not pick up reference turning points. This may be because of the strong assumption of a single common cycle that was required to perform the decomposition.

3.2.2 UC Model Results

.

For the UC approach the model (2.12) was estimated imposing three common trends and one common cycle. Table 3 reports the results of the maximum likelihood estimation of the model: the coefficients of the factor loading matrices, Θ_m, Θ_y and μ_0, and the standard deviations of the error terms on trend, cycle and irregular components.

---

[5] The calculations for the EV model were programmed in the GAUSS language while the UC model results come from Version 5.0 of the computer package Stamp (Koopman *et al.* (1995)). We are grateful to Andrew Harvey for allowing us access to a pre-release version of this program.

[6] This is attributable to a negative covariance between trend and cycle.

[7] The approximate dating of these turning points is also consistent with the timing of peaks in the CSO coincident index of economic activity.

Table 3*: Maximum likelihood estimation of UC Model (2.12)*

Table 3a*: Common Trend Components*

Sector	Θ_m			μ_0	$s(h)$	$s(e)$
Services	1.0	-	-	-	0.00719	0.0000
Production	1.275	1.0	-	-	0.01327	0.0008
Construction	2.278	0.349	1.0	-	0.02538	0.0079
Agriculture	-0.360	-1.628	0.632	7.694	0.01990	0.0003

Table 3b*: Common Cyclical Components*

Sector	Θ_y	$s(w)$
Services	1.0	0.00020
Production	35.3	0.00705
Construction	2.16	0.00043
Agriculture	70.40	0.01406

Notes: Estimation period: 1970Q1- 1993Q2; Model log-likelihood is 1504.47;
Cycle Frequency= 0.34153 Cycle damping factor ρ = 0.89795; Period of cycle: 4.60 years.
Θ_m , Θ_y *are estimated coefficients of factor loading matrices of trend and cycle respectively*
μ_0 *are estimated coefficients of the trend fixed values*
$s(h)$, $s(e)$ *and* $s(w)$ *are standard deviations of the error on the trend level, irregular and cyclical components respectively.*

Figure 2 plots the UC sectoral trends against the actual series.[8] The Services sector is characterised by a smooth near-linear trend, similar but less variable than the trend from the EV model. The trends for Services and Construction fit the series almost perfectly whereas there is a close but less perfect fit for Production and Agriculture. Two differences are apparent when comparing these trends with those of Figure 1. Firstly, they fit the the actual series more closely than the EV trends. Secondly, in all sectors they show less variability than the actual series. The ranking of the variance appears consistent with previous stylised facts with Table 3a reporting that the most volatile trend is Construction, with a component standard error of 0.0254 and the smoothest trend, Services, has the smallest estimated standard error of 0.0072.

The UC cycles are plotted in Figure 3, alongside the EV cycles. The common damping factor, ρ , is 0.898 and the cycles have a period of 4.6 years, which corresponds quite well with a standard business cycle period. The variances of the cycles vary, but in two cases are considerably smaller than those of the EV cycles and only in Agriculture are they larger. Construction has the largest variance (0.00537), followed by Agriculture (0.00476), Production (0.00131) and Services (0.00070). The relative ordering of these variances is more intuitively plausible than that from the EV cycles.

Figure 4 presents the UC cycles together with the EV cycles, rescaled so that they both have the same variance. This enables a direct comparison of the timing of the turning points in economic activity implied by the both sets of cycles. Since there is only one common cycle in the model and the loading matrix coefficients are all positive, all sectors in the UC model exhibit the same cyclical pattern, and this is quite similar to the EV cycles for Production and Services (although the variance is very different in the case of the Services sector). Conversely, the pattern of the UC cycles for Construction and Agriculture is the mirror image of that for the EV cycles. As a result, with the UC cycles, all

[8] All graphed components for the UC model are of smoothed Kalman filter estimates.

sectors show the marked peak in 1974 corresponding to the reference peak classified by both Artis, Bladen-Hovell and Zhang and Quah. This is followed by big troughs in 1976 and 1983/4 and then peaks in 1985 and 1993. None of these latter turning points matches the reference turning points very closely and they miss the 1990Q1 peak in aggregate economic activity identified in Quah (1994) or the 1989M4 peak classified by Artis, Bladen-Hovell and Zhang (1994).

In the UC model each sector also has an irregular component. However, in all cases, Table 3a shows that this has an extremely small variance compared with that of the other components, suggesting that the trend and cycle explain almost all of

Using the criteria of closeness of fit to the actual series and the volatility of the trend compared to the series, the trends derived from the UC approach seem more satisfactory than those from the EV approach. This is also reflected in the UC cycles whose variances are more plausible. The peaks and troughs are the same for all sectors and correspond a little more closely to the reference turning points.

The UC model also has an irregular component

4 Conclusions

This paper has compared the Vahid-Engle and Unobserved Components methodologies for decomposing a set of variables into trend and cycle components, allowing for common trends and common cycles. The approaches were applied to a four sector disaggregation of output for the UK, with three common trends and one common cycle. The results, derived using models with the same trend specification, but with different identifying restrictions, generate trends and cycles with quite different properties, with the UC trends fitting the original series more closely and the cycles having the same pattern but different variances, with peaks and troughs matching some of a set of reference turning points.

The results provide evidence on the relative importance of permanent versus transitory shocks. For the largest sector Services and also for Construction and Agriculture, transitory shocks dominate permanent shocks, with the ratio of the standard errors of the trend innovations to the cycle innovations being 0.04, 0.81 and 0.46 respectively. However, for the Production sector, which forms 28% of aggregate GDP, the ratio is 1.47 so the most important innovations in Production output are permanent in nature. These results show that, on the whole, transitory shocks are more important than permanent shocks although permanent shocks have a significant role in one important sector. These ratios are larger than the value of 0.9 reported by Clark (1987) using the UC framework applied to aggregate US GDP but considerably lower than ratios in the range of 5 to 6 reported by Nelson and Plosser (1982) using a Beveridge-Nelson decomposition. Engle and Issler (1995), looking at sectoral US GDP, are less clear on the relative importance of permanent versus transitory shocks.

The focus of this paper has been deliberately narrow. An interesting question would be to investigate the extent to which the common trends and cycles which we find can be associated with specific macroeconomic events such as oil price, productivity and nominal money shocks. This will be the subject of further research.

Figure 1: Sectoral Trends from the Vahid and Engle Model

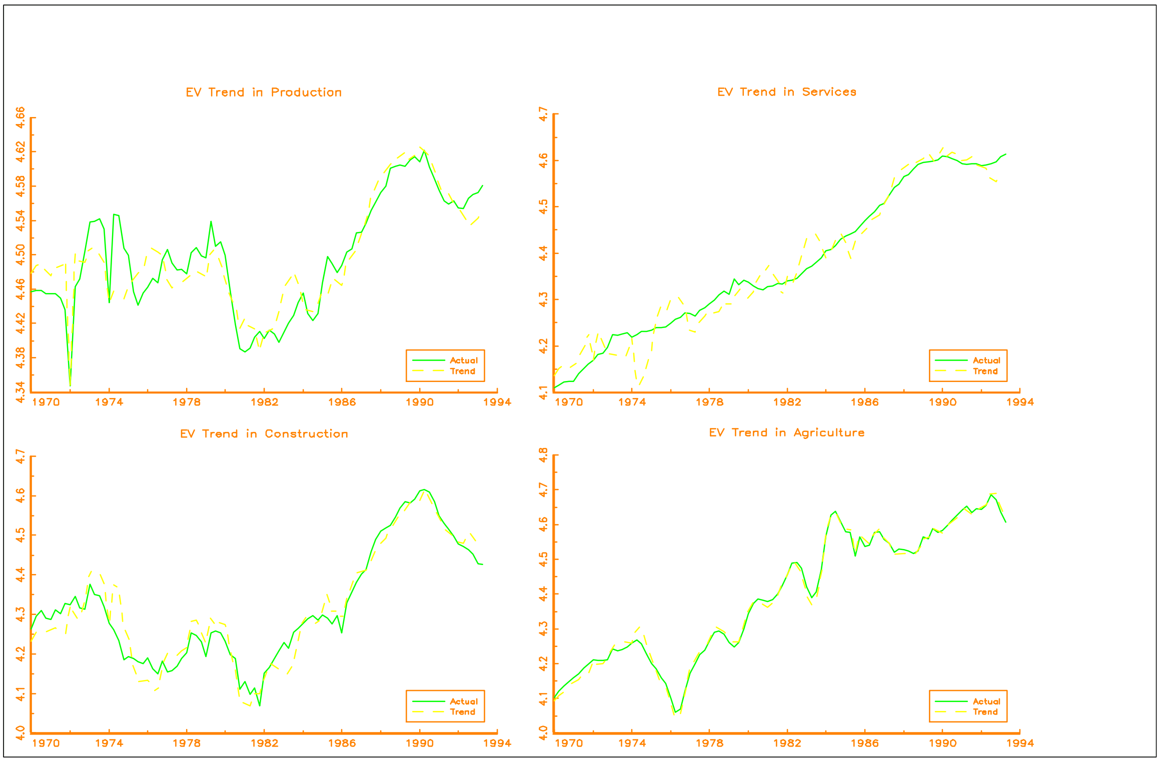


Figure 2: Sectoral Trends from the Unobserved Components Model

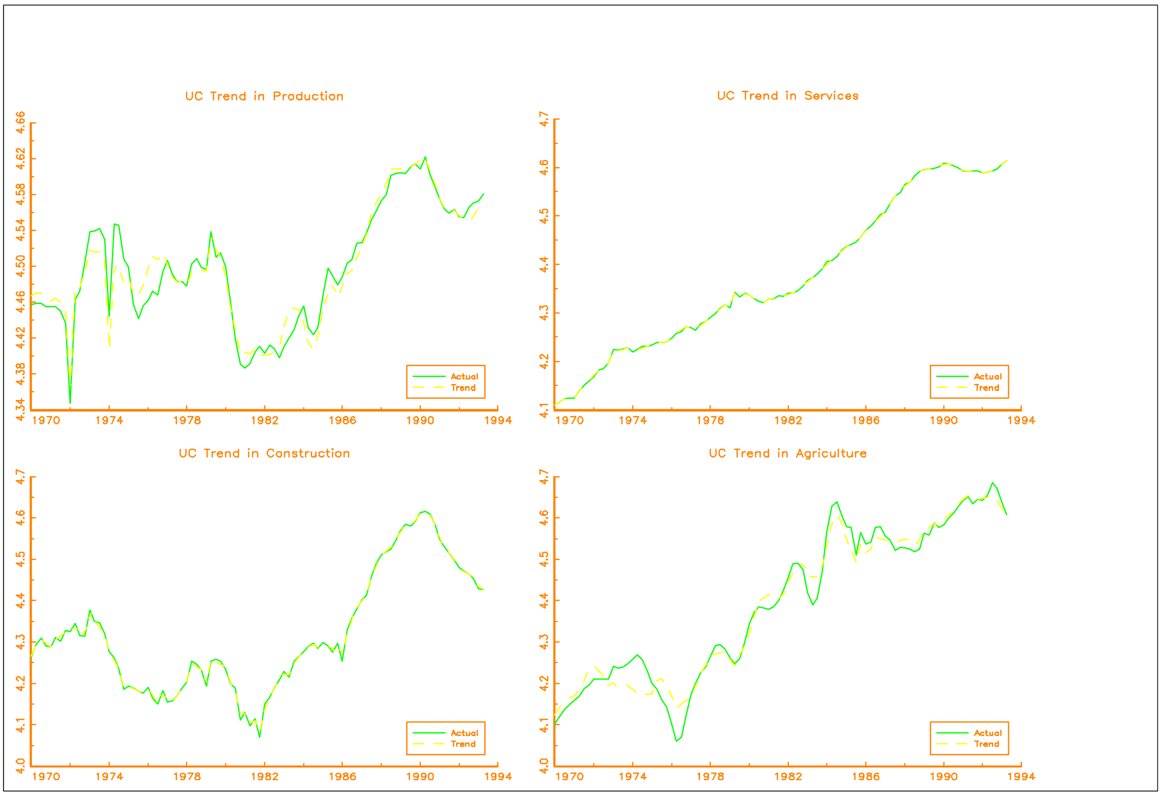


Figure 3: Sectoral Cycles from the EV and UC Models

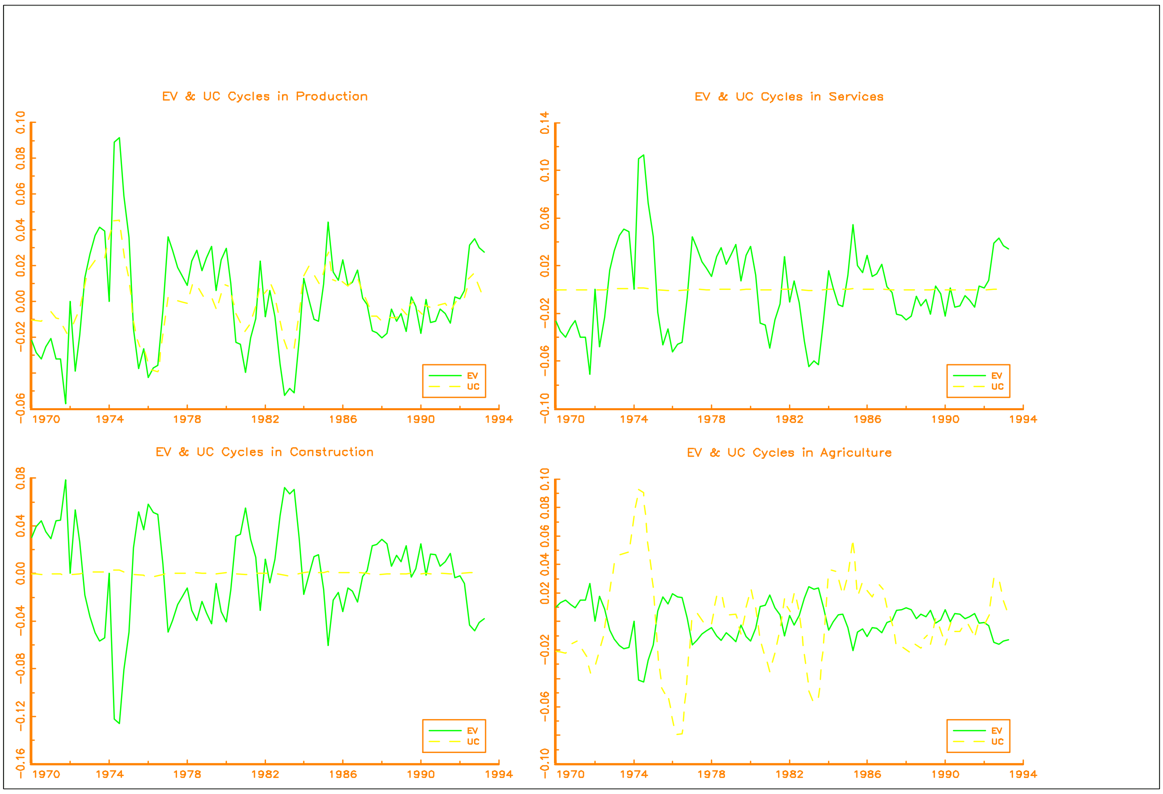
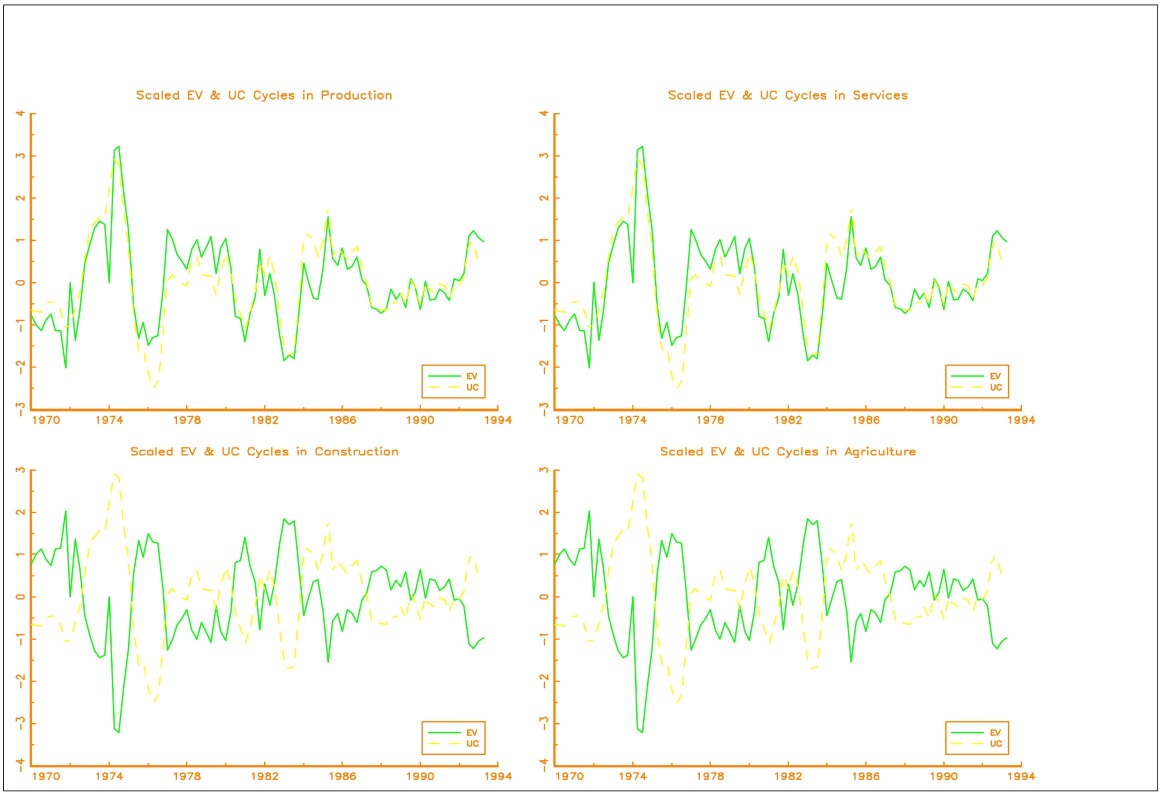


Figure 4: Scaled Sectoral Cycles from the EV and UC Models



References

Artis, M.J., R.C. Bladen-Hovell and W. Zhang (1994), 'Turning Points in the International Business Cycle: An Analysis of the OECD Leading Indicators for the G-7 Countries', *mimeo*.

Beveridge, S. and C.R. Nelson (1981), 'A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle', **Journal of Monetary Economics**, 7, 151-174.

Blanchard, O.J. and D. Quah (1989), 'The dynamics of aggregate demand and supply disturbances', **American Economic Review**, 79, 655-673.

Calcagnini, G. (1995), 'Common trends and common cycles in international labour productivity', **Economic Letters**, 48, 179-184.

Campbell, J.Y. and N.G. Mankiw (1987), 'Are output fluctuations transitory? ', **Quarterly Journal of Economics**, 102, 857-880.

Clark, P.K. (1987), 'The cyclical component of U.S. economic activity', **Quarterly Journal of Economics**, 102, 797-814.

Clark, P.K. (1989), 'Trend reversion in real output and unemployment', **Journal of Econometrics**, 40, 15-32.

Engle, R.F. and C.W.J. Granger (1987), 'Cointegration and error correction: representation, estimation and testing', **Econometrica**, 55, 251-276.

Engle, R.F. and J.V. Issler (1995), 'Estimating common sectoral cycles', **Journal of Monetary Economics**, 35, 83-113.

Engle, R.F. and S. Kozicki (1993), 'Testing for Common Features', **Journal of Business and Economic Statistics**, 11, no.4, 369-379.

Evans, G.W. (1989), 'Output and unemployment dynamics in the Unites States, 1950-1985', **Journal of Applied Econometrics**, 4, 213-237.

Evans, G.W. and L. Reichlin (1994), 'Information, forecasts, and measurement of the business cycle', **Journal of Monetary Economics**, 33, 233-254.

Harvey, A.C. (1985), 'Trends and cycles in macroeconomic time series', **Journal of Business and Economic Statistics**, 3, 216-227.

Harvey, A.C. (1989), **Forecasting, Structural Time Series Models and the Kalman Filter**, Cambridge University Press, Cambridge, UK.

Johansen, S. (1988): 'Statistical analysis of cointegrating vectors' **Journal of Economic Dynamics and Control**, 12, 231-44.

King, R.G., C.I. Plosser, J.H. Stock and M.W. Watson (1991), 'Stochastic trends and economic fluctuations', **American Economic Review**, 81, 819-840.

Koopman, S., Harvey, A.C., Doornik, J.A. and N. Shephard (1995), **Stamp 5.0: Structural Time Series Analyser, Modeller and Predictor**, Chapman & Hall, London.

Kydland, F.E. and R.C. Prescott (1988), 'Business cycles: Real facts and a monetary myth', **Federal Reserve Bank of Minneapolis Quarterly Review**, 14, 3-18.

Lee, K.C., M.H. Pesaran and R.G. Pierse (1992), 'Persistence of shocks and its sources in a multisectoral model of U.K. output growth', **Economic Journal**, 102, 342-356.

Long , J.B. and C.I. Plosser (1983), 'Real business cycles', **Journal of Political Economy**, 91, 39-69.

Long , J.B. and C.I. Plosser (1987), 'Sectoral vs aggregate shocks in the business cycle', **American Economic Review Papers and Proceedings**, 77, 2, 333-336.

Nelson, C.R. and C.I. Plosser (1982), 'Trends and random walks in macroeconomic time series: some evidence and implications', **Journal of Monetary Economics**, 10, 139-162.

Pesaran, M.H., R.G. Pierse and K.C. Lee (1993), 'Persistence, cointegration and aggregation: a disaggregated analysis of output fluctuations in the U.S. economy', **Journal of Econometrics**, 56, 57-88.

Quah, D. (1994), 'Measuring Some UK Business Cycles', *mimeo.*

Stock, J.H. and M.W. Watson (1988a), 'Testing for Common Trends', **Journal of the American Statistical Association**, 83, 1097-1107.

Stock, J.H. and M.W. Watson (1988b), 'Variable trends in economic time series', **Journal of Economic Perspectives**, 2, 147-174.

Vahid, F and R.F. Engle (1993), 'Common trends and common cycles', **Journal of Applied Econometrics**, 8, 341-360.

Watson, M.W. (1986), 'Univariate detrending methods with stochastic trends', **Journal of Monetary Economics**, 18, 49-75.